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1. Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio (i) 2: 3 internally, (ii) 2: 3 externally.

Solution:

Let the line segment joining the points P (-2, 3, 5) and Q (1, -4, 6) be PQ.

(i) 2: 3 internally

By using section formula,

We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n is given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

Upon comparing we have

$x_1 = -2$, $y_1 = 3$, $z_1 = 5$;

$x_2 = 1$, $y_2 = -4$, $z_2 = 6$ and

m = 2, n = 3

So, the coordinates of the point which divides the line segment joining the points P (– 2, 3, 5) and Q (1, – 4, 6) in the ratio 2 : 3 internally is given by:

$$\left(\frac{2\times 1 + 3\times(-2)}{2+3}, \frac{2\times(-4) + 3\times 3}{2+3}, \frac{2\times 6 + 3\times 5}{2+3}\right)$$

$$= \left(\frac{2-6}{5}, \frac{-8+9}{5}, \frac{12+15}{5}\right)$$

$$= \left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$$

Hence, the coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) is (-4/5, 1/5, 27/5)

(ii) 2: 3 externally

By using section formula,

We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio m: n is given by:

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Upon comparing we have

$x_1 = -2$, $y_1 = 3$, $z_1 = 5$;

$x_2 = 1$, $y_2 = -4$, $z_2 = 6$ and

m = 2, n = 3

So, the coordinates of the point which divides the line segment joining the points P (– 2, 3, 5) and Q (1, – 4, 6) in the ratio 2: 3 externally is given by:

$$\left(\frac{2\times1-3\times(-2)}{2-3},\frac{2\times(-4)-3\times3}{2-3},\frac{2\times6-3\times5}{2-3}\right)$$

$$=\left(\frac{2-(-6)}{-1},\frac{-8-9}{-1},\frac{12-15}{-1}\right)$$

$$=\left(\frac{8}{-1},\frac{-17}{-1},\frac{-3}{-1}\right)$$

= (-8, 17, 3)

∴ The co-ordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) is (-8, 17, 3).

2. Given that P (3, 2, – 4), Q (5, 4, – 6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.

Solution:

Let us consider Q divides PR in the ratio k: 1.

By using section formula,

We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n is given by:

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$$

Upon comparing we have,

x_1 = 3, y_1 = 2, z_1 = -4;

x_2 = 9, y_2 = 8, z_2 = -10 and

m = k, n = 1

So, we have

$$\left(\frac{9k+3}{k+1},\frac{8k+2}{k+1},\frac{-10k-4}{k+1}\right)=(5,4,-6)$$

$$\frac{9k+3}{k+1}=5,\frac{8k+2}{k+1}=4,\frac{-10k-4}{k+1}=-6$$

9k + 3 = 5 (k+1)

9k + 3 = 5k + 5

9k – 5k = 5 - 3

4k = 2

k = 2/4

= ½

Hence, the ratio in which Q divides PR is 1: 2.

3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).

Solution:

Let the line segment formed by joining the points P (-2, 4, 7) and Q (3, -5, 8) be PQ.
We know that any point on the YZ-plane is of the form (0, y, z).
So now, let R (0, y, z) divides the line segment PQ in the ratio k: 1.
Then,
Upon comparing we have,
$x_1 = -2$, $y_1 = 4$, $z_1 = 7$;
$x_2 = 3$, $y_2 = -5$, $z_2 = 8$ and
m = k, n = 1
By using the section formula,
We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n is given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

So we have,

$$\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right) = (0, y, z)$$

$$\frac{3k-2}{k+1} = 0$$

3k – 2 = 0
3k = 2
k = 2/3
Hence, the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8) is 2:3.

4. Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and C (0, 1/3, 2) are collinear.

Solution:

Let the point P divides AB in the ratio k: 1.
Upon comparing we have,
$x_1 = 2$, $y_1 = -3$, $z_1 = 4$;
$x_2 = -1$, $y_2 = 2$, $z_2 = 1$ and
m = k, n = 1
By using section formula,

We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n is given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

So we have,

The coordinates of P = $\left(\frac{-k+2}{k+1}, \frac{2k-3}{k+1}, \frac{k+4}{k+1}\right)$

Now, we check if for some value of k, the point coincides with the point C.

Put (-k+2)/(k+1) = 0

-k + 2 = 0

k = 2

When k = 2, then (2k-3)/(k+1) = (2(2)-3)/(2+1)

= (4-3)/3

= 1/3

And, (k+4)/(k+1) = (2+4)/(2+1)

= 6/3

= 2

∴ C (0, 1/3, 2) is a point which divides AB in the ratio 2: 1 and is same as P.

Hence, A, B, C are collinear.

5. Find the coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6).

Solution:

Let A (x_1, y_1, z_1) and B (x_2, y_2, z_2) trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

A divides the line segment PQ in the ratio 1: 2.

Upon comparing we have,

x_1 = 4, y_1 = 2, z_1 = -6;

x_2 = 10, y_2 = -16, z_2 = 6 and

m = 1, n = 2

By using the section formula,

We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m : n is given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

So we have,

The coordinates of A = $\left(\frac{1\times10+2\times4}{1+2}, \frac{1\times(-16)+2\times2}{1+2}, \frac{1\times6+2\times(-6)}{1+2}\right)$

= (18/3, -12/3, -6/3)

= (6, -4, -2)

Similarly, we know that B divides the line segment PQ in the ratio 2: 1.
Upon comparing we have,
$x_1 = 4$, $y_1 = 2$, $z_1 = -6$;
$x_2 = 10$, $y_2 = -16$, $z_2 = 6$ and
m = 2, n = 1
By using the section formula,
We know that the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio m: n is given by:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

So we have,

The coordinates of B = $\left(\frac{2\times10+1\times4}{2+1}, \frac{2\times(-16)+1\times2}{2+1}, \frac{2\times6+1\times(-6)}{2+1}\right)$
= (24/3, -30/3, 6/3)
= (8, -10, 2)

∴ The coordinates of the points which trisect the line segment joining the points P (4, 2, –6) and Q (10, –16, 6) are (6, -4, -2) and (8, -10, 2).