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1. Find the distance between the following pairs of points:
(i) (2, 3, 5) and (4, 3, 1)
(ii) (–3, 7, 2) and (2, 4, –1)
(iii) (–1, 3, – 4) and (1, –3, 4)
(iv) (2, –1, 3) and (–2, 1, 3)
Solution:
(i) (2, 3, 5) and (4, 3, 1)
Let P be (2, 3, 5) and Q be (4, 3, 1)
By using the formula,
Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = 2, y_1 = 3, z_1 = 5$
$x_2 = 4, y_2 = 3, z_2 = 1$
Distance PQ = $\sqrt{[(4 - 2)^2 + (3 - 3)^2 + (1 - 5)^2]}$
$= \sqrt{[(2)^2 + 0^2 + (-4)^2]}$
$= \sqrt{[4 + 0 + 16]}$
$= \sqrt{20}$
$= 2\sqrt{5}$
∴ The required distance is $2\sqrt{5}$ units.

(ii) (–3, 7, 2) and (2, 4, –1)
Let P be (– 3, 7, 2) and Q be (2, 4, – 1)
By using the formula,
Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = -3, y_1 = 7, z_1 = 2$
$x_2 = 2, y_2 = 4, z_2 = -1$
Distance PQ = $\sqrt{[(2 - (-3))^2 + (4 - 7)^2 + (-1 - 2)^2]}$
$= \sqrt{[(5)^2 + (-3)^2 + (-3)^2]}$
$= \sqrt{[25 + 9 + 9]}$
$= \sqrt{43}$
∴ The required distance is $\sqrt{43}$ units.

(iii) (–1, 3, – 4) and (1, –3, 4)
Let P be (– 1, 3, – 4) and Q be (1, – 3, 4)
By using the formula,
Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,
$x_1 = -1, y_1 = 3, z_1 = -4$
$x_2 = 1, y_2 = -3, z_2 = 4$
Distance PQ = $\sqrt{[(1-(-1))^2 + (-3-3)^2 + (4-(-4))^2]}$
$= \sqrt{[(2)^2 + (-6)^2 + (8)^2]}$
$= \sqrt{[4 + 36 + 64]}$
$= \sqrt{104}$
$= 2\sqrt{26}$
∴ The required distance is $2\sqrt{26}$ units.

(iv) (2, –1, 3) and (–2, 1, 3)
Let P be (2, – 1, 3) and Q be (– 2, 1, 3)
By using the formula,
Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = 2, y_1 = -1, z_1 = 3$
$x_2 = -2, y_2 = 1, z_2 = 3$
Distance PQ = $\sqrt{[(-2-2)^2 + (1-(-1))^2 + (3-3)^2]}$
$= \sqrt{[(-4)^2 + (2)^2 + (0)^2]}$
$= \sqrt{[16 + 4 + 0]}$
$= \sqrt{20}$
$= 2\sqrt{5}$
∴ The required distance is $2\sqrt{5}$ units.

2. Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.
Solution:
If three points are collinear, then they lie on a line.
Firstly let us calculate distance between the 3 points
i.e. PQ, QR and PR

Calculating PQ
P ≡ (– 2, 3, 5) and Q ≡ (1, 2, 3)
By using the formula,
Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = -2, y_1 = 3, z_1 = 5$
$x_2 = 1, y_2 = 2, z_2 = 3$
Distance PQ = $\sqrt{[(1-(-2))^2 + (2-3)^2 + (3-5)^2]}$
$= \sqrt{[(3)^2 + (-1)^2 + (-2)^2]}$

$= \sqrt{[9 + 1 + 4]}$
$= \sqrt{14}$

Calculating QR
Q ≡ (1, 2, 3) and R ≡ (7, 0, – 1)
By using the formula,
Distance QR = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$
So here,
$x_1 = 1, y_1 = 2, z_1 = 3$
$x_2 = 7, y_2 = 0, z_2 = – 1$
Distance QR = $\sqrt{[(7 – 1)^2 + (0 – 2)^2 + (-1 – 3)^2]}$
$= \sqrt{[(6)^2 + (-2)^2 + (-4)^2]}$
$= \sqrt{[36 + 4 + 16]}$
$= \sqrt{56}$
$= 2\sqrt{14}$

Calculating PR
P ≡ (– 2, 3, 5) and R ≡ (7, 0, – 1)
By using the formula,
Distance PR = $\sqrt{[(x_2 – x_1)^2 + (y_2 – y_1)^2 + (z_2 – z_1)^2]}$
So here,
$x_1 = – 2, y_1 = 3, z_1 = 5$
$x_2 = 7, y_2 = 0, z_2 = – 1$
Distance PR = $\sqrt{[(7 – (-2))^2 + (0 – 3)^2 + (-1 – 5)^2]}$
$= \sqrt{[(9)^2 + (-3)^2 + (-6)^2]}$
$= \sqrt{[81 + 9 + 36]}$
$= \sqrt{126}$
$= 3\sqrt{14}$

Thus, PQ = $\sqrt{14}$, QR = $2\sqrt{14}$ and PR = $3\sqrt{14}$
So, PQ + QR = $\sqrt{14} + 2\sqrt{14}$
$= 3\sqrt{14}$
= PR
∴ The points P, Q and R are collinear.

3. Verify the following:
(i) (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.
(ii) (0, 7, 10), (–1, 6, 6) and (– 4, 9, 6) are the vertices of a right angled triangle.
(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

Solution:

(i) (0, 7, –10), (1, 6, – 6) and (4, 9, – 6) are the vertices of an isosceles triangle.
Let us consider the points be
P(0, 7, –10), Q(1, 6, – 6) and R(4, 9, – 6)
If any 2 sides are equal, hence it will be an isosceles triangle
So firstly let us calculate the distance of PQ, QR

<u>Calculating PQ</u>
P ≡ (0, 7, – 10) and Q ≡ (1, 6, – 6)
By using the formula,
Distance PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = 0, y_1 = 7, z_1 = -10$
$x_2 = 1, y_2 = 6, z_2 = -6$
Distance PQ = $\sqrt{[(1 - 0)^2 + (6 - 7)^2 + (-6 - (-10))^2]}$
$= \sqrt{[(1)^2 + (-1)^2 + (4)^2]}$
$= \sqrt{[1 + 1 + 16]}$
$= \sqrt{18}$

<u>Calculating QR</u>
Q ≡ (1, 6, – 6) and R ≡ (4, 9, – 6)
By using the formula,
Distance QR = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = 1, y_1 = 6, z_1 = -6$
$x_2 = 4, y_2 = 9, z_2 = -6$
Distance QR = $\sqrt{[(4 - 1)^2 + (9 - 6)^2 + (-6 - (-6))^2]}$
$= \sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$
$= \sqrt{[9 + 9 + 0]}$
$= \sqrt{18}$
Hence, PQ = QR
18 = 18
2 sides are equal
∴ PQR is an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6) and (– 4, 9, 6) are the vertices of a right angled triangle.
Let the points be
P(0, 7, 10), Q(– 1, 6, 6) & R(– 4, 9, 6)
Firstly let us calculate the distance of PQ, OR and PR

<u>Calculating PQ</u>

$P \equiv (0, 7, 10)$ and $Q \equiv (-1, 6, 6)$

By using the formula,

Distance PQ $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = -1, y_2 = 6, z_2 = 6$

Distance PQ $= \sqrt{[(-1 - 0)^2 + (6 - 7)^2 + (6 - 10)^2]}$

$= \sqrt{[(-1)^2 + (-1)^2 + (-4)^2]}$

$= \sqrt{[1 + 1 + 16]}$

$= \sqrt{18}$

<u>Calculating QR</u>

$Q \equiv (1, 6, -6)$ and $R \equiv (4, 9, -6)$

By using the formula,

Distance QR $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 1, y_1 = 6, z_1 = -6$

$x_2 = 4, y_2 = 9, z_2 = -6$

Distance QR $= \sqrt{[(4 - 1)^2 + (9 - 6)^2 + (-6 - (-6))^2]}$

$= \sqrt{[(3)^2 + (3)^2 + (-6+6)^2]}$

$= \sqrt{[9 + 9 + 0]}$

$= \sqrt{18}$

<u>Calculating PR</u>

$P \equiv (0, 7, 10)$ and $R \equiv (-4, 9, 6)$

By using the formula,

Distance PR $= \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$

So here,

$x_1 = 0, y_1 = 7, z_1 = 10$

$x_2 = -4, y_2 = 9, z_2 = 6$

Distance PR $= \sqrt{[(-4 - 0)^2 + (9 - 7)^2 + (6 - 10)^2]}$

$= \sqrt{[(-4)^2 + (2)^2 + (-4)^2]}$

$= \sqrt{[16 + 4 + 16]}$

$= \sqrt{36}$

Now,

$PQ^2 + QR^2 = 18 + 18$

$= 36$

$= PR^2$

By using converse of Pythagoras theorem,
∴ The given vertices P, Q & R are the vertices of a right – angled triangle at Q.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.
Let the points be: A(–1, 2, 1), B(1, –2, 5), C(4, –7, 8) & D(2, –3, 4)
ABCD can be vertices of parallelogram only if opposite sides are equal.
i.e. AB = CD and BC = AD
Firstly let us calculate the distance

Calculating AB
A ≡ (– 1, 2, 1) and B ≡ (1, – 2, 5)
By using the formula,
Distance AB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = -1, y_1 = 2, z_1 = 1$
$x_2 = 1, y_2 = -2, z_2 = 5$
Distance AB = $\sqrt{[(1 - (-1))^2 + (-2 - 2)^2 + (5 - 1)^2]}$
$= \sqrt{[(2)^2 + (-4)^2 + (4)^2]}$
$= \sqrt{[4 + 16 + 16]}$
$= \sqrt{36}$
$= 6$

Calculating BC
B ≡ (1, – 2, 5) and C ≡ (4, – 7, 8)
By using the formula,
Distance BC = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = 1, y_1 = -2, z_1 = 5$
$x_2 = 4, y_2 = -7, z_2 = 8$
Distance BC = $\sqrt{[(4 - 1)^2 + (-7 - (-2))^2 + (8 - 5)^2]}$
$= \sqrt{[(3)^2 + (-5)^2 + (3)^2]}$
$= \sqrt{[9 + 25 + 9]}$
$= \sqrt{43}$

Calculating CD
C ≡ (4, – 7, 8) and D ≡ (2, – 3, 4)
By using the formula,
Distance CD = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,

$x_1 = 4, y_1 = -7, z_1 = 8$
$x_2 = 2, y_2 = -3, z_2 = 4$
Distance CD = $\sqrt{[(2-4)^2 + (-3-(-7))^2 + (4-8)^2]}$
$= \sqrt{[(-2)^2 + (4)^2 + (-4)^2]}$
$= \sqrt{[4 + 16 + 16]}$
$= \sqrt{36}$
$= 6$

<u>Calculating DA</u>
D ≡ (2, – 3, 4) and A ≡ (– 1, 2, 1)
By using the formula,
Distance DA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = 2, y_1 = -3, z_1 = 4$
$x_2 = -1, y_2 = 2, z_2 = 1$
Distance DA = $\sqrt{[(-1-2)^2 + (2-(-3))^2 + (1-4)^2]}$
$= \sqrt{[(-3)^2 + (5)^2 + (-3)^2]}$
$= \sqrt{[9 + 25 + 9]}$
$= \sqrt{43}$

Since AB = CD and BC = DA (given)
So, In ABCD both pairs of opposite sides are equal.
∴ ABCD is a parallelogram.

4. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).
Solution:
Let A (1, 2, 3) & B (3, 2, – 1)
Let point P be (x, y, z)
Since it is given that point P(x, y, z) is equal distance from point A(1, 2, 3) & B(3, 2, – 1)
i.e. PA = PB
Firstly let us calculate

<u>Calculating PA</u>
P ≡ (x, y, z) and A ≡ (1, 2, 3)
By using the formula,
Distance PA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = x, y_1 = y, z_1 = z$

$x_2 = 1, y_2 = 2, z_2 = 3$
Distance PA = $\sqrt{[(1 - x)^2 + (2 - y)^2 + (3 - z)^2]}$

<u>Calculating PB</u>
P ≡ (x, y, z) and B ≡ (3, 2, – 1)
By using the formula,
Distance PB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = x, y_1 = y, z_1 = z$
$x_2 = 3, y_2 = 2, z_2 = -1$
Distance PB = $\sqrt{[(3 - x)^2 + (2 - y)^2 + (-1 - z)^2]}$
Since PA = PB
Square on both the sides, we get
$PA^2 = PB^2$
$(1 - x)^2 + (2 - y)^2 + (3 - z)^2 = (3 - x)^2 + (2 - y)^2 + (-1 - z)^2$
$(1 + x^2 - 2x) + (4 + y^2 - 4y) + (9 + z^2 - 6z)$
$(9 + x^2 - 6x) + (4 + y^2 - 4y) + (1 + z^2 + 2z)$
$-2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$
$4x - 8z = 0$
$x - 2z = 0$
∴ The required equation is $x - 2z = 0$

5. Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (– 4, 0, 0) is equal to 10.
Solution:
Let A (4, 0, 0) & B (– 4, 0, 0)
Let the coordinates of point P be (x, y, z)

<u>Calculating PA</u>
P ≡ (x, y, z) and A ≡ (4, 0, 0)
By using the formula,
Distance PA = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = x, y_1 = y, z_1 = z$
$x_2 = 4, y_2 = 0, z_2 = 0$
Distance PA = $\sqrt{[(4 - x)^2 + (0 - y)^2 + (0 - z)^2]}$

<u>Calculating PB</u>
P ≡ (x, y, z) and B ≡ (– 4, 0, 0)

By using the formula,
Distance PB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$
So here,
$x_1 = x, y_1 = y, z_1 = z$
$x_2 = -4, y_2 = 0, z_2 = 0$
Distance PB = $\sqrt{[(-4 - x)^2 + (0 - y)^2 + (0 - z)^2]}$
Now it is given that:
PA + PB = 10
PA = 10 – PB
Square on both the sides, we get
$PA^2 = (10 - PB)^2$
$PA^2 = 100 + PB^2 - 20\ PB$
$(4 - x)^2 + (0 - y)^2 + (0 - z)^2$
$100 + (-4 - x)^2 + (0 - y)^2 + (0 - z)^2 - 20\ PB$
$(16 + x^2 - 8x) + (y^2) + (z^2)$
$100 + (16 + x^2 + 8x) + (y^2) + (z^2) - 20\ PB$
$20\ PB = 16x + 100$
$5\ PB = (4x + 25)$

Square on both the sides again, we get
$25\ PB^2 = 16x^2 + 200x + 625$
$25\ [(-4 - x)^2 + (0 - y)^2 + (0 - z)^2] = 16x^2 + 200x + 625$
$25\ [x^2 + y^2 + z^2 + 8x + 16] = 16x^2 + 200x + 625$
$25x^2 + 25y^2 + 25z^2 + 200x + 400 = 16x^2 + 200x + 625$
$9x^2 + 25y^2 + 25z^2 - 225 = 0$
∴ The required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$