EXERCISE 7.4

1. If $^nC_8 = {}^nC_2$, find $^nC_2$.

Solution:

Given $^nC_8 = {}^nC_2$

We know that if $^nC_r = {}^nC_p$ then either r = p or r = n – p

Here $^nC_8 = {}^nC_2$

$\Rightarrow 8 = n - 2$

On rearranging we get

$\Rightarrow n = 10$

Now,

$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!}$ ($\because {}^nC_r = \frac{n!}{r!(n-r)!}$)

$\Rightarrow {}^{10}C_2 = \frac{10\times9\times8!}{2\times1\times8!} = \frac{90}{2} = 45$

2. Determine n if
(i) $^{2n}C_3 : {}^nC_3 = 12 : 1$
(ii) $^{2n}C_3 : {}^nC_3 = 11 : 1$

Solution:

(i) Given: $^{2n}C_3 : {}^nC_3 = 12 : 1$

The above equation can be written as

$\Rightarrow \frac{2n_{C_3}}{n_{C_3}} = \frac{12}{1}$

Substituting the formula we get

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

Expanding the factorial we get

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2) \times (2n-3)!}{3!(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!(n-3)!}} = \frac{12}{1}$$

On simplifying

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)}{3!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times 2 \times (n-1)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

On multiplying we get

$$\Rightarrow \frac{4 \times n \times (2n-1)}{n \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

Simplifying and computing

$\Rightarrow 4 \times (2n - 1) = 12 \times (n - 2)$

$\Rightarrow 8n - 4 = 12n - 24$

$\Rightarrow 12n - 8n = 24 - 4$

$\Rightarrow 4n = 20$

$\therefore n = 5$

(ii) Given: ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)\times(2n-3)!}{3!(2n-3)!}}{\frac{n\times(n-1)\times(n-2)\times(n-3)!}{3!(n-3)!}} = \frac{11}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}} = \frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times2\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4\times(2n-1)}{(n-2)} = \frac{11}{1}$$

$\Rightarrow 4\times(2n-1) = 11\times(n-2)$

$\Rightarrow 8n - 4 = 11n - 22$

$\Rightarrow 11n - 8n = 22 - 4$

$\Rightarrow 3n = 18$

$\therefore n = 6$

3. How many chords can be drawn through 21 points on a circle?

Solution:

Given 21 points on a circle

We know that we require two points on the circle to draw a chord

∴ Number of chords is are

$\Rightarrow {}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21\times20\times19!}{2!\times19!} = \frac{21\times20}{2\times1} = \frac{420}{2} = 210$

∴ Total number of chords can be drawn are 210

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution:

Given 5 boys and 4 girls are in total

We can select 3 boys from 5 boys in ${}^5C_3$ ways

Similarly, we can select 3 boys from 54 girls in ${}^4C_3$ ways

∴ Number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3$

$\Rightarrow {}^5C_3 \times {}^4C_3 = \frac{5!}{3!(5-3)!} \times \frac{4!}{3!(4-3)!} = \frac{5!}{3!\times2!} \times \frac{4!}{3!\times1!}$

$\Rightarrow {}^5C_3 \times {}^4C_3 = 10 \times 4 = 40$

∴ Number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3 = 40$ ways

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Solution:

Given 6 red balls, 5 white balls and 5 blue balls

We can select 3 red balls from 6 red balls in ${}^6C_3$ ways

Similarly, we can select 3 white balls from 5 white balls in ${}^5C_3$ ways

Similarly, we can select 3 blue balls from 5 blue balls in ${}^5C_3$ ways

∴ Number of ways of selecting 9 balls is ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!} = \frac{6!}{3!\times3!} \times \frac{5!}{3!\times2!} \times \frac{5!}{3!\times2!}$

$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 =$

$\frac{6\times5\times4\times3!}{3!\times3!} \times \frac{5\times4\times3!}{3!\times2!} \times \frac{5\times4\times3!}{3!\times2!} = \frac{120}{3\times2\times1} \times \frac{20}{2\times1} \times \frac{20}{2\times1} = 20 \times 10 \times 10 = 2000$

∴ Number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour is ${}^6C_3 \times {}^5C_3 \times {}^5C_3 = 2000$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution:

Given a deck of 52 cards

There are 4 Ace cards in a deck of 52 cards.

According to question, we need to select 1 Ace card out the 4 Ace cards

∴ Number of ways to select 1 Ace from 4 Ace cards is $^4C_1$

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ Number of ways to select 4 cards from 48 cards is $^{48}C_4$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4!}{1!(4-1)!} \times \frac{48!}{4!(48-4)!} = \frac{4!}{1!\times 3!} \times \frac{48!}{4!\times 44!}$$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4\times 3!}{1!\times 3!} \times \frac{48\times 47\times 46\times 45\times 44!}{4!\times 44!} = \frac{4}{1} \times \frac{4669920}{24} = 4 \times 194580 = 778320$$

∴ Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination 778320.

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution:

Given 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers

There are 5 players how bowl, and we can require 4 bowlers in a team of 11

∴ Number of ways in which bowlers can be selected are: $^5C_4$

Now other players left are = 17 – 5(bowlers) = 12

Since we need 11 players in a team and already 4 bowlers are selected, we need to select 7 more players from 12.

∴ Number of ways we can select these players are: $^{12}C_7$

∴ Total number of combinations possible are: $^5C_4 \times {}^{12}C_7$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!} = \frac{5!}{4!\times 1!} \times \frac{12!}{7!\times 5!}$$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5\times 4!}{1!\times 4!} \times \frac{12\times 11\times 10\times 9\times 8\times 7!}{5!\times 7!} = \frac{5}{1} \times \frac{95040}{120} = 5 \times 792 = 3960$$

∴ Number of ways we can select a team of 11 players where 4 players are bowlers from 17 players are 3960

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution:

Given a bag contains 5 black and 6 red balls

Number of ways we can select 2 black balls from 5 black balls are $^5C_2$

Number of ways we can select 3 red balls from 6 red balls are $^6C_3$

Number of ways 2 black and 3 red balls can be selected are $^5C_2 \times {}^6C_3$

$$\therefore {}^5C_2 \times {}^6C_3 = \frac{5!}{2!(5-2)!} \times \frac{6!}{3!(6-3)!} = \frac{5!}{2! \times 3!} \times \frac{6!}{3! \times 3!}$$

$$\Rightarrow {}^5C_2 \times {}^6C_3 = \frac{5 \times 4 \times 3!}{2! \times 3!} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{20}{2} \times \frac{120}{6} = 10 \times 20 = 200$$

∴ Number of ways in which 2 black and 3 red balls can be selected from 5 black and 6 red balls are 200

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution:

Given 9 courses are available and 2 specific courses are compulsory for every student

Here 2 courses are compulsory out of 9 courses, so a student need to select 5 - 2 = 3 courses

∴ Number of ways in which 3 ways can be selected from 9 - 2(compulsory courses) = 7 are $^7C_3$

$$\therefore {}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \times 4!}$$

$$\Rightarrow {}^7C_3 = \frac{7 \times 6 \times 5 \times 4!}{3! \times 4!} = \frac{210}{6} = 35$$

∴ Number of ways a student selects 5 courses from 9 courses where 2 specific courses are compulsory are: 35