

EXERCISE 11.2

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In each of the following Exercises 1 to 6, find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum.

1. $y^2 = 12x$

Solution:

Given:

The equation is $y^2 = 12x$ Here we know that the coefficient of x is positive.

So, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we get,

$$4a = 12$$

$$a = 3$$

Thus, the co-ordinates of the focus = $(a, 0) = (3, 0)$ Since, the given equation involves y^2 , the axis of the parabola is the x -axis. \therefore The equation of directrix, $x = -a$, then,

$$x + 3 = 0$$

$$\text{Length of latus rectum} = 4a = 4 \times 3 = 12$$

2. $x^2 = 6y$

Solution:

Given:

The equation is $x^2 = 6y$ Here we know that the coefficient of y is positive.

So, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we get,

$$4a = 6$$

$$a = 6/4$$

$$= 3/2$$

Thus, the co-ordinates of the focus = $(0, a) = (0, 3/2)$ Since, the given equation involves x^2 , the axis of the parabola is the y -axis. \therefore The equation of directrix, $y = -a$, then,

$$y = -3/2$$

$$\text{Length of latus rectum} = 4a = 4(3/2) = 6$$

3. $y^2 = -8x$

Solution:

Given:

The equation is $y^2 = -8x$

Here we know that the coefficient of x is negative.

So, the parabola open towards the left.

On comparing this equation with $y^2 = -4ax$, we get,

$$-4a = -8$$

$$a = -8/-4 = 2$$

Thus, co-ordinates of the focus = $(-a,0) = (-2, 0)$

Since, the given equation involves y^2 , the axis of the parabola is the x -axis.

∴ Equation of directrix, $x = a$, then,

$$y = 2$$

$$\text{Length of latus rectum} = 4a = 4(2) = 8$$

4. $x^2 = -16y$

Solution:

Given:

The equation is $x^2 = -16y$

Here we know that the coefficient of y is negative.

So, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we get,

$$-4a = -16$$

$$a = -16/-4$$

$$= 4$$

Thus, co-ordinates of the focus = $(0,-a) = (0,-4)$

Since, the given equation involves x^2 , the axis of the parabola is the y -axis.

∴ The equation of directrix, $x = a$, then,

$$y = 4$$

$$\text{Length of latus rectum} = 4a = 4(4) = 16$$

5. $y^2 = 10x$

Solution:

Given:

The equation is $y^2 = 10x$

Here we know that the coefficient of x is positive.

So, the parabola open towards the right.

On comparing this equation with $y^2 = -4ax$, we get,

$$4a = 10$$

$$a = 10/4 = 5/2$$

Thus, co-ordinates of the focus = $(a,0) = (5/2, 0)$

Since, the given equation involves y^2 , the axis of the parabola is the x-axis.

∴ The equation of directrix, $x = -a$, then,

$$x = -5/2$$

$$\text{Length of latus rectum} = 4a = 4(5/2) = 10$$

6. $x^2 = -9y$

Solution:

Given:

The equation is $x^2 = -9y$

Here we know that the coefficient of y is negative.

So, the parabola open downwards.

On comparing this equation with $x^2 = -4ay$, we get,

$$-4a = -9$$

$$a = -9/-4 = 9/4$$

Thus, co-ordinates of the focus = $(0, -a) = (0, 9/4)$

Since, the given equation involves x^2 , the axis of the parabola is the y-axis.

∴ The equation of directrix, $y = a$, then,

$$y = 9/4$$

$$\text{Length of latus rectum} = 4a = 4(9/4) = 9$$

In each of the Exercises 7 to 12, find the equation of the parabola that satisfies the given conditions:

7. Focus $(6,0)$; directrix $x = -6$

Solution:

Given:

Focus $(6,0)$ and directrix $x = -6$

We know that the focus lies on the x-axis is the axis of the parabola.

So, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

It is also seen that the directrix, $x = -6$ is to the left of the y-axis,

While the focus $(6, 0)$ is to the right of the y-axis.

Hence, the parabola is of the form $y^2 = 4ax$.

Here, $a = 6$

∴ The equation of the parabola is $y^2 = 24x$.

8. Focus $(0,-3)$; directrix $y = 3$

Solution:

Given:

Focus $(0, -3)$ and directrix $y = 3$

We know that the focus lies on the y -axis, the y -axis is the axis of the parabola.

So, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

It is also seen that the directrix, $y = 3$ is above the x -axis,

While the focus $(0, -3)$ is below the x -axis.

Hence, the parabola is of the form $x^2 = -4ay$.

Here, $a = 3$

\therefore The equation of the parabola is $x^2 = -12y$.

9. Vertex $(0, 0)$; focus $(3, 0)$

Solution:

Given:

Vertex $(0, 0)$ and focus $(3, 0)$

We know that the vertex of the parabola is $(0, 0)$ and the focus lies on the positive x -axis.

[x -axis is the axis of the parabola.]

The equation of the parabola is of the form $y^2 = 4ax$.

Since, the focus is $(3, 0)$, $a = 3$

\therefore The equation of the parabola is $y^2 = 4 \times 3 \times x$,
 $y^2 = 12x$

10. Vertex $(0, 0)$; focus $(-2, 0)$

Solution:

Given:

Vertex $(0, 0)$ and focus $(-2, 0)$

We know that the vertex of the parabola is $(0, 0)$ and the focus lies on the positive x -axis.

[x -axis is the axis of the parabola.]

The equation of the parabola is of the form $y^2 = -4ax$.

Since, the focus is $(-2, 0)$, $a = 2$

\therefore The equation of the parabola is $y^2 = -4 \times 2 \times x$,
 $y^2 = -8x$

11. Vertex $(0, 0)$ passing through $(2, 3)$ and axis is along x -axis.

Solution:

We know that the vertex is $(0, 0)$ and the axis of the parabola is the x -axis

The equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

Given that the parabola passes through point $(2, 3)$, which lies in the first quadrant.

So, the equation of the parabola is of the form $y^2 = 4ax$, while point $(2, 3)$ must satisfy the equation $y^2 = 4ax$.

Then,

$$3^2 = 4a(2)$$

$$3^2 = 8a$$

$$9 = 8a$$

$$a = 9/8$$

Thus, the equation of the parabola is

$$y^2 = 4 (9/8)x$$
$$= 9x/2$$

$$2y^2 = 9x$$

∴ The equation of the parabola is $2y^2 = 9x$

12. Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis.

Solution:

We know that the vertex is (0, 0) and the parabola is symmetric about the y-axis.

The equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

Given that the parabola passes through point (5, 2), which lies in the first quadrant.

So, the equation of the parabola is of the form $x^2 = 4ay$, while point (5, 2) must satisfy the equation $x^2 = 4ay$.

Then,

$$5^2 = 4a(2)$$

$$25 = 8a$$

$$a = 25/8$$

Thus, the equation of the parabola is

$$x^2 = 4 (25/8)y$$

$$x^2 = 25y/2$$

$$2x^2 = 25y$$

∴ The equation of the parabola is $2x^2 = 25y$