## EXERCISE 7.3



**1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?**

**Solution:**

Total number of digits possible for choosing = 9

Number of places for which a digit has to be taken = 3

As there is no repetition allowed,

⇒ No. of permutations = ${}^9_3P = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9 \times 8 \times 7 \times 6!}{6!} = 504.$

**2. How many 4-digit numbers are there with no digit repeated?**

**Solution:**

To find four digit number (digits does not repeat)
Now we will have 4 places where 4 digits are to be put.
So, at thousand's place = There are 9 ways as 0 cannot be at thousand's place = 9 ways
At hundredth's place = There are 9 digits to be filled as 1 digit is already taken = 9 ways
At ten's place = There are now 8 digits to be filled as 2 digits are already taken = 8 ways
At unit's place = There are 7 digits that can be filled = 7 ways
Total Number of ways to fill the four places = 9 × 9 × 8 × 7 = 4536 ways.
So a total of 4536 four digit numbers can be there with no digits repeated.

**3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?**

**Solution:**

Even number means that last digit should be even,
Number of possible digits at one's place = 3 (2, 4 and 6)
⇒ Number of permutations= ${}^3_1P = \frac{3!}{(3-1)!} = 3$
One of digit is taken at one's place, Number of possible digits available = 5
⇒ Number of permutations= ${}^5_2P = \frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3!}{3!} = 20.$

Therefore, total number of permutations =3 × 20=60.

**4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?**

**Solution:**

Total number of digits possible for choosing = 5

Number of places for which a digit has to be taken = 4

As there is no repetition allowed,

⇒ Number of permutations = ${}^{5}_{4}P = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$

The number will be even when 2 and 4 are at one's place.

The possibility of (2, 4) at one's place = 2/5 = 0.4

Total number of even number = 120 × 0.4 = 48.

**5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?**

**Solution:**

Total number of people in committee = 8

Number of positions to be filled = 2

⇒ Number of permutations = ${}^{8}_{2}P = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 56.$

**6. Find n if $^{n-1}P_3 : {}^{n}P_3 = 1 : 9$.**

**Solution:**

Given equation can be written as

$$\frac{{}^{n-1}_{3}P}{{}^{n}_{4}P} = \frac{1}{9}$$

By substituting the values we get

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

On simplification

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$

$\Rightarrow$ n=9.

**7. Find r if**
**(i)$^5P_r = 2\,^6P_{r-1}$**
**(ii) $^5P_r = {^6P_{r-1}}$**

**Solution:**

(i) $^5_rP = 2\ {^6_{r-1}P}$

Substituting the values we get

$\Rightarrow \frac{5!}{(5-r)!} = 2\ \frac{6!}{(7-r)!}$

The above equation can be written as

$\Rightarrow \frac{(7-r)!}{(5-r)!} = 2\ \frac{6!}{5!}$

On simplifying we get

$\Rightarrow (7-r)(6-r) = 2(6)$

$\Rightarrow 42 - 13r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 10r - 3r + 30 = 0$

$\Rightarrow r(r-10) - 3(r-10) = 0$

$\Rightarrow (r-3)(r-10) = 0$

r= 3 or r=10

But r = 10 is rejected, as in $^5_rP$, r cannot be greater than 5.

Therefore, r = 3.

(ii) ${}^{5}_{r}P = {}^{6}_{r-1}P$

The above equation can be written as

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$

$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!}$

$\Rightarrow (7 - r)(6 - r) = 6$

$\Rightarrow 42 - 13r + r^2 = 6$

$\Rightarrow r^2 - 13r + 36 = 0$

$\Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow r(r - 9) - 4(r - 9) = 0$

$\Rightarrow (r-4)(r-9) = 0$

r= 4 or r=9

But r=9 is rejected, as in ${}^{5}_{r}P$ , r cannot be greater than 5.

Therefore, r=4.

**8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?**

**Solution:**

Total number of different letters in EQUATION = 8
Number of letters to be used to form a word = 8
$\Rightarrow$ Number of permutations = ${}^{8}_{8}P = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320.$

**9. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.**
**(i) 4 letters are used at a time,**
**(ii) All letters are used at a time,**
**(iii) all letters are used but first letter is a vowel?**

**Solution:**

(i) Number of letters to be used =4

⇒ Number of permutations = ${}^6_4P = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360.$

(ii) Number of letters to be used = 6

⇒ Number of permutations = ${}^6_6P = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720.$

(iii) Number of vowels in MONDAY = 2 (O and A)

⇒ Number of permutations in vowel = ${}^2_1P = 2$

Now, remaining places = 5

Remaining letters to be used =5

⇒ Number of permutations = ${}^5_5P = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120.$

Therefore, total number of permutations = 2 × 120 =240.

**10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?**

**Solution:**

Total number of letters in MISSISSIPPI =11

Letter Number of occurrence

| | |
|---|---|
| M | 1 |
| I | 4 |
| S | 4 |
| P | 2 |

⇒ Number of permutations = $\frac{11!}{1!4!4!2!} = 34650$

We take that 4 I's come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations = $\frac{8!}{1!4!2!} = 840$

Therefore, total number of permutations where four I's don't come together = 34650-840=33810

**11. In how many ways can the letters of the word PERMUTATIONS be arranged if the**

**(i) Words start with P and end with S,**
**(ii) Vowels are all together,**
**(iii) There are always 4 letters between P and S?**

**Solution:**

(i) Total number of letters in PERMUTATIONS =12
Only repeated letter is T; 2times
First and last letter of the word are fixed as P and S respectively.
Number of letters remaining =12 – 2 = 10
⇒ Number of permutations = $\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

(ii) Number of vowels in PERMUTATIONS = 5 (E, U, A, I, O)
Now, we consider all the vowels together as one.
Number of permutations of vowels = 120
Now total number of letters = 12 – 5 + 1= 8
⇒ Number of permutations = $\frac{{}^{8}_{8}P}{2!} = \frac{8!}{2(8-8)!} = \frac{8!}{2} = 20160.$
Therefore, total number of permutations = 120 × 20160 = 2419200

(iii) Number of places are as 1 2 3 4 5 6 7 8 9 10 11 12
There should always be 4 letters between P and S.
Possible places of P and S are 1 and 6, 2and 7, 3 and 8, 4 and 9, 5 and 10, 6 and 11, 7 and 12
Possible ways =7,
Also, P and S can be interchanged,
No. of permutations =2 × 7 =14
Remaining 10 places can be filled with 10 remaining letters,
∴ No. of permutations = $\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$
Therefore, total number of permutations = 14 × 1814400 =25401600.