EXERCISE 11.3 PAGE NO: 255

In each of the Exercises 1 to 9, find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse.

1. $x^2/36 + y^2/16 = 1$

Solution:

Given:

The equation is $x^2/36 + y^2/16 = 1$

Here, the denominator of $x^2/36$ is greater than the denominator of $y^2/16$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

a = 6 and b = 4.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(36-16)}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Then,

The coordinates of the foci are $(2\sqrt{5}, 0)$ and (-6, 0).

The coordinates of the vertices are (6, 0) and (-6, 0)

Length of major axis = 2a = 2 (6) = 12

Length of minor axis = 2b = 2 (4) = 8

Eccentricity, $e^{c/a} = 2\sqrt{5}/6 = \sqrt{5}/3$

Length of latus rectum = $2b^2/a = (2\times16)/6 = 16/3$

2. $x^2/4 + y^2/25 = 1$

Solution:

Given:

The equation is $x^2/4 + y^2/25 = 1$

Here, the denominator of $y^2/25$ is greater than the denominator of $x^2/4$.

So, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get

a = 5 and b = 2.

$c = \sqrt{(a^2 + b^2)}$

$= \sqrt{(25-4)}$

$= \sqrt{21}$

Then,

The coordinates of the foci are $(0, \sqrt{21})$ and $(0, -\sqrt{21})$.

The coordinates of the vertices are (0, 5) and (0, -5)
Length of major axis = 2a = 2 (5) = 10
Length of minor axis = 2b = 2 (2) = 4
Eccentricity, $e^{c/a} = \sqrt{21}/5$
Length of latus rectum = $2b^2/a = (2\times2^2)/5 = (2\times4)/5 = 8/5$

3. $x^2/16 + y^2/9 = 1$
Solution:
Given:
The equation is $x^2/16 + y^2/9 = 1$ or $x^2/4^2 + y^2/3^2 = 1$
Here, the denominator of $x^2/16$ is greater than the denominator of $y^2/9$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
a = 4 and b = 3.
$c = \sqrt{(a^2 + b^2)}$
$= \sqrt{(16-9)}$
$= \sqrt{7}$
Then,
The coordinates of the foci are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$.
The coordinates of the vertices are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$
Length of major axis = 2a = 2 (4) = 8
Length of minor axis = 2b = 2 (3) = 6
Eccentricity, $e^{c/a} = \sqrt{7}/4$
Length of latus rectum = $2b^2/a = (2\times3^2)/4 = (2\times9)/4 = 18/4 = 9/2$

4. $x^2/25 + y^2/100 = 1$
Solution:
Given:
The equation is $x^2/25 + y^2/100 = 1$
Here, the denominator of $y^2/100$ is greater than the denominator of $x^2/25$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
b = 5 and a =10.
$c = \sqrt{(a^2 + b^2)}$
$= \sqrt{(100-25)}$
$= \sqrt{75}$
$= 5\sqrt{3}$
Then,
The coordinates of the foci are $(0, 5\sqrt{3})$ and $(0, -5\sqrt{3})$.

The coordinates of the vertices are (0, √10) and (0, -√10)
Length of major axis = 2a = 2 (10) = 20
Length of minor axis = 2b = 2 (5) = 10
Eccentricity, $e^{c/a}$ = 5√3/10 = √3/2
Length of latus rectum = $2b^2/a$ = $(2\times5^2)/10$ = (2×25)/10 = 5

5. $x^2/49 + y^2/36 = 1$
Solution:
Given:
The equation is $x^2/49 + y^2/36 = 1$
Here, the denominator of $x^2/49$ is greater than the denominator of $y^2/36$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
b = 6 and a =7
c = $\sqrt{(a^2 + b^2)}$
= √(49-36)
= √13
Then,
The coordinates of the foci are (√13, 0) and (-√3, 0).
The coordinates of the vertices are (7, 0) and (-7, 0)
Length of major axis = 2a = 2 (7) = 14
Length of minor axis = 2b = 2 (6) = 12
Eccentricity, $e^{c/a}$ = √13/7
Length of latus rectum = $2b^2/a$ = $(2\times6^2)/7$ = (2×36)/7 = 72/7

6. $x^2/100 + y^2/400 = 1$
Solution:
Given:
The equation is $x^2/100 + y^2/400 = 1$
Here, the denominator of $y^2/400$ is greater than the denominator of $x^2/100$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
b = 10 and a =20.
c = $\sqrt{(a^2 + b^2)}$
= √(400-100)
= √300
= 10√3
Then,
The coordinates of the foci are (0, 10√3) and (0, -10√3).

The coordinates of the vertices are (0, 20) and (0, -20)
Length of major axis = 2a = 2 (20) = 40
Length of minor axis = 2b = 2 (10) = 20
Eccentricity, $e^{c/a} = 10\sqrt{3}/20 = \sqrt{3}/2$
Length of latus rectum = $2b^2/a = (2\times10^2)/20 = (2\times100)/20 = 10$

7. $36x^2 + 4y^2 = 144$
Solution:
Given:
The equation is $36x^2 + 4y^2 = 144$ or $x^2/4 + y^2/36 = 1$ or $x^2/2^2 + y^2/6^2 = 1$
Here, the denominator of $y^2/6^2$ is greater than the denominator of $x^2/2^2$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
b = 2 and a = 6.
$c = \sqrt{(a^2 + b^2)}$
$= \sqrt{(36-4)}$
$= \sqrt{32}$
$= 4\sqrt{2}$
Then,
The coordinates of the foci are $(0, 4\sqrt{2})$ and $(0, -4\sqrt{2})$.
The coordinates of the vertices are (0, 6) and (0, -6)
Length of major axis = 2a = 2 (6) = 12
Length of minor axis = 2b = 2 (2) = 4
Eccentricity, $e^{c/a} = 4\sqrt{2}/6 = 2\sqrt{2}/3$
Length of latus rectum = $2b^2/a = (2\times2^2)/6 = (2\times4)/6 = 4/3$

8. $16x^2 + y^2 = 16$
Solution:
Given:
The equation is $16x^2 + y^2 = 16$ or $x^2/1 + y^2/16 = 1$ or $x^2/1^2 + y^2/4^2 = 1$
Here, the denominator of $y^2/4^2$ is greater than the denominator of $x^2/1^2$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
b =1 and a =4.
$c = \sqrt{(a^2 + b^2)}$
$= \sqrt{(16-1)}$
$= \sqrt{15}$
Then,
The coordinates of the foci are $(0, \sqrt{15})$ and $(0, -\sqrt{15})$.

The coordinates of the vertices are (0, 4) and (0, -4)
Length of major axis = 2a = 2 (4) = 8
Length of minor axis = 2b = 2 (1) = 2
Eccentricity, $e^{c/a}$ = √15/4
Length of latus rectum = $2b^2/a = (2\times1^2)/4 = 2/4 = ½$

9. $4x^2 + 9y^2 = 36$
Solution:
Given:
The equation is $4x^2 + 9y^2 = 36$ or $x^2/9 + y^2/4 = 1$ or $x^2/3^2 + y^2/2^2 = 1$
Here, the denominator of $x^2/3^2$ is greater than the denominator of $y^2/2^2$.
So, the major axis is along the x-axis, while the minor axis is along the y-axis.
On comparing the given equation with $x^2/a^2 + y^2/b^2 = 1$, we get
a =3 and b =2.
$c = \sqrt{(a^2 + b^2)}$
$= \sqrt{(9-4)}$
$= \sqrt{5}$
Then,
The coordinates of the foci are (√5, 0) and (-√5, 0).
The coordinates of the vertices are (3, 0) and (-3, 0)
Length of major axis = 2a = 2 (3) = 6
Length of minor axis = 2b = 2 (2) = 4
Eccentricity, $e^{c/a}$ = √5/3
Length of latus rectum = $2b^2/a = (2\times2^2)/3 = (2\times4)/3 = 8/3$

In each of the following Exercises 10 to 20, find the equation for the ellipse that satisfies the given conditions:

10. Vertices (± 5, 0), foci (± 4, 0)
Solution:
Given:
Vertices (± 5, 0) and foci (± 4, 0)
Here, the vertices are on the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, a = 5 and c = 4.
It is known that $a^2 = b^2 + c^2$.
So, $5^2 = b^2 + 4^2$
$25 = b^2 + 16$

$b^2 = 25 - 16$
$b = \sqrt{9}$
$= 3$
∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2 = 1$ or $x^2/25 + y^2/9 = 1$

11. Vertices (0, ± 13), foci (0, ± 5)
Solution:
Given:
Vertices (0, ± 13) and foci (0, ± 5)
Here, the vertices are on the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, a =13 and c = 5.
It is known that $a^2 = b^2 + c^2$.
$13^2 = b^2+5^2$
$169 = b^2 + 15$
$b^2 = 169 - 125$
$b = \sqrt{144}$
$= 12$
∴ The equation of the ellipse is $x^2/12^2 + y^2/13^2 = 1$ or $x^2/144 + y^2/169 = 1$

12. Vertices (± 6, 0), foci (± 4, 0)
Solution:
Given:
Vertices (± 6, 0) and foci (± 4, 0)
Here, the vertices are on the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, a = 6 and c = 4.
It is known that $a^2 = b^2 + c^2$.
$6^2 = b^2+4^2$
$36 = b^2 + 16$
$b^2 = 36 - 16$
$b = \sqrt{20}$
∴ The equation of the ellipse is $x^2/6^2 + y^2/(\sqrt{20})^2 = 1$ or $x^2/36 + y^2/20 = 1$

13. Ends of major axis (± 3, 0), ends of minor axis (0, ±2)
Solution:
Given:

Ends of major axis (± 3, 0) and ends of minor axis (0, ±2)
Here, the major axis is along the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, a = 3 and b = 2.
∴ The equation for the ellipse $x^2/3^2 + y^2/2^2 = 1$ or $x^2/9 + y^2/4 = 1$

14. Ends of major axis (0, ±√5), ends of minor axis (±1, 0)
Solution:
Given:
Ends of major axis (0, ±√5) and ends of minor axis (±1, 0)
Here, the major axis is along the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, a = √5 and b = 1.
∴ The equation for the ellipse $x^2/1^2 + y^2/(\sqrt{5})^2 = 1$ or $x^2/1 + y^2/5 = 1$

15. Length of major axis 26, foci (±5, 0)
Solution:
Given:
Length of major axis is 26 and foci (±5, 0)
Since the foci are on the x-axis, the major axis is along the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, 2a = 26
a = 13 and c = 5.
It is known that $a^2 = b^2 + c^2$.
$13^2 = b^2+4^2$
$169 = b^2 + 25$
$b^2 = 169 – 25$
$b = \sqrt{144}$
$= 12$
∴ The equation of the ellipse is $x^2/13^2 + y^2/12^2 = 1$ or $x^2/169 + y^2/144 = 1$

16. Length of minor axis 16, foci (0, ±6).
Solution:
Given:
Length of minor axis is 16 and foci (0, ±6).
Since the foci are on the x-axis, the major axis is along the x-axis.

So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, $2b = 16$
$b = 8$ and $c = 6$.
It is known that $a^2 = b^2 + c^2$.
$a^2 = 8^2 + 6^2$
$= 64 + 36$
$= 100$
$b = \sqrt{100}$
$= 10$
∴ The equation of the ellipse is $x^2/8^2 + y^2/10^2 = 1$ or $x^2/64 + y^2/100 = 1$

17. Foci (±3, 0), a = 4
Solution:
Given:
Foci (±3, 0) and a = 4
Since the foci are on the x-axis, the major axis is along the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, c = 3 and a = 4.
It is known that $a^2 = b^2 + c^2$.
$a^2 = 8^2 + 6^2$
$= 64 + 36$
$= 100$
$16 = b^2 + 9$
$b^2 = 16 – 9$
$= 7$
∴ The equation of the ellipse is $x^2/16 + y^2/7 = 1$

18. b = 3, c = 4, centre at the origin; foci on the x axis.
Solution:
Given:
b = 3, c = 4, centre at the origin and foci on the x axis.
Since the foci are on the x-axis, the major axis is along the x-axis.
So, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
Then, b = 3 and c = 4.
It is known that $a^2 = b^2 + c^2$.
$a^2 = 3^2 + 4^2$

$= 9 + 16$
$=25$
$a = \sqrt{25}$
$= 5$
∴ The equation of the ellipse is $x^2/5^2 + y^2/3^2$ or $x^2/25 + y^2/9 = 1$

19. Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).
Solution:
Given:
Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).
Since the centre is at (0, 0) and the major axis is on the y- axis, the equation of the ellipse will be of the form $x^2/a^2 + y^2/b^2 = 1$, where 'a' is the semi-major axis.
The ellipse passes through points (3, 2) and (1, 6).
So, by putting the values $x = 3$ and $y = 2$, we get,
$3^2/b^2 + 2^2/a^2 = 1$
$9/b^2 + 4/a^2$…. (1)
And by putting the values $x = 1$ and $y = 6$, we get,
$1^1/b^2 + 6^2/a^2 = 1$
$1/b^2 + 36/a^2 = 1$ …. (2)
On solving equation (1) and (2), we get
$b^2 = 10$ and $a^2 = 40$.
∴ The equation of the ellipse is $x^2/10 + y^2/40 = 1$ or $4x^2 + y^2 = 40$

20. Major axis on the x-axis and passes through the points (4,3) and (6,2).
Solution:
Given:
Major axis on the x-axis and passes through the points (4, 3) and (6, 2).
Since the major axis is on the x-axis, the equation of the ellipse will be the form
$x^2/a^2 + y^2/b^2 = 1$…. (1) [Where 'a' is the semi-major axis.]

The ellipse passes through points (4, 3) and (6, 2).
So by putting the values $x = 4$ and $y = 3$ in equation (1), we get,
$16/a^2 + 9/b^2 = 1$ …. (2)
Putting, $x = 6$ and $y = 2$ in equation (1), we get,
$36/a^2 + 4/b^2 = 1$ …. (3)
From equation (2)
$16/a^2 = 1 – 9/b^2$
$1/a^2 = (1/16 (1 – 9/b^2))$ …. (4)

Substituting the value of $1/a^2$ in equation (3) we get,
$36/a^2 + 4/b^2 = 1$
$36(1/a^2) + 4/b^2 = 1$
$36[1/16 (1 – 9/b^2)] + 4/b^2 = 1$
$36/16 (1 – 9/b^2) + 4/b^2 = 1$
$9/4 (1 – 9/b^2) + 4/b^2 = 1$
$9/4 – 81/4b^2 + 4/b^2 = 1$
$-81/4b^2 + 4/b^2 = 1 – 9/4$
$(-81+16)/4b^2 = (4-9)/4$
$-65/4b^2 = -5/4$
$-5/4(13/b^2) = -5/4$
$13/b^2 = 1$
$1/b^2 = 1/13$
$b^2 = 13$
Now substitute the value of b^2 in equation (4) we get,
$1/a^2 = 1/16(1 – 9/b^2)$
$= 1/16(1 – 9/13)$
$= 1/16((13-9)/13)$
$= 1/16(4/13)$
$= 1/52$
$a^2 = 52$
Equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$
By substituting the values of a^2 and b^2 in above equation we get,
$x^2/52 + y^2/13 = 1$