

EXERCISE 8.2

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Find the coefficient of

1. x⁵ in (x + 3)⁸

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here x^{5} is the T_{r+1} term so a = x, b = 3 and n = 8 $T_{r+1} = {}^{8}C_{r} x^{8-r} 3^{r}$(i) For finding out x^{5} We have to equate $x^{5} = x^{8-r}$ $\Rightarrow r = 3$ Putting value of r in (i) we get $T_{3+1} = {}^{8}C_{3} x^{8-3} 3^{3}$

$$T_4 = \frac{8!}{3! \, 5!} \times x^5 \times 27$$

= 1512 x⁵ Hence the coefficient of x⁵= 1512

2. $a^{5}b^{7}$ in $(a - 2b)^{12}$.

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here a = a, b = -2b & n = 12Substituting the values, we get $T_{r+1} = {}^{12}C_{r} a^{12-r} (-2b)^{r}$(i) To find a^{5} We equate $a^{12-r} = a^{5}$ r = 7Putting r = 7 in (i) $T_{8} = {}^{12}C_{7} a^{5} (-2b)^{7}$ $T_{8} = {}^{12!} \frac{12!}{7!5!} \times a^{5} \times (-2)^{7} b^{7}$ $= -101376 a^{5} b^{7}$ Hence the coefficient of $a^{5}b^{7}$ = -101376



Write the general term in the expansion of 3. $(x^2 - y)^6$

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ (i) Here $a = x^{2}$, n = 6 and b = -yPutting values in (i) $T_{r+1} = {}^{6}C_{r} x {}^{2(6-r)} (-1)^{r} y^{r}$ $= \frac{6!}{r! (6-r)!} \times x^{12-2r} \times (-1)^{r} \times y^{r}$ $= -1^{r} \frac{6!}{r! (6-r)!} \times x^{12-2r} \times y^{r}$ $= -1^{r} 6c_{r} .x^{12-2r} .y^{r}$

4. $(x^2 - y x)^{12}$, $x \neq 0$.

Solution:

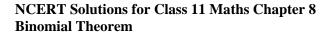
The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here n = 12, $a = x^{2}$ and b = -y xSubstituting the values we get $T_{n+1} = {}^{12}C_{r} \times x^{2(12-r)} (-1)^{r} y^{r} x^{r}$ $= \frac{12!}{r!(12-r)!} \times x^{24-2r} - 1^{r} y^{r} x^{r}$

 $= \frac{-1^{r}}{r!(12-r)!} x^{24-r} y^{r}$ $= -1^{r} x^{12} C_{r} x^{24-2r} y^{r}$

5. Find the 4th term in the expansion of $(x - 2y)^{12}$.

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here a= x, n =12, r= 3 and b = -2y By substituting the values we get $T_{4} = {}^{12}C_{3} x^{9} (-2y)^{3}$





$$= \frac{12!}{3!9!} \times x^{9} \times -8 \times y^{3}$$
$$= -\frac{12 \times 11 \times 10 \times 8}{3 \times 2 \times 1} \times x^{9} y^{3}$$
$$= -1760 x^{9} y^{3}$$

6. Find the 13th term in the expansion of

$$\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$$

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$

Here a=9x,
$$b = -\frac{1}{3\sqrt{x}} = -\frac{1}{3\sqrt{x}}$$
 n =18 and r = 12

Putting values

$$T_{13} = \frac{18!}{12! \, 6!} \, 9x^{18-12} \left(-\frac{1}{3\sqrt{x}} \right)^{12}$$
$$= \frac{(18 \times 17 \times 16 \times 15 \times 14 \times 13 \times 12!)}{12! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 3^{12} \times x^6 \times \frac{1}{x^6} \times \frac{1}{3^{12}}$$

= 18564

Find the middle terms in the expansions of

7.
$$\left(3-\frac{x^3}{6}\right)^{\prime}$$

Solution: Here n = 7 so there would be two middle terms given by

$$\left(\frac{n+1}{2}^{th}\right)$$
 term = 4 th and $\left(\frac{n+1}{2}+1\right)$ th term = 5th

We have

$$a = 3, n = 7 \text{ and } b = -\frac{x^3}{6}$$



For T₄, r= 3

The term will be

$$\begin{split} T_{r+1} &= {}^{n}C_{r} \; a^{n-r} \; b^{r} \\ T_{4} &= \frac{7!}{3!} \; 3^{4} \; \left(-\frac{x^{3}}{6}\right)^{3} \\ &= -\frac{7 \times 6 \times 5 \times 4}{3 \times 2 \times 1} \times 3^{4} \times \frac{x^{9}}{2^{3} 3^{3}} \\ &= -\frac{105}{8} \; x^{9} \end{split}$$

For T₅ term, r = 4

The term T_{r+1} in the binomial expansion is given by

$$T_{r+1} = {}^{n}C_{r} a^{n+r} b^{r}$$

$$T_{5} = \frac{7!}{4! \, 3!} \, 3^{3} \left(-\frac{x^{3}}{6}\right)^{4}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{4! \, 3!} \times \frac{3^{3}}{2^{4} 3^{4}} \times x^{3} = \frac{35 \, x^{12}}{48}$$

$$8. \left(\frac{x}{3} + 9y\right)^{10}$$

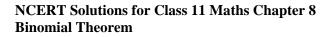


Solution:

Here n is even so the middle term will be given by $(\frac{n+1}{2})^{\text{th}}$ term = 6th term The general term T_{r+1} in the binomial expansion is given by T_{r+1} = ⁿC_r a^{n-r} b^r

Now
$$a = \frac{x}{3}$$
, $b = 9y$, $n = 10$ and $r = 5$

Substituting the values





$$T_{6} = \frac{10!}{5! \, 5!} \times \left(\frac{x}{3}\right)^{5} \times (9y)^{5}$$
$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{x^{5}}{3^{5}} \times 3^{10} \times y^{5}$$

9. In the expansion of $(1 + a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Solution:

= 61236 x⁵y⁵

We know that the general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here n = m + n, a = 1 and b = aSubstituting the values in the general form $T_{r+1} = {}^{m+n} C_r \ 1^{m+n-r} a^r$ $= {}^{m+n} C_r a^r$(i) Now we have that the general term for the expression is, $T_{r+1} = {}^{m+n} C_r a^r$ Now, For coefficient of a^m $T_{m+1} = {}^{m+n} C_m a^m$ Hence, for coefficient of a^m , value of r = mSo, the coefficient is $^{m+n}C_m$ Similarly, Coefficient of aⁿ is ^{m+n} C_n (m+n)! $^{m+n}C_m = m!n!$ (m+n)! And also, $^{m+n}C_{n} = \frac{m!n!}{m!n!}$

(m+n)!

The coefficient of a^m and aⁿ are same that is mini

10. The coefficients of the $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3 : 5. Find n and r.

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here the binomial is $(1+x)^{n}$ with a = 1, b = x and n = nThe $(r+1)^{th}$ term is given by $T_{(r+1)} = {}^{n}C_{r} 1^{n-r} x^{r}$



 $T_{(r+1)} = {}^{n}C_{r} x^{r}$ The coefficient of $(r+1)^{th}$ term is ${}^{n}C_{r}$ The rth term is given by (r-1)th term $T_{(r+1-1)} = {}^{n}C_{r-1} x^{r-1}$ $T_r = {}^{n}C_{r-1} x^{r-1}$ \therefore the coefficient of rth term is ⁿC_{r-1} For (r-1)th term we will take (r-2)th term $T_{r-2+1} = {}^{n}C_{r-2} x^{r-2}$ $T_{r-1} = {}^{n}C_{r-2} x^{r-2}$ \therefore the coefficient of (r-1)th term is ⁿC_{r-2} Given that the coefficient of $(r-1)^{th}$, r^{th} and $r+1^{th}$ term are in ratio 1:3:5 Therefore, $\frac{\text{the coefficient of } r - 1^{\text{th}} \text{ term}}{\text{coefficient of } r^{\text{th}} \text{ term}} = \frac{1}{3}$ $n_{\substack{C\\\frac{r-2}{n-c}\\r-1}}=\frac{1}{3}$ $\Rightarrow \frac{\frac{n!}{(r-2)!(n-r+2)!}}{n!} = \frac{1}{3}$ On rearranging we get $\frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{1}{3}$ By multiplying $\Rightarrow \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)!} = \frac{1}{3}$ $\Rightarrow \frac{(r-1)(n-r+1)!}{(n-r+2)(n-r+1)!} = \frac{1}{3}$ On simplifying we get



$$\Rightarrow \frac{(r-1)}{(n-r+2)} = \frac{1}{3}$$

 \Rightarrow 3r - 3 = n - r + 2

Also

 $\frac{\text{the coefficient of } r^{\text{th}} \text{ term}}{\text{coefficient of } r + 1^{\text{th}} \text{ term}} = \frac{3}{5}$

$$\xrightarrow{\frac{n!}{(r-1)!(n-r+1)!}}_{\Rightarrow \frac{n!}{r!(n-r)!}} = \frac{3}{5}$$

On rearranging we get

$$\xrightarrow{n!}_{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{3}{5}$$

By multiplying

 $\frac{r(r-1)!(n-r)!}{\Rightarrow (r-1)!(n-r+1)!} = \frac{3}{5}$ $\frac{r(n-r)!}{\Rightarrow (n-r+1)!} = \frac{3}{5}$ $\frac{r(n-r)!}{\Rightarrow (n-r+1)(n-r)!} = \frac{3}{5}$

On simplifying we get

$$\frac{r}{\Rightarrow (n-r+1)} = \frac{3}{5}$$
Also

 $\frac{\text{the coefficient of } r^{\text{th}} \text{ term}}{\text{coefficient of } r + 1^{\text{th}} \text{ term}} = \frac{3}{5}$

$$\xrightarrow{\frac{n!}{(r-1)!(n-r+1)!}}_{\overrightarrow{n!}} = \frac{3}{5}$$

On rearranging we get \Rightarrow 5r = 3n - 3r + 3 NCERT Solutions for Class 11 Maths Chapter 8 Binomial Theorem



⇒ 8r - 3n - 3 = 0....2We have 1 and 2 as $n - 4r \pm 5 = 0....1$ 8r - 3n - 3 = 0....2Multiplying equation 1 by number 2 2n - 8r + 10 = 0....3Adding equation 2 and 3 2n - 8r + 10 = 0-3n - 8r - 3 = 0⇒ -n = -7n = 7 and r = 3

11. Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ The general term for binomial $(1+x)^{2n}$ is $T_{r+1} = {}^{2n}C_r x^r \dots 1$ To find the coefficient of xⁿ r = n $T_{n+1} = {}^{2n}C_n x^n$ The coefficient of $x^n = {}^{2n}C_n$ The general term for binomial $(1+x)^{2n-1}$ is $T_{r+1} = {}^{2n-1}C_r x^r$ To find the coefficient of xⁿ Putting n = r $T_{r+1} = {}^{2n-1}C_r x^n$ The coefficient of $x^n = {}^{2n-1}C_n$ We have to prove Coefficient of x^n in $(1+x)^{2n} = 2$ coefficient of x^n in $(1+x)^{2n-1}$ Consider LHS = ${}^{2n}C_n$



$$=\frac{2n!}{n!(2n-n)!}$$

$$=\frac{2\pi}{n!(n)!}$$

Again consider RHS = $2 \times {}^{2n-1}C_n$

$$= 2 \times \frac{(2n-1)!}{n! (2n-1-n)!}$$

$$= 2 \times \frac{(2n-1)!}{n!(n-1)!}$$

Now multiplying and dividing by n we get

$$= 2 \times \frac{(2n-1)!}{n! (n-1)!} \times \frac{n}{n}$$
$$= \frac{2n(2n-1)!}{n! n(n-1)!}$$
$$= \frac{2n!}{n! n!}$$

From above equations LHS = RHS

Hence the proof.

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

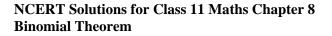
Solution:

The general term T_{r+1} in the binomial expansion is given by $T_{r+1} = {}^{n}C_{r} a^{n-r} b^{r}$ Here a = 1, b = x and n = mPutting the value $T_{r+1} = {}^{m}C_{r} 1^{m-r} x^{r}$ $= {}^{m}C_{r} x^{r}$ We need coefficient of x^{2} \therefore putting r = 2

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 $T_{2+1} = {}^{m}C_{2} x^{2}$ The coefficient of $x^{2} = {}^{m}C_{2}$ Given that coefficient of $x^{2} = {}^{m}C_{2} = 6$ $\Rightarrow \frac{m!}{2!(m-2)!} = 6$ $\Rightarrow \frac{m(m-1)(m-2)!}{2 \times 1 \times (m-2)!} = 6$ $\Rightarrow m (m - 1) = 12$ $\Rightarrow m^{2} - m - 12 = 0$ $\Rightarrow m^{2} - 4m + 3m - 12 = 0$ $\Rightarrow m (m - 4) + 3 (m - 4) = 0$ $\Rightarrow (m+3) (m - 4) = 0$ $\Rightarrow m = -3, 4$ We need positive value of m so m = 4