EXERCISE 10.2 PAGE NO: 219

In Exercises 1 to 8, find the equation of the line which satisfy the given conditions:

1. Write the equations for the x-and y-axes.
Solution:
The y-coordinate of every point on x-axis is 0.
∴ Equation of x-axis is y = 0.
The x-coordinate of every point on y-axis is 0.
∴ Equation of y-axis is x = 0.

2. Passing through the point (– 4, 3) with slope 1/2
Solution:
Given:
Point (-4, 3) and slope, m = 1/2
We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y – y_0 = m (x – x_0)$
So, $y – 3 = 1/2 (x – (-4))$
$y – 3 = 1/2 (x + 4)$
$2(y – 3) = x + 4$
$2y – 6 = x + 4$
$x + 4 – (2y – 6) = 0$
$x + 4 – 2y + 6 = 0$
$x – 2y + 10 = 0$
∴ The equation of the line is $x – 2y + 10 = 0$.

3. Passing through (0, 0) with slope m.
Solution:
Given:
Point (0, 0) and slope, m = m
We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y – y_0 = m (x – x_0)$
So, $y – 0 = m (x – 0)$
$y = mx$
$y – mx = 0$
∴ The equation of the line is $y – mx = 0$.

4. Passing through (2, $2\sqrt{3}$) and inclined with the x-axis at an angle of 75°.
Solution:

Given: point $(2, 2\sqrt{3})$ and $\theta = 75°$
Equation of line: $(y - y_1) = m (x - x_1)$
where, m = slope of line = $\tan \theta$ and (x_1, y_1) are the points through which line passes
$\therefore m = \tan 75°$
$75° = 45° + 30°$
Applying the formula:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° . \tan 30°} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\tan 75° = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Let us rationalizing we get,

$$\tan 75° = \frac{3 + 1 + 2\sqrt{3}}{3 - 1} = 2 + \sqrt{3}$$

We know that the point (x, y) lies on the line with slope m through the fixed point $(x_1, y1)$, if and only if, its coordinates satisfy the equation $y – y_1 = m (x – x_1)$
Then, $y – 2\sqrt{3} = (2 + \sqrt{3}) (x – 2)$
$y – 2\sqrt{3} = 2 x - 4 + \sqrt{3} x - 2 \sqrt{3}$
$y = 2 x - 4 + \sqrt{3} x$
$(2 + \sqrt{3}) x – y - 4 = 0$
$\therefore$ The equation of the line is $(2 + \sqrt{3}) x – y - 4 = 0$.

5. Intersecting the x-axis at a distance of 3 units to the left of origin with slope –2.
Solution:
Given:
Slope, m = -2
We know that if a line L with slope m makes x-intercept d, then equation of L is
$y = m(x − d)$.
If the distance is 3 units to the left of origin then d = -3
So, $y = (-2) (x – (-3))$
$y = (-2) (x + 3)$
$y = -2x – 6$
$2x + y + 6 = 0$

∴ The equation of the line is 2x + y + 6 = 0.

6. Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30º with positive direction of the x-axis.

Solution:

Given: θ = 30°

We know that slope, m = tan θ

m = tan30° = (1/√3)

We know that the point (x, y) on the line with slope m and y-intercept c lies on the line if and only if y = mx + c.

If distance is 2 units above the origin, c = +2

So, y = (1/√3)x + 2

y = (x + 2√3) / √3

√3 y = x + 2√3

x - √3 y + 2√3 = 0

∴ The equation of the line is x - √3 y + 2√3 = 0.

7. Passing through the points (–1, 1) and (2, – 4).

Solution:

Given:

Points (-1, 1) and (2, -4)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-4 - 1}{2 - (-1)}(x - (-1))$$

y – 1 = -5/3 (x + 1)

3 (y – 1) = (-5) (x + 1)

3y – 3 = -5x – 5

3y – 3 + 5x + 5 = 0

5x + 3y + 2 = 0

∴ The equation of the line is 5x + 3y + 2 = 0.

8. Perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is 30º.

Solution:

Given: p = 5 and ω = 30°

We know that the equation of the line having normal distance p from the origin and angle

ω which the normal makes with the positive direction of x-axis is given by x cos ω + y sin ω = p.
Substituting the values in the equation, we get
x cos30° + y sin30° = 5
x(√3 / 2) + y(1/2) = 5
√3 x + y = 5(2) = 10
√3 x + y – 10 = 0
∴ The equation of the line is √3 x + y – 10 = 0.

9. The vertices of ΔPQR are P (2, 1), Q (–2, 3) and R (4, 5). Find equation of the median through the vertex R.
Solution:
Given:
Vertices of ΔPQR i.e. P (2, 1), Q (-2, 3) and R (4, 5)
Let RL be the median of vertex R.
So, L is a midpoint of PQ.
We know that the midpoint formula is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.
∴ L = $\left(\frac{2-2}{2}, \frac{1+3}{2}\right)$ = (0, 2)
We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$
$\therefore y - 5 = \frac{2-5}{0-4}(x-4)$
y – 5 = -3/-4 (x-4)
(-4) (y – 5) = (-3) (x – 4)
-4y + 20 = -3x + 12
-4y + 20 + 3x – 12 = 0
3x – 4y + 8 = 0
∴ The equation of median through the vertex R is 3x – 4y + 8 = 0.

10. Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).
Solution:
Given:
Points are (2, 5) and (-3, 6).
We know that slope, m = $(y_2 – y_1)/(x_2 – x_1)$
= (6 - 5)/(-3 - 2)
= 1/-5 = -1/5

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.
Then, m = (-1/m)
= -1/(-1/5)
= 5

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y – y_0 = m (x – x_0)$
Then, y – 5 = 5(x – (-3))
y – 5 = 5x + 15
5x + 15 – y + 5 = 0
5x – y + 20 = 0
∴ The equation of the line is 5x – y + 20 = 0

11. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.
Solution:
We know that the coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$\left(\frac{1(2)+n(1)}{1+n}, \frac{1(3)+n(0)}{1+n}\right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

We know that slope, $m = (y_2 – y_1)/(x_2 – x_1)$
= (3 - 0)/(2 - 1)
= 3/1
= 3

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.
Then, m = (-1/m) = -1/3

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y – y_0 = m (x – x_0)$
Here, the point is

$$\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$$

$$\left(y - \frac{3}{1+n}\right) = \frac{-1}{3}\left(x - \frac{2+n}{1+n}\right)$$

3((1 + n) y – 3) = (-(1 + n) x + 2 + n)
3(1 + n) y – 9 = - (1 + n) x + 2 + n

$(1 + n) x + 3(1 + n) y – n – 9 – 2 = 0$
$(1 + n) x + 3(1 + n) y – n – 11 = 0$
∴ The equation of the line is $(1 + n) x + 3(1 + n) y – n – 11 = 0$.

12. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).
Solution:
Given: the line cuts off equal intercepts on the coordinate axes i.e. a = b.
We know that equation of the line intercepts a and b on x-and y-axis, respectively, which is
$x/a + y/b = 1$
So, $x/a + y/a = 1$
$x + y = a$ … (1)
Given: point (2, 3)
$2 + 3 = a$
$a = 5$
Substitute value of 'a' in (1), we get
$x + y = 5$
$x + y – 5 = 0$
∴ The equation of the line is $x + y – 5 = 0$.

13. Find equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.
Solution:
We know that equation of the line making intercepts a and b on x-and y-axis, respectively, is $x/a + y/b = 1$. … (1)
Given: sum of intercepts = 9
$a + b = 9$
$b = 9 - a$
Now, substitute value of b in the above equation, we get
$x/a + y/(9 - a) = 1$
Given: the line passes through the point (2, 2),
So, $2/a + 2/(9 - a) = 1$
$[2(9 - a) + 2a] / a(9 - a) = 1$
$[18 – 2a + 2a] / a(9 - a) = 1$
$18/a(9 - a) = 1$
$18 = a (9 – a)$
$18 = 9a – a^2$
$a^2 – 9a + 18 = 0$

Upon factorizing, we get
$a^2 – 3a – 6a + 18 = 0$
$a (a – 3) – 6 (a – 3) = 0$
$(a – 3) (a – 6) = 0$
$a = 3$ or $a = 6$
Let us substitute in (1),
Case 1 (a = 3):
Then $b = 9 – 3 = 6$
$x/3 + y/6 = 1$
$2x + y = 6$
$2x + y – 6 = 0$

Case 2 (a = 6):
Then $b = 9 – 6 = 3$
$x/6 + y/3 = 1$
$x + 2y = 6$
$x + 2y – 6 = 0$
∴ The equation of the line is $2x + y – 6 = 0$ or $x + 2y – 6 = 0$.

14. Find equation of the line through the point (0, 2) making an angle 2π/3 with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.
Solution:
Given:
Point (0, 2) and $\theta = 2\pi/3$
We know that $m = \tan \theta$
$m = \tan (2\pi/3) = -\sqrt{3}$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y – y_0 = m (x – x_0)$
$y – 2 = -\sqrt{3} (x – 0)$
$y – 2 = -\sqrt{3} x$
$\sqrt{3} x + y – 2 = 0$

Given, equation of line parallel to above obtained equation crosses the y-axis at a distance of 2 units below the origin.
So, the point = (0, -2) and $m = -\sqrt{3}$

From point slope form equation,

$y - (-2) = -\sqrt{3}\,(x - 0)$
$y + 2 = -\sqrt{3}\,x$
$\sqrt{3}\,x + y + 2 = 0$
∴ The equation of line is $\sqrt{3}\,x + y - 2 = 0$ and the line parallel to it is $\sqrt{3}\,x + y + 2 = 0$.

15. The perpendicular from the origin to a line meets it at the point (–2, 9), find the equation of the line.
Solution:
Given:
Points are origin (0, 0) and (-2, 9).
We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$
$= (9 - 0)/(-2-0)$
$= -9/2$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.
$m = (-1/m) = -1/(-9/2) = 2/9$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m\,(x - x_0)$
$y - 9 = (2/9)\,(x - (-2))$
$9(y - 9) = 2(x + 2)$
$9y - 81 = 2x + 4$
$2x + 4 - 9y + 81 = 0$
$2x - 9y + 85 = 0$
∴ The equation of line is $2x - 9y + 85 = 0$.

16. The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L= 125.134 when C = 110, express L in terms of C.
Solution:
Let us assume 'L' along X-axis and 'C' along Y-axis, we have two points (124.942, 20) and (125.134, 110) in XY-plane.
We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$C - 20 = \frac{110 - 20}{125.134 - 124.942}(L - 124.942)$$

$C - 20 = \frac{90}{0.192}(L - 124.942)$

$0.192\,(C - 20) = 90\,(L - 124.942)$

$L = \frac{0.192}{90}(C - 20) + 124.942$

$\therefore$ The required relation is $L = \frac{0.192}{90}(C - 20) + 124.942$

17. The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs. 14/litre and 1220 litres of milk each week at Rs. 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs. 17/litre?

Solution:

Assuming the relationship between selling price and demand is linear.

Let us assume selling price per litre along X-axis and demand along Y-axis, we have two points (14, 980) and (16, 1220) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$

$y - 980 = \frac{240}{2}(x - 14)$

$y - 980 = 120\,(x - 14)$

$y = 120\,(x - 14) + 980$

When x = Rs 17/litre,

$y = 120\,(17 - 14) + 980$

$y = 120(3) + 980$

$y = 360 + 980 = 1340$

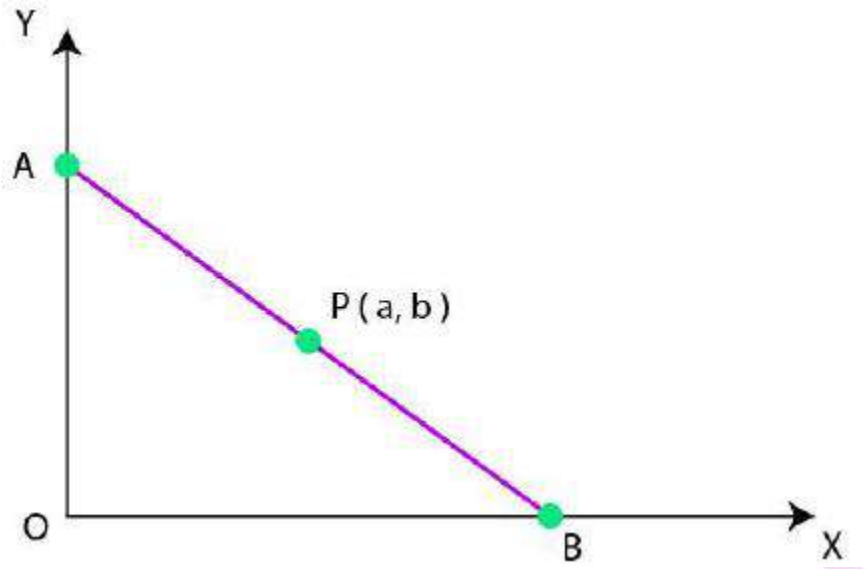
$\therefore$ The owner can sell 1340 litres weekly at Rs. 17/litre.

18. P (a, b) is the mid-point of a line segment between axes. Show that equation of the line is x/a + y/b = 2

Solution:

Let AB be a line segment whose midpoint is P (a, b).

Let the coordinates of A and B be (0, y) and (x, 0) respectively.



We know that the midpoint is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Since P is the midpoint of (a, b),

$$\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = (a, b)$$

$$\left(\frac{x}{2}, \frac{y}{2}\right) = (a, b)$$

$a = x/2$ and $b = y/2$

$x = 2a$ and $y = 2b$

A = (0, 2b) and B = (2a, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2b = \frac{0 - 2b}{2a - 0}(x - 0)$$

$$y - 2b = \frac{-2b}{2a}(x)$$

$$y - 2b = \frac{-b}{a}(x)$$

$a (y – 2b) = -bx$

$ay – 2ab = -bx$

$bx + ay = 2ab$

Divide both the sides with ab, then

$\frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$

$\frac{x}{a}+\frac{y}{b}=2$

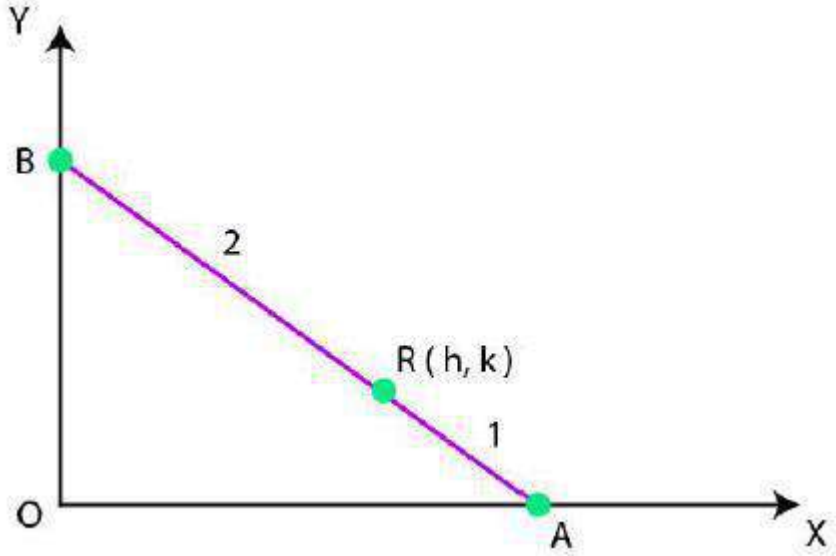
Hence proved.

19. Point R (h, k) divides a line segment between the axes in the ratio 1: 2. Find the equation of the line.

Solution:

Let us consider, AB be the line segment such that r (h, k) divides it in the ratio 1: 2.

So the coordinates of A and B be (0, y) and (x, 0) respectively.



We know that the coordinates of a point dividing the line segment joins the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n is

$(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n})$

$(\frac{1(0)+2(x)}{1+2},\frac{1(y)+2(0)}{1+2})=(h,k)$

$(\frac{2x}{3},\frac{y}{3})=(h,k)$

h = 2x/3 and k = y/3

x = 3h/2 and y = 3k

∴ A = (0, 3k) and B = (3h/2, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-3k=\frac{0-3k}{\frac{3h}{2}-0}(x-0)$

3h(y - 3k) = -6kx
3hy - 9hk = -6kx
6kx + 3hy = 9hk
Let us divide both the sides by 9hk, we get,
2x/3h + y/3k = 1
∴ The equation of the line is given by 2x/3h + y/3k = 1

20. By using the concept of equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear.
Solution:
According to the question,
If we have to prove that the given three points (3, 0), (– 2, – 2) and (8, 2) are collinear, then we have to also prove that the line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

By using the formula,
The equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{-2 - 0}{-2 - 3}(x - 3)$$

$$y = \frac{-2}{-5}(x - 3)$$

-5y = -2 (x – 3)
-5y = -2x + 6
2x – 5y = 6
If 2x – 5y = 6 passes through (8, 2),
2x – 5y = 2(8) – 5(2)
= 16 – 10
= 6
= RHS
The line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).
Hence proved. The given three points are collinear.