## MISCELLANEOUS EXERCISE PAGE NO: 156

**1. How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?**

**Solution:**

The word DAUGHTER has 3 vowels A, E, U and 5 consonants D, G, H, T and R.

The three vowels can be chosen in ${}^3C_2$ as only two vowels are to be chosen.

Similarly, the five consonants can be chosen in ${}^5C_3$ ways.

∴ Number of choosing 2 vowels and 5 consonants would be ${}^3C_2 \times {}^5C_3$

$$= \frac{3!}{2!\,(3-2)!} \times \frac{5!}{3!\,(5-3)!} = \frac{3!}{2!1!} \times \frac{5!}{3!2!}$$

= 30

∴ Total number of ways of is 30

Each of these 5 letters can be arranged in 5 ways to form different words = ${}^5P_5$

$$\Rightarrow \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Total number of words formed would be = 30 × 120 = 3600

**2. How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?**

**Solution:**

In the word EQUATION there are 5 vowels (A, E, I, O, U) and 3 consonants (Q, T, N)

The numbers of ways in which 5 vowels can be arranged are ${}^5C_5$

$$\Rightarrow \frac{5!}{(5-5)!} = \frac{5\times4\times3\times2\times1}{0!} = \frac{120}{1} = 120 \quad \ldots\ldots\ldots\ldots \text{(i)}$$

Similarly, the numbers of ways in which 3 consonants can be arranged are ${}^3P_3$

$$\Rightarrow \frac{3!}{(3-3)!} = \frac{3\times2\times1}{0!} = \frac{6}{1} = 6 \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

There are two ways in which vowels and consonants can appear together

(AEIOU) (QTN) or (QTN) (AEIOU)

∴ The total number of ways in which vowel and consonant can appear together are 2 × ${}^5C_5$ × ${}^3C_3$

∴ 2 × 120 × 6 = 1440

**3. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can**

**this be done when the committee consists of:**
**(i) Exactly 3 girls?**
**(ii) At least 3 girls?**
**(iii) At most 3 girls?**

**Solution:**
(i) Given exactly 3 girls
Total numbers of girls are 4
Out of which 3 are to be chosen
∴ Number of ways in which choice would be made = ${}^4C_3$
Numbers of boys are 9 out of which 4 are to be chosen which is given by ${}^9C_4$
Total ways of forming the committee with exactly three girls
$= {}^4C_3 \times {}^9C_4$

$$= \frac{4!}{3!(4-3)!} \times \frac{9!}{4!(9-4)!} = \frac{4!}{3!1!} \times \frac{9!}{4!5!} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1\times5\times4\times3\times2\times1} = 504$$

(ii) Given at least 3 girls
There are two possibilities of making committee choosing at least 3 girls
There are 3 girls and 4 boys or there are 4 girls and 3 boys
Choosing three girls we have done in (i)
Choosing four girls and 3 boys would be done in ${}^4C_4$ ways
And choosing 3 boys would be done in ${}^9C_3$
Total ways = ${}^4C_4 \times {}^9C_3$

$$= \frac{4!}{4!\,(4-4)!} \times \frac{9!}{3!\,(9-3)!} = \frac{4!}{4!\,0!} \times \frac{9!}{3!\,6!} = \frac{9\times8\times7\times6!}{3\times2\times1\times6!} = 84$$

Total numbers of ways of making the committee are
504 + 84 = 588

(iii) Given at most 3 girls
In this case the numbers of possibilities are
0 girl and 7 boys
1 girl and 6 boys
2 girls and 5 boys
3 girls and 4 boys
Number of ways to choose 0 girl and 7 boys = ${}^4C_0 \times {}^9C_7$

$$= \frac{4!}{0!\,(4-0)!} \times \frac{9!}{7!\,2!} = \frac{4!}{4!} \times \frac{9 \times 8 \times 7!}{7! \times 2 \times 1} = \frac{72}{2} = 36$$

Number of ways of choosing 1 girl and 6 boys = ${}^4C_1 \times {}^9C_6$

$$\frac{4!}{1!\,3!} \times \frac{9!}{6!\,3!} = \frac{4 \times 3!}{3!} \times \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1} = 336$$

Number of ways of choosing 2 girls and 5 boys = ${}^4C_2 \times {}^9C_5$

$$\frac{4!}{2!\,2!} \times \frac{9!}{5!\,4!} = \frac{4!}{2 \times 1 \times 2 \times 1} \times \frac{9 \times 7 \times 8 \times 6 \times 5!}{5!\,4!} = 756$$

Number of choosing 3 girls and 4 boys has been done in (1)
= 504
Total number of ways in which committee can have at most 3 girls are = 36 + 336 + 756 + 504 = 1632

**4. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?**

**Solution:**
In dictionary words are listed alphabetically, so to find the words
Listed before E should start with letter either A, B, C or D
But the word EXAMINATION doesn`t have B, C or D
Hence the words should start with letter A
The remaining 10 places are to be filled by the remaining letters of the word EXAMINATION which are E, X, A, M, 2N, T, 2I, 0
Since the letters are repeating the formula used would be

$$= \frac{n!}{p_1!\,p_2!\,p_{3!}}$$

Where n is remaining number of letters $p_1$ and $p_2$ are number of times the repeated terms occurs.

$$= \frac{10!}{2!\,2!} = 907200$$

The number of words in the list before the word starting with E
= words starting with letter A = 907200

**5. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?**

**Solution:**

The number is divisible by 10 if the unit place has 0 in it.

The 6-digit number is to be formed out of which unit place is fixed as 0

The remaining 5 places can be filled by 1, 3, 5, 7 and 9

Here n = 5

And the numbers of choice available are 5

So, the total ways in which the rest the places can be filled are ${}^5P_5$

$$= \frac{5!}{(5-5)!} \times 1 = \frac{5!}{1} \times 1 = 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120$$

**6. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?**

**Solution:**

We know that there are 5 vowels and 21 consonants in English alphabets.

Choosing two vowels out of 5 would be done in ${}^5C_2$ ways

Choosing 2 consonants out of 21 can be done in ${}^{21}C_2$ ways

The total number of ways selecting 2 vowels and 2 consonants

$= {}^5C_2 \times {}^{21}C_2$

$$\Rightarrow \frac{5!}{2!\,3!} \times \frac{21!}{2!\,19!} = \frac{5 \times 4 \times 3!}{2!\,3!} \times \frac{21 \times 20 \times 19!}{2 \times 1 \times 19!} = 2100$$

Each of these four letters can be arranged in four ways ${}^4P_4$

$$\Rightarrow \frac{4!}{0!} = 4 \times 3 \times 2 \times 1 = 24 \text{ ways}$$

Total numbers of words that can be formed are

$24 \times 2100 = 50400$

**7. In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?**

**Solution:**

The student can choose 3 questions from part I and 5 from part II

Or

4 questions from part I and 4 from part II

5 questions from part 1 and 3 from part II

3 questions from part I and 5 from part II can be chosen in

$= {}^5C_3 \times {}^7C_5$

$$= \frac{5!}{3!\,2!} \times \frac{7!}{5!\,2!} = \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = 210$$

4 questions from part I and 4 from part II can be chosen in

$= {}^5C_4 \times {}^7C_4$

$$= \frac{5!}{4!\,1!} \times \frac{7!}{4!\,3!} = \frac{5 \times 4!}{4!} \times \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} = 175$$

5 questions from part 1 and 3 from part II can be chosen in

$= {}^5C_5 \times {}^7C_3$

$$= \frac{5!}{5!\,0!} \times \frac{7!}{3!\,4!} = 1 \times \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

Now the total number of ways in which a student can choose the questions are

$= 210 + 175 + 35 = 420$

**8. Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.**

**Solution:**

We have a deck of cards has 4 kings.

The numbers of remaining cards are 52.

Ways of selecting a king from the deck = ${}^4C_1$

Ways of selecting the remaining 4 cards from 48 cards= ${}^{48}C_4$

Total number of selecting the 5 cards having one king always

$= {}^4C_1 \times {}^{48}C_4$

$$= \frac{4!}{1!\,3!} \times \frac{48!}{4!\,44!} = \frac{4 \times 3!}{3!} \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{4 \times 3 \times 2 \times 1 \times 44!} = 778320$$

**9. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?**

**Solution:**

Given there are total 9 people

Women occupies even places that means they will be sitting on 2nd, 4th, 6th and 8th place where as men will be sitting on 1st, 3rd, 5th,7th and 9th place.
4 women can sit in four places and ways they can be seated= $^4P_4$

$$= \frac{4!}{(4-4)!} = \frac{4 \times 3 \times 2 \times 1}{0!} = 24$$

5 men can occupy 5 seats in 5 ways
The numbers of ways in which these can be seated = $^5P_5$

$$= \frac{5!}{(5-5)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{1} = 120$$

The total numbers of sitting arrangements possible are
24 × 120 = 2880

**10. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?**

**Solution:**
In this question we get 2 options that is
(i) Either all 3 will go
Then remaining students in class are: 25 – 3 = 22
Number of students remained to be chosen for party = 7
Number of ways choosing the remaining 22 students = $^{22}C_7$

$$= \frac{22!}{7!15!} = 170544$$

(ii) None of them will go
The students going will be 10
Remaining students eligible for going = 22
Number of ways in which these 10 students can be selected are $^{22}C_{10}$

$$= \frac{22!}{10!\,12!} = 646646$$

Total numbers of ways in which students can be chosen are
= 170544 + 646646 = 817190

**11. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?**

**Solution:**

In the given word ASSASSINATION, there are 4 'S'. Since all the 4 'S' have to be arranged together so let as take them as one unit.
The remaining letters are= 3 'A', 2 'I', 2 'N', T
The number of letters to be arranged are 9 (including 4 'S')
Using the formula $\frac{n!}{p_1!p_2!p_3!}$ where n is number of terms and $p_1$, $p_2$ $p_3$ are the number of times the repeating letters repeat themselves.
Here $p_1$= 3, $p_2$= 2, $p_3$ = 2
Putting the values in formula we get

$$\frac{10!}{3!\,2!\,2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 2 \times 1 \times 1} = 151200$$