EXERCISE 7.1

1. How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that
(i) Repetition of the digits is allowed?
(ii) Repetition of the digits is not allowed?

Solution:
(i) Let the 3-digit number be ABC, where C is at the units place, B at the tens place and A at the hundreds place.
Now when repetition is allowed,
The number of digits possible at C is 5. As repetition is allowed, the number of digits possible at B and A is also 5 at each.
Hence, the total number possible 3-digit numbers $=5 \times 5 \times 5 = 125$

(ii) Let the 3-digit number be ABC, where C is at the units place, B at the tens place and A at the hundreds place.
Now when repetition is not allowed,
The number of digits possible at C is 5. Let's suppose one of 5 digits occupies place C, now as the repletion is not allowed, the possible digits for place B are 4 and similarly there are only 3 possible digits for place A.
Therefore, The total number of possible 3-digit numbers$=5 \times 4 \times 3=60$

2. How many 3-digits even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

Solution:
Let the 3-digit number be ABC, where C is at the unit's place, B at the tens place and A at the hundreds place.
As the number has to even, the digits possible at C are 2 or 4 or 6. That is number of possible digits at C is 3.
Now, as the repetition is allowed, the digits possible at B is 6. Similarly, at A, also, the number of digits possible is 6.
Therefore, The total number possible 3 digit numbers $= 6 \times 6 \times 3 = 108$.

3. How many 4-letter code can be formed using the first 10 letters of the English

alphabet, if no letter can be repeated?

Solution:
Let the 4 digit code be 1234.
At the first place, the number of letters possible is 10.
Let’s suppose any 1 of the ten occupies place 1.
Now, as the repetition is not allowed, the number of letters possible at place 2 is 9. Now at 1 and 2, any 2 of the 10 alphabets have been taken. The number of alphabets left for place 3 is 8 and similarly the number of alphabets possible at 4 is 7.
Therefore the total number of 4 letter codes=$10 \times 9 \times 8 \times 7$=5040.

4. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

Solution:
Let the five-digit number be ABCDE. Given that first 2 digits of each number is 67.
Therefore, the number is 67CDE.
As the repetition is not allowed and 6 and 7 are already taken, the digits available for place C are 0,1,2,3,4,5,8,9. The number of possible digits at place C is 8. Suppose one of them is taken at C, now the digits possible at place D is 7. And similarly, at E the possible digits are 6.
∴The total five-digit numbers with given conditions = $8 \times 7 \times 6 = 336$.

5. A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

Solution:
Given A coin is tossed 3 times and the outcomes are recorded
The possible outcomes after a coin toss are head and tail.
The number of possible outcomes at each coin toss is 2.
∴The total number of possible outcomes after 3 times = $2 \times 2 \times 2 = 8$.

6. Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

Solution:
Given 5 flags of different colours

We know the signal requires 2 flags.
The number of flags possible for upper flag is 5.
Now as one of the flag is taken, the number of flags remaining for lower flag in the signal is 4.
The number of ways in which signal can be given = $5 \times 4 = 20$.

EXERCISE 7.2 PAGE NO: 140

1. Evaluate
(i) 8!
(ii) 4! – 3!

Solution:
(i) Consider 8!
We know that 8! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1
= 40320

(ii) Consider 4!-3!
4!-3! = (4 × 3!) - 3!
Above equation can be written as
= 3! (4-1)
= 3 × 2 × 1 × 3
= 18

2. Is 3! + 4! = 7!?

Solution:
Consider LHS 3! + 4!
Computing left hand side, we get
3! + 4! = (3 × 2 × 1) + (4 × 3 × 2 × 1)
= 6 + 24
= 30
Again consider RHS and computing we get
7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040
Therefore LHS ≠ RHS
Therefore 3! + 4! ≠ 7!

3. Compute

$$\frac{8!}{6! \times 2!}$$

Solution:

Given $\frac{8!}{6! \times 2!}$

Expanding all the factorials and simplifying we get

$$\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1}$$

$$\Rightarrow \frac{8!}{6! \times 2!} = \frac{8 \times 7}{2} = 28$$

4. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$ **find x.**

Solution:

Consider LHS and by computing we get

$$\frac{1}{6!} + \frac{1}{7!} = \frac{1}{6!} + \frac{1}{7 \times 6!}$$

$$\Rightarrow \frac{7+1}{7 \times 6!} = \frac{8}{7!}$$

Equating LHS to RHS to get the value of x

$$\frac{8}{7!} = \frac{x}{8!}$$

$$\Rightarrow \frac{8}{7!} = \frac{x}{8 \times 7!}$$

On rearranging we get

$$\Rightarrow 8 \times 8 = x$$

$$\Rightarrow x = 64.$$

5. Evaluate

$\frac{n!}{(n-r)!}$,

When
(i) n = 6, r = 2
(ii) n = 9, r = 5

Solution:

(i) Given n = 6 and r = 2

Putting the value of n and r we get

$$\frac{6!}{(6-2)!}$$

$$\Rightarrow \frac{6!}{4!} = \frac{6 \times 5 \times 4!}{4!} = 6 \times 5 = 30.$$

(ii) Given n = 9 and r = 5

Putting the value of n and r we get

$$\frac{9!}{(9-5)!}$$

$$\Rightarrow \frac{9!}{4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!} = 9 \times 8 \times 7 \times 6 \times 5 = 15120.$$

EXERCISE 7.3 PAGE NO: 148

1. How many 3-digit numbers can be formed by using the digits 1 to 9 if no digit is repeated?

Solution:

Total number of digits possible for choosing = 9

Number of places for which a digit has to be taken = 3

As there is no repetition allowed,

⇒ No. of permutations = ${}^{9}_{3}P = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\times8\times7\times6!}{6!} = 504.$

2. How many 4-digit numbers are there with no digit repeated?

Solution:
To find four digit number (digits does not repeat)
Now we will have 4 places where 4 digits are to be put.
So, at thousand's place = There are 9 ways as 0 cannot be at thousand's place = 9 ways
At hundredth's place = There are 9 digits to be filled as 1 digit is already taken = 9 ways
At ten's place = There are now 8 digits to be filled as 2 digits are already taken = 8 ways
At unit's place = There are 7 digits that can be filled = 7 ways
Total Number of ways to fill the four places = 9 × 9 × 8 × 7 = 4536 ways.
So a total of 4536 four digit numbers can be there with no digits repeated.

3. How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 6, 7, if no digit is repeated?

Solution:
Even number means that last digit should be even,
Number of possible digits at one's place = 3 (2, 4 and 6)
⇒ Number of permutations= ${}^{3}_{1}P = \frac{3!}{(3-1)!} = 3$
One of digit is taken at one's place, Number of possible digits available = 5
⇒ Number of permutations= ${}^{5}_{2}P = \frac{5!}{(5-2)!} = \frac{5\times4\times3!}{3!} = 20.$

Therefore, total number of permutations =3 × 20=60.

4. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated. How many of these will be even?

Solution:

Total number of digits possible for choosing = 5
Number of places for which a digit has to be taken = 4
As there is no repetition allowed,
⇒ Number of permutations = ${}^{5}_{4}P = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120.$
The number will be even when 2 and 4 are at one's place.
The possibility of (2, 4) at one's place = 2/5 = 0.4
Total number of even number = 120 × 0.4 = 48.

5. From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

Solution:

Total number of people in committee = 8
Number of positions to be filled = 2
⇒ Number of permutations = ${}^{8}_{2}P = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 56.$

6. Find n if ${}^{n-1}P_3 : {}^{n}P_3 = 1 : 9$.

Solution:

Given equation can be written as

$$\frac{{}^{n-1}_{3}P}{{}^{n}_{4}P} = \frac{1}{9}$$

By substituting the values we get

$$\Rightarrow \frac{\frac{(n-1)!}{(n-4)!}}{\frac{n!}{(n-4)!}} = \frac{1}{9}$$

On simplification

$\Rightarrow \frac{(n-1)!}{n!} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$

$\Rightarrow$ n=9.

7. Find r if
(i)$^5P_r = 2\,^6P_{r-1}$
(ii) $^5P_r = {}^6P_{r-1}$

Solution:

(i) $^5_rP = 2\ {}^6_{r-1}P$

Substituting the values we get

$\Rightarrow \frac{5!}{(5-r)!} = 2\ \frac{6!}{(7-r)!}$

The above equation can be written as

$\Rightarrow \frac{(7-r)!}{(5-r)!} = 2\ \frac{6!}{5!}$

On simplifying we get

$\Rightarrow (7-r)(6-r) = 2(6)$

$\Rightarrow 42 - 13r + r^2 = 12$

$\Rightarrow r^2 - 13r + 30 = 0$

$\Rightarrow r^2 - 10r - 3r + 30 = 0$

$\Rightarrow r(r-10) - 3(r-10) = 0$

$\Rightarrow (r-3)(r-10) = 0$

r= 3 or r=10

But r = 10 is rejected, as in ^5_rP, r cannot be greater than 5.

Therefore, r = 3.

(ii) ${}^{5}_{r}P = {}^{6}_{r-1}P$

The above equation can be written as

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$

$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!}$

$\Rightarrow (7 - r)(6 - r) = 6$

$\Rightarrow 42 - 13r + r^2 = 6$

$\Rightarrow r^2 - 13r + 36 = 0$

$\Rightarrow r^2 - 9r - 4r + 36 = 0$

$\Rightarrow r(r - 9) - 4(r - 9) = 0$

$\Rightarrow (r-4)(r-9) = 0$

r= 4 or r=9

But r=9 is rejected, as in ${}^{5}_{r}P$, r cannot be greater than 5.

Therefore, r=4.

8. How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once?

Solution:
Total number of different letters in EQUATION = 8
Number of letters to be used to form a word = 8
$\Rightarrow$ Number of permutations = ${}^{8}_{8}P = \frac{8!}{(8-8)!} = \frac{8!}{0!} = 40320.$

9. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if.
(i) 4 letters are used at a time,
(ii) All letters are used at a time,
(iii) all letters are used but first letter is a vowel?

Solution:

(i) Number of letters to be used =4

⇒ Number of permutations = ${}^{6}_{4}P = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360.$

(ii) Number of letters to be used = 6

⇒ Number of permutations = ${}^{6}_{6}P = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 720.$

(iii) Number of vowels in MONDAY = 2 (O and A)

⇒ Number of permutations in vowel = ${}^{2}_{1}P = 2$

Now, remaining places = 5

Remaining letters to be used =5

⇒ Number of permutations = ${}^{5}_{5}P = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 120.$

Therefore, total number of permutations = 2 × 120 =240.

10. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Solution:

Total number of letters in MISSISSIPPI =11

Letter Number of occurrence

M	1
I	4
S	4
P	2

⇒ Number of permutations = $\frac{11!}{1!4!4!2!} = 34650$

We take that 4 I's come together, and they are treated as 1 letter,

∴ Total number of letters=11 – 4 + 1 = 8

⇒ Number of permutations = $\frac{8!}{1!4!2!} = 840$

Therefore, total number of permutations where four I's don't come together = 34650-840=33810

11. In how many ways can the letters of the word PERMUTATIONS be arranged if the

(i) Words start with P and end with S,
(ii) Vowels are all together,
(iii) There are always 4 letters between P and S?

Solution:

(i) Total number of letters in PERMUTATIONS =12
Only repeated letter is T; 2times
First and last letter of the word are fixed as P and S respectively.
Number of letters remaining =12 – 2 = 10

⇒ Number of permutations = $\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

(ii) Number of vowels in PERMUTATIONS = 5 (E, U, A, I, O)
Now, we consider all the vowels together as one.
Number of permutations of vowels = 120
Now total number of letters = 12 – 5 + 1= 8

⇒ Number of permutations = $\frac{{}^{8}_{8}P}{2!} = \frac{8!}{2(8-8)!} = \frac{8!}{2} = 20160.$

Therefore, total number of permutations = 120 × 20160 = 2419200

(iii) Number of places are as 1 2 3 4 5 6 7 8 9 10 11 12
There should always be 4 letters between P and S.
Possible places of P and S are 1 and 6, 2and 7, 3 and 8, 4 and 9, 5 and 10, 6 and 11, 7 and 12
Possible ways =7,
Also, P and S can be interchanged,
No. of permutations =2 × 7 =14
Remaining 10 places can be filled with 10 remaining letters,

∴ No. of permutations = $\frac{{}^{10}_{10}P}{2!} = \frac{10!}{2(10-10)!} = \frac{10!}{2} = 1814400$

Therefore, total number of permutations = 14 × 1814400 =25401600.

EXERCISE 7.4

1. If $^nC_8 = {^nC_2}$, find $^nC_2$.

Solution:

Given $^nC_8 = {^nC_2}$

We know that if $^nC_r = {^nC_p}$ then either r = p or r = n – p

Here $^nC_8 = {^nC_2}$

⇒ 8 = n – 2

On rearranging we get

⇒ n = 10

Now,

$\therefore {^nC_2} = {^{10}C_2} = \frac{10!}{2!(10-2)!}$ ($\because {^nC_r} = \frac{n!}{r!(n-r)!}$)

$\Rightarrow {^{10}C_2} = \frac{10\times9\times8!}{2\times1\times8!} = \frac{90}{2} = 45$

2. Determine n if
(i) $^{2n}C_3 : {^nC_3} = 12: 1$
(ii) $^{2n}C_3 : {^nC_3} = 11: 1$

Solution:

(i) Given: $^{2n}C_3 : {^nC_3} = 12: 1$

The above equation can be written as

$\Rightarrow \frac{2n_{C_3}}{n_{C_3}} = \frac{12}{1}$

Substituting the formula we get

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

Expanding the factorial we get

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2) \times (2n-3)!}{3!(2n-3)!}}{\frac{n \times (n-1) \times (n-2) \times (n-3)!}{3!(n-3)!}} = \frac{12}{1}$$

On simplifying

$$\Rightarrow \frac{\frac{2n \times (2n-1) \times (2n-2)}{3!}}{\frac{n \times (n-1) \times (n-2)}{3!}} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times (2n-2)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{2n \times (2n-1) \times 2 \times (n-1)}{n \times (n-1) \times (n-2)} = \frac{12}{1}$$

On multiplying we get

$$\Rightarrow \frac{4 \times n \times (2n-1)}{n \times (n-2)} = \frac{12}{1}$$

$$\Rightarrow \frac{4 \times (2n-1)}{(n-2)} = \frac{12}{1}$$

Simplifying and computing

$\Rightarrow 4 \times (2n - 1) = 12 \times (n - 2)$

$\Rightarrow 8n - 4 = 12n - 24$

$\Rightarrow 12n - 8n = 24 - 4$

$\Rightarrow 4n = 20$

$\therefore n = 5$

(ii) Given: ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$

$$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n!}{3!(2n-3)!}}{\frac{n!}{3!(n-3)!}} = \frac{12}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)\times(2n-3)!}{3!(2n-3)!}}{\frac{n\times(n-1)\times(n-2)\times(n-3)!}{3!(n-3)!}} = \frac{11}{1}$$

$$\Rightarrow \frac{\frac{2n\times(2n-1)\times(2n-2)}{3!}}{\frac{n\times(n-1)\times(n-2)}{3!}} = \frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times(2n-2)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{2n\times(2n-1)\times 2\times(n-1)}{n\times(n-1)\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4\times n\times(2n-1)}{n\times(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{4\times(2n-1)}{(n-2)} = \frac{11}{1}$$

$\Rightarrow 4\times(2n-1) = 11\times(n-2)$

$\Rightarrow 8n - 4 = 11n - 22$
$\Rightarrow 11n - 8n = 22 - 4$
$\Rightarrow 3n = 18$
$\therefore n = 6$

3. How many chords can be drawn through 21 points on a circle?

Solution:
Given 21 points on a circle
We know that we require two points on the circle to draw a chord
∴ Number of chords is are

$\Rightarrow {}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21\times20\times19!}{2!\times19!} = \frac{21\times20}{2\times1} = \frac{420}{2} = 210$

∴ Total number of chords can be drawn are 210

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution:

Given 5 boys and 4 girls are in total

We can select 3 boys from 5 boys in ${}^5C_3$ ways

Similarly, we can select 3 boys from 54 girls in ${}^4C_3$ ways

∴ Number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3$

$\Rightarrow {}^5C_3 \times {}^4C_3 = \frac{5!}{3!(5-3)!} \times \frac{4!}{3!(4-3)!} = \frac{5!}{3!\times2!} \times \frac{4!}{3!\times1!}$

$\Rightarrow {}^5C_3 \times {}^4C_3 = 10 \times 4 = 40$

∴ Number of ways a team of 3 boys and 3 girls can be selected is ${}^5C_3 \times {}^4C_3 = 40$ ways

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Solution:

Given 6 red balls, 5 white balls and 5 blue balls

We can select 3 red balls from 6 red balls in ${}^6C_3$ ways

Similarly, we can select 3 white balls from 5 white balls in ${}^5C_3$ ways

Similarly, we can select 3 blue balls from 5 blue balls in ${}^5C_3$ ways

∴ Number of ways of selecting 9 balls is ${}^6C_3 \times {}^5C_3 \times {}^5C_3$

$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!} = \frac{6!}{3!\times3!} \times \frac{5!}{3!\times2!} \times \frac{5!}{3!\times2!}$

$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3 =$

$\frac{6\times5\times4\times3!}{3!\times3!} \times \frac{5\times4\times3!}{3!\times2!} \times \frac{5\times4\times3!}{3!\times2!} = \frac{120}{3\times2\times1} \times \frac{20}{2\times1} \times \frac{20}{2\times1} = 20 \times 10 \times 10 = 2000$

∴ Number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour is ${}^6C_3 \times {}^5C_3 \times {}^5C_3 = 2000$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution:

Given a deck of 52 cards

There are 4 Ace cards in a deck of 52 cards.

According to question, we need to select 1 Ace card out the 4 Ace cards

∴ Number of ways to select 1 Ace from 4 Ace cards is $^4C_1$

⇒ More 4 cards are to be selected now from 48 cards (52 cards – 4 Ace cards)

∴ Number of ways to select 4 cards from 48 cards is $^{48}C_4$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4!}{1!(4-1)!} \times \frac{48!}{4!(48-4)!} = \frac{4!}{1!\times 3!} \times \frac{48!}{4!\times 44!}$$

$$\Rightarrow {}^4C_1 \times {}^{48}C_4 = \frac{4\times 3!}{1!\times 3!} \times \frac{48\times 47\times 46\times 45\times 44!}{4!\times 44!} = \frac{4}{1} \times \frac{4669920}{24} = 4 \times 194580 = 778320$$

∴ Number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination 778320.

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution:

Given 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers

There are 5 players how bowl, and we can require 4 bowlers in a team of 11

∴ Number of ways in which bowlers can be selected are: $^5C_4$

Now other players left are = 17 – 5(bowlers) = 12

Since we need 11 players in a team and already 4 bowlers are selected, we need to select 7 more players from 12.

∴ Number of ways we can select these players are: $^{12}C_7$

∴ Total number of combinations possible are: $^5C_4 \times {}^{12}C_7$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!(5-4)!} \times \frac{12!}{7!(12-7)!} = \frac{5!}{4!\times 1!} \times \frac{12!}{7!\times 5!}$$

$$\Rightarrow {}^5C_4 \times {}^{12}C_7 = \frac{5\times 4!}{1!\times 4!} \times \frac{12\times 11\times 10\times 9\times 8\times 7!}{5!\times 7!} = \frac{5}{1} \times \frac{95040}{120} = 5 \times 792 = 3960$$

∴ Number of ways we can select a team of 11 players where 4 players are bowlers from 17 players are 3960

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution:

Given a bag contains 5 black and 6 red balls

Number of ways we can select 2 black balls from 5 black balls are ${}^5C_2$

Number of ways we can select 3 red balls from 6 red balls are ${}^6C_3$

Number of ways 2 black and 3 red balls can be selected are ${}^5C_2 \times {}^6C_3$

$\therefore {}^5C_2 \times {}^6C_3 = \frac{5!}{2!(5-2)!} \times \frac{6!}{3!(6-3)!} = \frac{5!}{2! \times 3!} \times \frac{6!}{3! \times 3!}$

$\Rightarrow {}^5C_2 \times {}^6C_3 = \frac{5 \times 4 \times 3!}{2! \times 3!} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{20}{2} \times \frac{120}{6} = 10 \times 20 = 200$

$\therefore$ Number of ways in which 2 black and 3 red balls can be selected from 5 black and 6 red balls are 200

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution:

Given 9 courses are available and 2 specific courses are compulsory for every student

Here 2 courses are compulsory out of 9 courses, so a student need to select 5 - 2 = 3 courses

$\therefore$ Number of ways in which 3 ways can be selected from 9 - 2(compulsory courses) = 7 are ${}^7C_3$

$\therefore {}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3! \times 4!}$

$\Rightarrow {}^7C_3 = \frac{7 \times 6 \times 5 \times 4!}{3! \times 4!} = \frac{210}{6} = 35$

$\therefore$ Number of ways a student selects 5 courses from 9 courses where 2 specific courses are compulsory are: 35

MISCELLANEOUS EXERCISE

1. How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word DAUGHTER?

Solution:

The word DAUGHTER has 3 vowels A, E, U and 5 consonants D, G, H, T and R.
The three vowels can be chosen in $^3C_2$ as only two vowels are to be chosen.
Similarly, the five consonants can be chosen in $^5C_3$ ways.
∴ Number of choosing 2 vowels and 5 consonants would be $^3C_2 \times ^5C_3$

$$= \frac{3!}{2!\,(3-2)!} \times \frac{5!}{3!\,(5-3)!} = \frac{3!}{2!1!} \times \frac{5!}{3!2!}$$

= 30
∴ Total number of ways of is 30
Each of these 5 letters can be arranged in 5 ways to form different words = $^5P_5$

$$\Rightarrow \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5!}{1} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Total number of words formed would be = 30 × 120 = 3600

2. How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Solution:

In the word EQUATION there are 5 vowels (A, E, I, O, U) and 3 consonants (Q, T, N)
The numbers of ways in which 5 vowels can be arranged are $^5C_5$

$$\Rightarrow \frac{5!}{(5-5)!} = \frac{5\times4\times3\times2\times1}{0!} = \frac{120}{1} = 120 \quad \text{............... (i)}$$

Similarly, the numbers of ways in which 3 consonants can be arranged are $^3P_3$

$$\Rightarrow \frac{3!}{(3-3)!} = \frac{3\times2\times1}{0!} = \frac{6}{1} = 6 \quad \text{................. (ii)}$$

There are two ways in which vowels and consonants can appear together
(AEIOU) (QTN) or (QTN) (AEIOU)
∴ The total number of ways in which vowel and consonant can appear together are 2 × $^5C_5$ × $^3C_3$
∴ 2 × 120 × 6 = 1440

3. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can

this be done when the committee consists of:
(i) Exactly 3 girls?
(ii) At least 3 girls?
(iii) At most 3 girls?

Solution:
(i) Given exactly 3 girls
Total numbers of girls are 4
Out of which 3 are to be chosen
∴ Number of ways in which choice would be made = $^4C_3$
Numbers of boys are 9 out of which 4 are to be chosen which is given by $^9C_4$
Total ways of forming the committee with exactly three girls
= $^4C_3 \times {}^9C_4$

$$= \frac{4!}{3!(4-3)!} \times \frac{9!}{4!(9-4)!} = \frac{4!}{3!1!} \times \frac{9!}{4!5!} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1\times5\times4\times3\times2\times1} = 504$$

(ii) Given at least 3 girls
There are two possibilities of making committee choosing at least 3 girls
There are 3 girls and 4 boys or there are 4 girls and 3 boys
Choosing three girls we have done in (i)
Choosing four girls and 3 boys would be done in $^4C_4$ ways
And choosing 3 boys would be done in $^9C_3$
Total ways = $^4C_4 \times {}^9C_3$

$$= \frac{4!}{4!\,(4-4)!} \times \frac{9!}{3!\,(9-3)!} = \frac{4!}{4!\,0!} \times \frac{9!}{3!\,6!} = \frac{9\times8\times7\times6!}{3\times2\times1\times6!} = 84$$

Total numbers of ways of making the committee are
504 + 84 = 588

(iii) Given at most 3 girls
In this case the numbers of possibilities are
0 girl and 7 boys
1 girl and 6 boys
2 girls and 5 boys
3 girls and 4 boys
Number of ways to choose 0 girl and 7 boys = $^4C_0 \times {}^9C_7$

$$= \frac{4!}{0!\,(4-0)!} \times \frac{9!}{7!\,2!} = \frac{4!}{4!} \times \frac{9 \times 8 \times 7!}{7! \times 2 \times 1} = \frac{72}{2} = 36$$

Number of ways of choosing 1 girl and 6 boys = ${}^4C_1 \times {}^9C_6$

$$\frac{4!}{1!\,3!} \times \frac{9!}{6!\,3!} = \frac{4 \times 3!}{3!} \times \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1} = 336$$

Number of ways of choosing 2 girls and 5 boys = ${}^4C_2 \times {}^9C_5$

$$\frac{4!}{2!\,2!} \times \frac{9!}{5!\,4!} = \frac{4!}{2 \times 1 \times 2 \times 1} \times \frac{9 \times 7 \times 8 \times 6 \times 5!}{5!\,4!} = 756$$

Number of choosing 3 girls and 4 boys has been done in (1)
= 504
Total number of ways in which committee can have at most 3 girls are = 36 + 336 + 756 + 504 = 1632

4. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Solution:
In dictionary words are listed alphabetically, so to find the words
Listed before E should start with letter either A, B, C or D
But the word EXAMINATION doesn`t have B, C or D
Hence the words should start with letter A
The remaining 10 places are to be filled by the remaining letters of the word EXAMINATION which are E, X, A, M, 2N, T, 2I, 0
Since the letters are repeating the formula used would be

$$= \frac{n!}{p_1!\,p_2!\,p_3!}$$

Where n is remaining number of letters p_1 and p_2 are number of times the repeated terms occurs.

$$= \frac{10!}{2!\,2!} = 907200$$

The number of words in the list before the word starting with E
= words starting with letter A = 907200

5. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Solution:

The number is divisible by 10 if the unit place has 0 in it.

The 6-digit number is to be formed out of which unit place is fixed as 0

The remaining 5 places can be filled by 1, 3, 5, 7 and 9

Here n = 5

And the numbers of choice available are 5

So, the total ways in which the rest the places can be filled are $^5P_5$

$$= \frac{5!}{(5-5)!} \times 1 = \frac{5!}{1} \times 1 = 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 120$$

6. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

Solution:

We know that there are 5 vowels and 21 consonants in English alphabets.

Choosing two vowels out of 5 would be done in $^5C_2$ ways

Choosing 2 consonants out of 21 can be done in $^{21}C_2$ ways

The total number of ways selecting 2 vowels and 2 consonants

$= {}^5C_2 \times {}^{21}C_2$

$$\Rightarrow \frac{5!}{2!\,3!} \times \frac{21!}{2!\,19!} = \frac{5 \times 4 \times 3!}{2!\,3!} \times \frac{21 \times 20 \times 19!}{2 \times 1 \times 19!} = 2100$$

Each of these four letters can be arranged in four ways $^4P_4$

$$\Rightarrow \frac{4!}{0!} = 4 \times 3 \times 2 \times 1 = 24 \text{ ways}$$

Total numbers of words that can be formed are

24 × 2100 = 50400

7. In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Solution:

The student can choose 3 questions from part I and 5 from part II

Or

4 questions from part I and 4 from part II

5 questions from part 1 and 3 from part II

3 questions from part I and 5 from part II can be chosen in

$= {}^5C_3 \times {}^7C_5$

$$= \frac{5!}{3!\,2!} \times \frac{7!}{5!\,2!} = \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{7 \times 6 \times 5!}{5! \times 2 \times 1} = 210$$

4 questions from part I and 4 from part II can be chosen in

$= {}^5C_4 \times {}^7C_4$

$$= \frac{5!}{4!\,1!} \times \frac{7!}{4!\,3!} = \frac{5 \times 4!}{4!} \times \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} = 175$$

5 questions from part 1 and 3 from part II can be chosen in

$= {}^5C_5 \times {}^7C_3$

$$= \frac{5!}{5!\,0!} \times \frac{7!}{3!\,4!} = 1 \times \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

Now the total number of ways in which a student can choose the questions are

= 210 + 175 + 35 = 420

8. Determine the number of 5-card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Solution:

We have a deck of cards has 4 kings.

The numbers of remaining cards are 52.

Ways of selecting a king from the deck = ${}^4C_1$

Ways of selecting the remaining 4 cards from 48 cards= ${}^{48}C_4$

Total number of selecting the 5 cards having one king always

$= {}^4C_1 \times {}^{48}C_4$

$$= \frac{4!}{1!\,3!} \times \frac{48!}{4!\,44!} = \frac{4 \times 3!}{3!} \times \frac{48 \times 47 \times 46 \times 45 \times 44!}{4 \times 3 \times 2 \times 1 \times 44!} = 778320$$

9. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Solution:

Given there are total 9 people

Women occupies even places that means they will be sitting on 2nd, 4th, 6th and 8th place where as men will be sitting on 1st, 3rd, 5th,7th and 9th place.
4 women can sit in four places and ways they can be seated= ${}^{4}P_{4}$

$$=\frac{4!}{(4-4)!}=\frac{4\times 3\times 2\times 1}{0!}=24$$

5 men can occupy 5 seats in 5 ways
The numbers of ways in which these can be seated = ${}^{5}P_{5}$

$$=\frac{5!}{(5-5)!}=\frac{5\times 4\times 3\times 2\times 1}{1}=120$$

The total numbers of sitting arrangements possible are
24 × 120 = 2880

10. From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?

Solution:
In this question we get 2 options that is
(i) Either all 3 will go
Then remaining students in class are: 25 – 3 = 22
Number of students remained to be chosen for party = 7
Number of ways choosing the remaining 22 students = ${}^{22}C_{7}$

$$=\frac{22!}{7!15!}=170544$$

(ii) None of them will go
The students going will be 10
Remaining students eligible for going = 22
Number of ways in which these 10 students can be selected are ${}^{22}C_{10}$

$$=\frac{22!}{10!\,12!}=646646$$

Total numbers of ways in which students can be chosen are
= 170544 + 646646 = 817190

11. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Solution:

In the given word ASSASSINATION, there are 4 'S'. Since all the 4 'S' have to be arranged together so let as take them as one unit.

The remaining letters are= 3 'A', 2 'I', 2 'N', T

The number of letters to be arranged are 9 (including 4 'S')

Using the formula $\frac{n!}{p_1!p_2!p_3!}$ where n is number of terms and p_1, p_2 p_3 are the number of times the repeating letters repeat themselves.

Here p_1= 3, p_2= 2, p_3 = 2

Putting the values in formula we get

$$\frac{10!}{3!\,2!\,2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 2 \times 2 \ \times 1 \times 1} = 151200$$