

MISCELLANEOUS EXERCISE

PAGE NO: 129

Solve the inequalities in Exercises 1 to 6

1. $2 \leq 3x - 4 \leq 5$

Solution:

According to the question,

The inequality given is,

$$2 \leq 3x - 4 \leq 5$$

$$\Rightarrow 2 \leq 3x - 4 \leq 5$$

$$\Rightarrow 2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$$

$$\Rightarrow 6 \leq 3x \leq 9$$

$$\Rightarrow 6/3 \leq 3x/3 \leq 9/3$$

$$\Rightarrow 2 \leq x \leq 3$$

Hence, all real numbers x greater than or equal to 2 but less than or equal to 3 are solution of given equality.

$$x \in [2, 3]$$

2. $6 \leq -3(2x - 4) < 12$

Solution:

According to the question,

The inequality given is,

$$6 \leq -3(2x - 4) < 12$$

$$\Rightarrow 6 \leq -3(2x - 4) < 12$$

Dividing the inequality by 3 we get.

$$\Rightarrow 2 \leq -(2x - 4) < 4$$

Multiplying the inequality by -1.

$$\Rightarrow -2 \geq 2x - 4 > -4 \text{ [multiplying the inequality with -1 changes the inequality sign.]}$$

$$\Rightarrow -2 + 4 \geq 2x - 4 + 4 > -4 + 4$$

$$\Rightarrow 2 \geq 2x > 0$$

Dividing the inequality by 2

$$\Rightarrow 0 < x \leq 1$$

Hence, all real numbers x greater than 0 but less than or equal to 1 are solution of given equality.

$$x \in (0, 1]$$

3. $-3 \leq 4 - 7x/2 \leq 18$

Solution:

According to the question,

The inequality given is,

$$-3 \leq 4 - 7x/2 \leq 18$$

$$\Rightarrow -3 - 4 \leq 4 - 7x/2 - 4 \leq 18 - 4$$

$$\Rightarrow -7 \leq -7x/2 \leq 18 - 14$$

Multiplying the inequality by -2.

$$\Rightarrow (-7) \times (-2) \geq -\frac{7x}{2} \times (-2) \geq 14 \times (-2)$$

$$\Rightarrow 14 \geq 7x \geq -28$$

$$\Rightarrow -28 \leq 7x \leq 14$$

Dividing the inequality by 7

$$\Rightarrow -4 \leq x \leq 2$$

Hence, all real numbers x greater than or equal to -4 but less than or equal to 2 are solution of given equality.

$$x \in [-4, 2]$$

4. $-15 \leq 3(x - 2)/5 \leq 0$

Solution:

According to the question,

The inequality given is,

$$-15 \leq 3(x - 2)/5 \leq 0$$

$$\Rightarrow -15 < 3(x - 2)/5 \leq 0$$

Multiplying the inequality by 5.

$$\Rightarrow -15 \times 5 < \frac{3(x - 2)}{5} \times 5 \leq 0 \times 5$$

$$\Rightarrow -75 < 3(x - 2) \leq 0$$

Dividing the inequality by 3 we get

$$\Rightarrow -\frac{75}{3} < \frac{3(x - 2)}{3} \leq \frac{0}{3}$$

$$\Rightarrow -25 < x - 2 \leq 0$$

$$\Rightarrow -25 + 2 < x - 2 + 2 \leq 0 + 2$$

$$\Rightarrow -23 < x \leq 2$$

Hence, all real numbers x greater than -23 but less than or equal to 2 are solution of

given equality.

$$x \in (-23, 2]$$

5. $-12 < 4 - 3x/(-2) \leq 2$

Solution:

According to the question,

The inequality given is,

$$-12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 - 4 < 4 - \frac{3x}{-5} - 4 \leq 2 - 4$$

$$\Rightarrow -16 < \frac{-3x}{-5} \leq -2$$

$$\Rightarrow -16 < \frac{3x}{5} \leq -2$$

Multiplying the inequality by 5.

$$\Rightarrow -16 \times 5 < \frac{3x}{5} \times 5 \leq -2 \times 5$$

$$\Rightarrow -80 < 3x \leq -10$$

$$\Rightarrow -\frac{80}{3} < x \leq -\frac{10}{3}$$

Hence, all real numbers x greater than $-80/3$ but less than or equal to $-10/3$ are solution of given equality.

$$x \in (-80/3, -10/3]$$

6. $7 \leq (3x + 11)/2 \leq 11$

Solution:

According to the question,

The inequality given is,

$$7 \leq \frac{(3x + 11)}{2} \leq 11$$

$$\Rightarrow 7 \leq \frac{(3x + 11)}{2} \leq 11$$

Multiplying the inequality by 2.

$$\Rightarrow 7 \times 2 \leq \frac{(3x + 11)}{2} \times 2 \leq 11 \times 2$$

$$\Rightarrow 14 \leq 3x + 11 \leq 22$$

$$\Rightarrow 14 - 11 \leq 3x + 11 - 11 \leq 22 - 11$$

$$\Rightarrow 3 \leq 3x \leq 11$$

$$\Rightarrow 1 \leq x \leq 11/3$$

Hence, all real numbers x greater than or equal to -4 but less than or equal to 2 are solution of given equality.

$$x \in [1, 11/3]$$

Solve the inequalities in Exercises 7 to 11 and represent the solution graphically on number line.

7. $5x + 1 > -24$, $5x - 1 < 24$

Solution:

According to the question,

The inequalities given are,

$$5x + 1 > -24 \text{ and } 5x - 1 < 24$$

$$5x + 1 > -24$$

$$\Rightarrow 5x > -24 - 1$$

$$\Rightarrow 5x > -25$$

$$\Rightarrow x > -5 \dots\dots\dots (i)$$

$$5x - 1 < 24$$

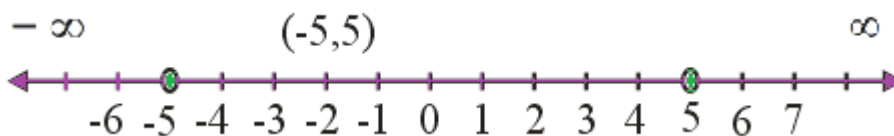
$$\Rightarrow 5x < 24 + 1$$

$$\Rightarrow 5x < 25$$

$$\Rightarrow x < 5 \dots\dots\dots (ii)$$

From equations (i) and (ii),

We can infer that the solution of given inequalities is $(-5, 5)$.



8. $2(x - 1) < x + 5$, $3(x + 2) > 2 - x$

Solution:

According to the question,

The inequalities given are,

$$2(x - 1) < x + 5 \text{ and } 3(x + 2) > 2 - x$$

$$2(x - 1) < x + 5$$

$$\Rightarrow 2x - 2 < x + 5$$

$$\Rightarrow 2x - x < 5 + 2$$

$$\Rightarrow x < 7 \dots\dots\dots (i)$$

$$3(x + 2) > 2 - x$$

$$\Rightarrow 3x + 6 > 2 - x$$

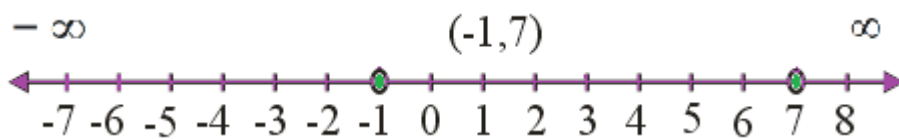
$$\Rightarrow 3x + x > 2 - 6$$

$$\Rightarrow 4x > -4$$

$$\Rightarrow x > -1 \dots\dots\dots (ii)$$

From equations (i) and (ii),

We can infer that the solution of given inequalities is $(-1, 7)$.



9. $3x - 7 > 2(x - 6)$, $6 - x > 11 - 2x$

Solution:

According to the question,

The inequalities given are,

$$3x - 7 > 2(x - 6) \text{ and } 6 - x > 11 - 2x$$

$$3x - 7 > 2(x - 6)$$

$$\Rightarrow 3x - 7 > 2x - 12$$

$$\Rightarrow 3x - 2x > 7 - 12$$

$$\Rightarrow x > -5 \dots\dots\dots (i)$$

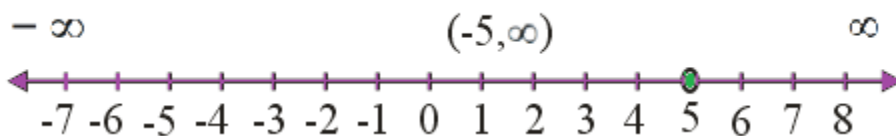
$$6 - x > 11 - 2x$$

$$\Rightarrow 2x - x > 11 - 6$$

$$\Rightarrow x > 5 \dots\dots\dots(ii)$$

From equations (i) and (ii),

We can infer that the solution of given inequalities is $(5, \infty)$.



10. $5(2x - 7) - 3(2x + 3) \leq 0$, $2x + 19 \leq 6x + 47$

Solution:

According to the question,

The inequalities given are,

$$5(2x - 7) - 3(2x + 3) \leq 0 \text{ and } 2x + 19 \leq 6x + 47$$

$$5(2x - 7) - 3(2x + 3) \leq 0$$

$$\Rightarrow 10x - 35 - 6x - 9 \leq 0$$

$$\Rightarrow 4x - 44 \leq 0$$

$$\Rightarrow 4x \leq 44$$

$$\Rightarrow x \leq 11 \text{(i)}$$

$$2x + 19 \leq 6x + 47$$

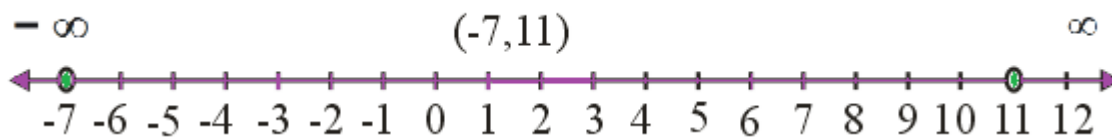
$$\Rightarrow 6x - 2x \geq 19 - 47$$

$$\Rightarrow 4x \geq -28$$

$$\Rightarrow x \geq -7 \text{(ii)}$$

From equations (i) and (ii),

We can infer that the solution of given inequalities is $(-7, 11)$.



11. A solution is to be kept between 68° F and 77° F . What is the range in temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) conversion formula is given by $F = (9/5)C + 32$?

Solution:

According to the question,

The solution has to be kept between 68° F and 77° F

So, we get, $68^\circ < F < 77^\circ$

Substituting,

$$F = \frac{9}{5}C + 32$$

$$\Rightarrow 68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow 68 - 32 < \frac{9}{5}C + 32 - 32 < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < \frac{9}{5}C \times \frac{5}{9} < 45 \times \frac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

Hence, we get,

The range of temperature in degree Celsius is between 20°C to 25°C .

12. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Solution:

According to the question,

8% of solution of boric acid = 640 litres

Let the amount of 2% boric acid solution added = x litres

Then we have,

Total mixture = $x + 640$ litres

We know that,

The resulting mixture has to be more than 4% but less than 6% boric acid.

\therefore 2% of $x + 8\%$ of 640 $>$ 4% of $(x + 640)$ and

2% of $x + 8\%$ of 640 $<$ 6% of $(x + 640)$

2% of $x + 8\%$ of 640 $>$ 4% of $(x + 640)$

$$\Rightarrow (2/100) \times x + (8/100) \times 640 > (4/100) \times (x + 640)$$

$$\Rightarrow 2x + 5120 > 4x + 2560$$

$$\Rightarrow 5120 - 2560 > 4x - 2x$$

$$\Rightarrow 2560 > 2x$$

$$\Rightarrow x < 1280 \dots(i)$$

2% of $x + 8\%$ of 640 $<$ 6% of $(x + 640)$

$$\Rightarrow (2/100) \times x + (8/100) \times 640 < (6/100) \times (x + 640)$$

$$\Rightarrow 2x + 5120 < 6x + 3840$$

$$\Rightarrow 6x - 2x > 5120 - 3840$$

$$\Rightarrow 4x > 1280$$

$$\Rightarrow x > 320 \dots(ii)$$

From (i) and (ii)

$$320 < x < 1280$$

Therefore, the number of litres of 2% of boric acid solution that has to be added will be more than 320 litres but less than 1280 litres.

13. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid

content?

Solution:

According to the question,

45% of solution of acid = 1125 litres

Let the amount of water added = x litres

Resulting mixture = $x + 1125$ litres

We know that,

The resulting mixture has to be more than 25% but less than 30% acid content.

Amount of acid in resulting mixture = 45% of 1125 litres.

\therefore 45% of 1125 < 30% of $(x + 1125)$ and 45% of 1125 > 25% of $(x + 1125)$

45% of 1125 < 30% of $(x + 1125)$

$$\Rightarrow \frac{45}{100} \times 1125 < \frac{30}{100} \times (x + 1125)$$

$$\Rightarrow 45 \times 1125 < 30x + 30 \times 1125$$

$$\Rightarrow (45 - 30) \times 1125 < 30x$$

$$\Rightarrow 15 \times 1125 < 30x$$

$$\Rightarrow x > 562.5 \dots\dots\dots(i)$$

45% of 1125 > 25% of $(x + 1125)$

$$\Rightarrow \frac{45}{100} \times 1125 > \frac{25}{100} \times (x + 1125)$$

$$\Rightarrow 45 \times 1125 > 25x + 25 \times 1125$$

$$\Rightarrow (45 - 25) \times 1125 > 25x$$

$$\Rightarrow 25x < 20 \times 1125$$

$$\Rightarrow x < 900 \dots\dots(ii)$$

$$\therefore 562.5 < x < 900$$

Therefore, the number of litres of water that has to be added will have to be more than 562.5 litres but less than 900 litres.

14. IQ of a person is given by the formula

$$IQ = \frac{MA}{CA} \times 100,$$

Where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 years old children, find the range of their mental age.

Solution:

According to the question,

Chronological age = CA = 12 years

IQ for age group of 12 is $80 \leq IQ \leq 140$.

We get that,

$$80 \leq IQ \leq 140$$

Substituting,

$$IQ = \frac{MA}{CA} \times 100$$

We get,

$$\Rightarrow 80 \leq \frac{MA}{CA} \times 100 \leq 140$$

$$\Rightarrow 80 \leq \frac{MA}{12} \times 100 \leq 140$$

$$\Rightarrow 80 \times \frac{12}{100} \leq \frac{MA}{12} \times 100 \leq 140 \times \frac{12}{100}$$

$$\Rightarrow 9.6 \leq MA \leq 16.8$$

\therefore Range of mental age of the group of 12 years old children is $9.6 \leq MA \leq 16.8$