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1. Reduce the following equations into slope - intercept form and find their slopes and the y - intercepts.
(i) $x + 7y = 0$
(ii) $6x + 3y – 5 = 0$
(iii) $y = 0$
Solution:
(i) $x + 7y = 0$
Given:
The equation is $x + 7y = 0$
Slope – intercept form is represented in the form ‘$y = mx + c$’, where m is the slope and c is the y intercept
So, the above equation can be expressed as
$y = -1/7x + 0$
∴ The above equation is of the form $y = mx + c$, where $m = -1/7$ and $c = 0$.

(ii) $6x + 3y – 5 = 0$
Given:
The equation is $6x + 3y – 5 = 0$
Slope – intercept form is represented in the form ‘$y = mx + c$’, where m is the slope and c is the y intercept
So, the above equation can be expressed as
$3y = -6x + 5$
$y = -6/3x + 5/3$
$= -2x + 5/3$
∴ The above equation is of the form $y = mx + c$, where $m = -2$ and $c = 5/3$.

(iii) $y = 0$
Given:
The equation is $y = 0$
Slope – intercept form is given by ‘$y = mx + c$’, where m is the slope and c is the y intercept
$y = 0 \times x + 0$
∴ The above equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.

2. Reduce the following equations into intercept form and find their intercepts on the axes.
(i) $3x + 2y – 12 = 0$

(ii) 4x – 3y = 6
(iii) 3y + 2 = 0
Solution:
(i) 3x + 2y – 12 = 0
Given:
The equation is 3x + 2y – 12 = 0
Equation of line in intercept form is given by x/a + y/b = 1, where 'a' and 'b' are intercepts on x axis and y – axis respectively.
So, 3x + 2y = 12
now let us divide both sides by 12, we get
3x/12 + 2y/12 = 12/12
x/4 + y/6 = 1
∴ The above equation is of the form x/a + y/b = 1, where a = 4, b = 6
Intercept on x – axis is 4
Intercept on y – axis is 6

(ii) 4x – 3y = 6
Given:
The equation is 4x – 3y = 6
Equation of line in intercept form is given by x/a + y/b = 1, where 'a' and 'b' are intercepts on x axis and y – axis respectively.
So, 4x – 3y = 6
Now let us divide both sides by 6, we get
4x/6 – 3y/6 = 6/6
2x/3 – y/2 = 1
x/(3/2) + y/(-2) = 1
∴ The above equation is of the form x/a + y/b = 1, where a = 3/2, b = -2
Intercept on x – axis is 3/2
Intercept on y – axis is -2

(iii) 3y + 2 = 0
Given:
The equation is 3y + 2 = 0
Equation of line in intercept form is given by x/a + y/b = 1, where 'a' and 'b' are intercepts on x axis and y – axis respectively.
So, 3y = -2
Now, let us divide both sides by -2, we get
3y/-2 = -2/-2
3y/-2 = 1

$y/(-2/3) = 1$
∴ The above equation is of the form $x/a + y/b = 1$, where $a = 0$, $b = -2/3$
Intercept on x – axis is 0
Intercept on y – axis is -2/3

3. Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.
(i) $x - \sqrt{3}y + 8 = 0$
(ii) $y - 2 = 0$
(iii) $x - y = 4$
Solution:
(i) $x - \sqrt{3}y + 8 = 0$
Given:
The equation is $x - \sqrt{3}y + 8 = 0$
Equation of line in normal form is given by $x \cos \theta + y \sin \theta = p$ where 'θ' is the angle between perpendicular and positive x axis and 'p' is perpendicular distance from origin.
So now, $x - \sqrt{3}y + 8 = 0$
$x - \sqrt{3}y = -8$
Divide both the sides by $\sqrt{(1^2 + (\sqrt{3})^2)} = \sqrt{(1 + 3)} = \sqrt{4} = 2$
$x/2 - \sqrt{3}y/2 = -8/2$
$(-1/2)x + \sqrt{3}/2y = 4$
This is in the form of: $x \cos 120^o + y \sin 120^o = 4$
∴ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 120°$ and $p = 4$.
Perpendicular distance of line from origin = 4
Angle between perpendicular and positive x – axis = 120°

(ii) $y - 2 = 0$
Given:
The equation is $y - 2 = 0$
Equation of line in normal form is given by $x \cos \theta + y \sin \theta = p$ where 'θ' is the angle between perpendicular and positive x axis and 'p' is perpendicular distance from origin.
So now, $0 \times x + 1 \times y = 2$
Divide both sides by $\sqrt{(0^2 + 1^2)} = \sqrt{1} = 1$
$0 (x) + 1 (y) = 2$
This is in the form of: $x \cos 90^o + y \sin 90^o = 2$
∴ The above equation is of the form $x \cos \theta + y \sin \theta = p$, where $\theta = 90°$ and $p = 2$.
Perpendicular distance of line from origin = 2
Angle between perpendicular and positive x – axis = 90°

(iii) x – y = 4
Given:
The equation is x – y + 4 = 0
Equation of line in normal form is given by x cos θ + y sin θ = p where 'θ' is the angle between perpendicular and positive x axis and 'p' is perpendicular distance from origin.
So now, x – y = 4
Divide both the sides by $\sqrt{(1^2 + 1^2)} = \sqrt{(1+1)} = \sqrt{2}$
$x/\sqrt{2} - y/\sqrt{2} = 4/\sqrt{2}$
$1/\sqrt{2}x + (-1/\sqrt{2})y = 2\sqrt{2}$
This is in the form: $x \cos 315^o + y \sin 315^o = 2\sqrt{2}$
∴ The above equation is of the form x cos θ + y sin θ = p, where θ = 315° and $p = 2\sqrt{2}$.
Perpendicular distance of line from origin = $2\sqrt{2}$
Angle between perpendicular and positive x – axis = 315°

4. Find the distance of the point (–1, 1) from the line 12(x + 6) = 5(y – 2).
Solution:
Given:
The equation of the line is 12(x + 6) = 5(y – 2).
12x + 72 = 5y – 10
12x – 5y + 82 = 0 … (1)
Now, compare equation (1) with general equation of line Ax + By + C = 0, where A = 12, B = –5, and C = 82
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Given point $(x_1, y_1) = (-1, 1)$
∴ Distance of point (–1, 1) from the given line is

$$d = \frac{|12 \times (-1) + (-5) \times 1 + 82|}{\sqrt{12^2 + (-5)^2}} = \frac{|-12 - 5 + 82|}{\sqrt{144 + 25}} = \frac{|65|}{\sqrt{169}} = \frac{65}{13} \text{units}$$

= 5 units
∴ The distance is 5units.

5. Find the points on the x-axis, whose distances from the line x/3 + y/4 = 1 are 4 units.
Solution:
Given:
The equation of line is x/3 + y/4 = 1
4x + 3y = 12

4x + 3y – 12 = 0 …. (1)
Now, compare equation (1) with general equation of line Ax + By + C = 0, where A = 4, B = 3, and C = -12
Let (a, 0) be the point on the x-axis, whose distance from the given line is 4 units.
So, the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$4 = \frac{|4a - 12|}{\sqrt{16 + 9}} = \frac{|4a - 12|}{5}$$

|4a – 12| = 4 × 5
± (4a – 12) = 20
4a – 12 = 20 or - (4a – 12) = 20
4a = 20 + 12 or 4a = -20 + 12
a = 32/4 or a = -8/4
a = 8 or a = -2
∴ The required points on the x – axis are (-2, 0) and (8, 0)

6. Find the distance between parallel lines
(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0
(ii) l(x + y) + p = 0 and l (x + y) – r = 0
Solution:
(i) 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0
Given:
The parallel lines are 15x + 8y – 34 = 0 and 15x + 8y + 31 = 0.
By using the formula,
The distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Where, A = 15, B = 8, $C_1 = -34$, $C_2 = 3$
Distance between parallel lines is

$$d = \frac{|-34 - 31|}{\sqrt{15^2 + 8^2}}$$

$$= \frac{|-65|}{\sqrt{225 + 64}}$$

= 65/√289
= 65/17
∴ The distance between parallel lines is 65/17

(ii) l(x + y) + p = 0 and l (x + y) – r = 0
Given:
The parallel lines are l (x + y) + p = 0 and l (x + y) – r = 0.
lx + ly + p = 0 and lx + ly – r = 0
by using the formula,
The distance (d) between parallel lines Ax + By + C_1 = 0 and Ax + By + C_2 = 0 is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Where, A = l, B = l, C_1 = p, C_2 = -r
Distance between parallel lines is

$$d = \frac{|p - (-r)|}{\sqrt{l^2 + l^2}}$$

$$= \frac{|p + r|}{\sqrt{2}l}$$

$$= \frac{|p+r|}{l\sqrt{2}}$$

∴ The distance between parallel lines is |p+q|/l√2

7. Find equation of the line parallel to the line 3x − 4y + 2 = 0 and passing through the point (–2, 3).
Solution:
Given:
The line is 3x – 4y + 2 = 0
So, y = 3x/4 + 2/4
= 3x/4 + ½
Which is of the form y = mx + c, where m is the slope of the given line.
The slope of the given line is 3/4
We know that parallel line have same slope.
∴ Slope of other line = m = 3/4
Equation of line having slope m and passing through (x_1, y_1) is given by
y – y_1 = m (x – x_1)
∴ Equation of line having slope 3/4 and passing through (-2, 3) is
y – 3 = ¾ (x – (-2))
4y – 3 × 4 = 3x + 3 × 2

3x – 4y = 18

∴ The equation is 3x – 4y = 18

8. Find equation of the line perpendicular to the line x – 7y + 5 = 0 and having x intercept 3.

Solution:

Given:

The equation of line is x – 7y + 5 = 0

So, y = 1/7x + 5/7 [which is of the form y = mx + c, where m is the slope of the given line.]

Slope of the given line is 1/7

Slope of the line perpendicular to the line having slope m is -1/m

Slope of the line perpendicular to the line having a slope of 1/7 is -1/(1/7) = -7

So, the equation of line with slope -7 and x intercept 3 is given by y = m(x – d)

y = -7 (x – 3)

y = -7x + 21

7x + y = 21

∴ The equation is 7x + y = 21

9. Find angles between the lines √3x + y = 1 and x + √3y = 1.

Solution:

Given:

The lines are √3x + y = 1 and x + √3y = 1

So, y = -√3x + 1 … (1) and

y = -1/√3x + 1/√3 …. (2)

Slope of line (1) is m_1 = -√3, while the slope of line (2) is m_2 = -1/√3

Let θ be the angle between two lines

So,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\sqrt{3} - \left(-\frac{1}{\sqrt{3}}\right)}{1 + \left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right| = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$$

$= 1/\sqrt{3}$

θ = 30°

∴ The angle between the given lines is either 30° or 180°- 30° = 150°

10. The line through the points (h, 3) and (4, 1) intersects the line 7x − 9y −19 = 0. At

right angle. Find the value of h.
Solution:
Let the slope of the line passing through (h, 3) and (4, 1) be m_1
Then, $m_1 = (1-3)/(4-h) = -2/(4-h)$
Let the slope of line $7x – 9y – 19 = 0$ be m_2
$7x – 9y – 19 = 0$
So, $y = 7/9x – 19/9$
$m_2 = 7/9$
Since, the given lines are perpendicular
$m_1 \times m_2 = -1$
$-2/(4-h) \times 7/9 = -1$
$-14/(36-9h) = -1$
$-14 = -1 \times (36 – 9h)$
$36 – 9h = 14$
$9h = 36 – 14$
$h = 22/9$
∴ The value of h is 22/9

11. Prove that the line through the point (x_1, y_1) and parallel to the line Ax + By + C = 0 is A (x – x_1) + B (y – y_1) = 0.
Solution:
Let the slope of line $Ax + By + C = 0$ be m
$Ax + By + C = 0$
So, $y = -A/Bx – C/B$
$m = -A/B$
By using the formula,
Equation of the line passing through point (x_1, y_1) and having slope $m = -A/B$ is
$y – y_1 = m (x – x_1)$
$= -A/B (x – x_1)$
$B (y – y_1) = -A (x – x_1)$
∴ $A(x – x_1) + B(y – y_1) = 0$
So, the line through point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A (x – x_1) + B (y – y_1) = 0$
Hence proved.

12. Two lines passing through the point (2, 3) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.
Solution:
Given: $m_1 = 2$

Let the slope of the first line be m_1
And let the slope of the other line be m_2.
Angle between the two lines is 60°.
So,

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 60° = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

i.e.,

$$\sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\sqrt{3}\,(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}\,(1 + 2m_2) = -(2 - m_2)$$

$$\sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$m_2(2\sqrt{3} + 1) = 2 - \sqrt{3} \text{ or } m_2(2\sqrt{3} - 1) = -(2 + \sqrt{3})$$

$$m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

So now let us consider

Case 1: When

$$m_2 = \frac{2-\sqrt{3}}{(2\sqrt{3}+1)}$$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y - 3 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right)(x - 2)$$

$$(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$$

∴ Equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = 8\sqrt{3} - 1$

Case 2: When

$$m_2 = \frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}$$

The equation of the line passing through point (2, 3) and having a slope m_2 is

$$y-3=\frac{-(2+\sqrt{3})}{(2\sqrt{3}-1)}(x-2)$$

$$(2\sqrt{3}-1)y-3(2\sqrt{3}-1)=-(2+\sqrt{3})x+2(2+\sqrt{3})$$

$$(2\sqrt{3}-1)y+(2+\sqrt{3})x=4+2\sqrt{3}+6\sqrt{3}-3$$

$$(2\sqrt{3}-1)y+(2+\sqrt{3})x=8\sqrt{3}+1$$

∴ Equation of the other line is $(2\sqrt{3}-1)y+(2+\sqrt{3})x=8\sqrt{3}+1$

13. Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

Solution:

Given:

The right bisector of a line segment bisects the line segment at 90°.

End-points of the line segment AB are given as A (3, 4) and B (–1, 2).

Let mid-point of AB be (x, y)

x = (3+4)y/2

y = (-1+2)/2

(x, y) = (7/2, 1/2)

Let the slope of line AB be m_1

m_1 = (2 – 4)/(-1 – 3)

= -2/(-4)

= 1/2

And let the slope of the line perpendicular to AB be m_2

m_2 = -1/(1/2)

= -2

The equation of the line passing through (1, 3) and having a slope of –2 is

(y – 3) = -2 (x – 1)

y – 3 = - 2x + 2

2x + y = 5

∴ The required equation of the line is 2x + y = 5

14. Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line 3x – 4y – 16 = 0.

Solution:

Let us consider the co-ordinates of the foot of the perpendicular from (-1, 3) to the line 3x – 4y – 16 = 0 be (a, b)

So, let the slope of the line joining (-1, 3) and (a, b) be m_1
$m_1 = (b-3)/(a+1)$

And let the slope of the line 3x – 4y – 16 = 0 be m_2
y = 3/4x – 4
$m_2 = 3/4$
Since these two lines are perpendicular, $m_1 \times m_2 = -1$
(b-3)/(a+1) × (3/4) = -1
(3b-9)/(4a+4) = -1
3b – 9 = -4a – 4
4a + 3b = 5 …….(1)
Point (a, b) lies on the line 3x – 4y = 16
3a – 4b = 16 ……..(2)
Solving equations (1) and (2), we get
a = 68/25 and b = -49/25
∴ The co-ordinates of the foot of perpendicular is (68/25, -49/25)

15. The perpendicular from the origin to the line y = mx + c meets it at the point (–1, 2). Find the values of m and c.
Solution:
Given:
The perpendicular from the origin meets the given line at (–1, 2).
The equation of line is y = mx + c
The line joining the points (0, 0) and (–1, 2) is perpendicular to the given line.
So, the slope of the line joining (0, 0) and (–1, 2) = 2/(-1) = -2
Slope of the given line is m.
m × (-2) = -1
m = 1/2
Since, point (-1, 2) lies on the given line,
y = mx + c
2 = 1/2 × (-1) + c
c = 2 + 1/2 = 5/2
∴ The values of m and c are 1/2 and 5/2 respectively.

16. If p and q are the lengths of perpendiculars from the origin to the lines x cos θ − y sin θ = k cos 2θ and x sec θ + y cosec θ = k, respectively, prove that $p^2 + 4q^2 = k^2$
Solution:
Given:
The equations of given lines are

x cos θ – y sin θ = k cos 2θ …………………… (1)
x sec θ + y cosec θ = k ………………….. (2)
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

So now, compare equation (1) to the general equation of line i.e., Ax + By + C = 0, we get
A = cos θ, B = -sin θ, and C = -k cos 2θ
It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$p = \frac{|A\times0+B\times0+C|}{\sqrt{A^2+B^2}} = \frac{|C|}{\sqrt{A^2+B^2}} = \frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}} = k\cos 2\theta$$

p = k cos 2θ
Let us square on both sides we get,
$P^2 = k^2 \cos^2 2\theta$ ………………… (3)
Now, compare equation (2) to the general equation of line i.e., Ax + By + C = 0, we get
A = sec θ, B = cosec θ, and C = -k
It is given that q is the length of the perpendicular from (0, 0) to line (2)

$$q = \frac{|A \times 0 + B \times 0 + C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|-k|}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}$$

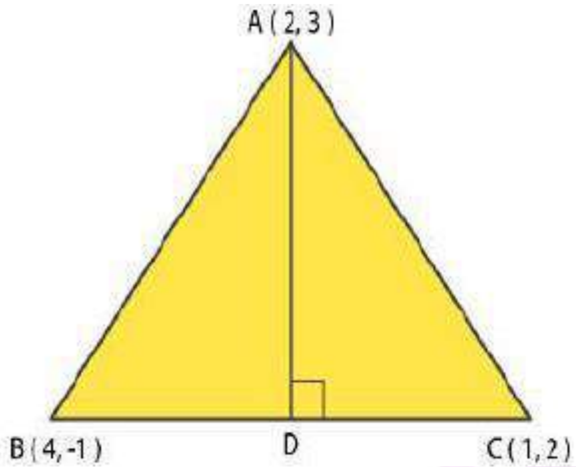
$$= \frac{k}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}$$

$$= \frac{k}{\sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}}} = \frac{k\cos\theta\sin\theta}{\sqrt{\cos^2\theta + \sin^2\theta}} = k\cos\theta\sin\theta$$

q = k cos θ sin θ
Multiply both sides by 2, we get
2q = 2k cos θ sin θ = k × 2sin θ cos θ
2q = k sin 2θ
Squaring both sides, we get
$4q^2 = k^2 \sin^2 2\theta$ …………………(4)

Now add (3) and (4) we get
$p^2 + 4q^2 = k^2 \cos^2 2\theta + k^2 \sin^2 2\theta$
$p^2 + 4q^2 = k^2 (\cos^2 2\theta + \sin^2 2\theta)$ [Since, $\cos^2 2\theta + \sin^2 2\theta = 1$]
$\therefore p^2 + 4q^2 = k^2$
Hence proved.

17. In the triangle ABC with vertices A (2, 3), B (4, –1) and C (1, 2), find the equation and length of altitude from the vertex A.
Solution:



Let AD be the altitude of triangle ABC from vertex A.
So, AD is perpendicular to BC
Given:
Vertices A (2, 3), B (4, –1) and C (1, 2)
Let slope of line BC = m_1
$m_1 = (- 1 - 2)/(4 - 1)$
$m_1 = -1$
Let slope of line AD be m_2
AD is perpendicular to BC
$m_1 \times m_2 = -1$
$-1 \times m_2 = -1$
$m_2 = 1$
The equation of the line passing through point (2, 3) and having a slope of 1 is
$y – 3 = 1 \times (x – 2)$
$y – 3 = x – 2$
$y – x = 1$
Equation of the altitude from vertex A = $y – x = 1$
Length of AD = Length of the perpendicular from A (2, 3) to BC
Equation of BC is

y + 1 = -1 × (x – 4)
y + 1 = -x + 4
x + y – 3 = 0 …………………(1)
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now compare equation (1) to the general equation of line i.e., Ax + By + C = 0, we get

Length of AD = $\frac{|1\times2+1\times3-3|}{\sqrt{1^2+1^2}} = \frac{|2|}{\sqrt{2}} = \sqrt{2}$ units

[where, A = 1, B = 1 and C = -3]
∴ The equation and the length of the altitude from vertex A are y – x = 1 and √2 units respectively.

18. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $1/p^2 = 1/a^2 + 1/b^2$
Solution:
Equation of a line whose intercepts on the axes are a and b is x/a + y/b = 1
bx + ay = ab
bx + ay – ab = 0 ……………….(1)
Perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Now compare equation (1) to the general equation of line i.e., Ax + By + C = 0, we get
A = b, B = a and C = -ab
If p is the length of the perpendicular from point (x_1, y_1) = (0, 0) to line (1), we get

$$p = \frac{|A \times 0 + B \times 0 - ab|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-ab|}{\sqrt{a^2+b^2}}$$

Now square on both the sides we get

$$p^2 = \frac{(-ab)^2}{a^2 + b^2}$$

$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2}$$

$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$$

∴ $1/p^2 = 1/a^2 + 1/b^2$

Hence proved.