EXERCISE 13.1

1. Evaluate the Given limit: $\lim_{x\to 3} x+3$

Solution:

Given

$\lim_{x\to 3} x+3$

Substituting x = 3, we get

= 3 + 3

= 6

2. Evaluate the Given limit: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Solution:

Given limit: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Substituting x = π, we get

$\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$ = (π – 22 / 7)

3. Evaluate the Given limit: $\lim_{r\to 1}\pi r^2$

Solution:

Given limit: $\lim_{r\to 1}\pi r^2$

Substituting r = 1, we get

$\lim_{r\to 1}\pi r^2$ = $\pi(1)^2$

= π

4. Evaluate the Given limit: $\lim_{x\to 4}\frac{4x+3}{x-2}$

Solution:

Given limit: $\lim_{x\to 4}\frac{4x+3}{x-2}$

Substituting x = 4, we get

$\lim_{x\to 4}\frac{4x+3}{x-2}$ = [4(4) + 3] / (4 – 2)

= (16 + 3) / 2

= 19 / 2

5. Evaluate the Given limit: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Solution:

Given limit: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Substituting x = -1, we get

$\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

= $[(-1)^{10} + (-1)^5 + 1] / (-1 – 1)$

= (1 – 1 + 1) / - 2

= - 1 / 2

6. Evaluate the Given limit: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Solution:

Given limit: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

= $[(0 + 1)^5 – 1] / 0$

=0

Since, this limit is undefined

Substitute x + 1 = y, then x = y – 1

$\lim_{y\to 1}\frac{(y)^5-1}{y-1}$

= $\lim_{y\to 1}\frac{(y)^5-1^5}{y-1}$

We know that,

$\lim_{x\to a}\frac{x^n-a^n}{x-a}$ = na^{n-1}

Hence,

$\lim_{y\to 1}\frac{(y)^5-1^5}{y-1}$

=$5(1)^{5-1}$

= $5(1)^4$

= 5

7. Evaluate the Given limit: $\lim_{x\to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

Solution:

By evaluating the limit at x = 2, we get

$\lim_{x\to 2} \frac{3x^2-x-10}{x^2-4} = [3(2)^2 - x - 10] / 4 - 4$

= 0

Now, by factorising numerator, we get

$\lim_{x\to 2} \frac{3x^2-x-10}{x^2-4} = \lim_{x\to 2} \frac{3x^2-6x+5x-10}{x^2-2^2}$

We know that,

$a^2 - b^2 = (a - b)(a + b)$

$= \lim_{x\to 2} \frac{(x-2)(3x+5)}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{(3x+5)}{(x+2)}$

By substituting x = 2, we get,

= [3(2) + 5] / (2 + 2)

= 11 / 4

8. Evaluate the Given limit: $\lim_{x\to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

Solution:

First substitute x = 3 in the given limit, we get

$$= \lim_{x \to 3} \frac{(3)^4 - 81}{2(3)^2 - 5 \times 3 - 3}$$

= (81 – 81) / (18 – 18)
= 0 / 0
Since the limit is of the form 0 / 0, we need to factorise the numerator and denominator

$$\lim_{x \to 3} \frac{(x^2 - 9)(x^2 + 9)}{2x^2 - 6x + x - 3} = \lim_{x \to 3} \frac{(x - 3)(x + 3)(x^2 + 9)}{(2x + 1)(x - 3)}$$

$$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = \lim_{x \to 3} \frac{(x + 3)(x^2 + 9)}{(2x + 1)}$$

Now substituting x = 3, we get

$$= \frac{(3 + 3)(3^2 + 9)}{(2 \times 3 + 1)}$$

= 108 / 7
Hence,

$$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = 108 / 7$$

9. Evaluate the Given limit: $\lim_{x \to 0} \frac{ax + b}{cx + 1}$
Solution:

$$\lim_{x \to 0} \frac{ax + b}{cx + 1}$$

= [a (0) + b] / c (0) + 1
= b / 1
= b

10. Evaluate the Given limit: $\lim_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$
Solution:

$\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$ = (1 – 1) / (1 – 1)

= 0

Let the value of $z^{1/6}$ be x

$(z^{1/6})^2 = x^2$

$z^{1/3} = x^2$

Now, substituting $z^{1/3} = x^2$ we get

$\lim_{x\to 1}\frac{x^2-1}{x-1} = \frac{x^2-1^2}{x-1}$

We know that,

$\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$

$\lim_{x\to 1}\frac{x^2-1^2}{x-1} = 2\,(1)^{2-1}$

= 2

11. Evaluate the Given limit: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Solution:

Given limit: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

Substituting x = 1

$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}$

= $[a(1)^2 + b(1) + c] / [c(1)^2 + b(1) + a]$

= (a + b + c) / (a + b + c)

Given

$[a+b+c\neq 0]$

= 1

12. Evaluate the Given limit: $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Solution:

By substituting x = - 2, we get

$$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2} = 0 / 0$$

Now,

$$\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2} = \frac{\frac{2+x}{2x}}{x+2}$$

= 1 / 2x

= 1 / 2(-2)

= - 1 / 4

13. Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax}{bx}$

Solution:

Given $\lim_{x\to 0}\frac{\sin ax}{bx}$

Formula used here

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

By applying the limits in the given expression

$$\lim_{x\to 0}\frac{\sin ax}{bx} = \frac{0}{0}$$

By multiplying and dividing by 'a' in the given expression, we get

$$\lim_{x\to 0}\frac{\sin ax}{bx}\times\frac{a}{a}$$

We get,

$$\lim_{x\to 0}\frac{\sin ax}{ax}\times\frac{a}{b}$$

We know that,

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$= \frac{a}{b}\lim_{ax\to 0}\frac{\sin ax}{ax} = \frac{a}{b}\times 1$$

= a / b

14. Evaluate the given limit: $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

Solution:

$\lim_{x \to 0} \frac{\sin ax}{\sin bx} = 0 / 0$

By multiplying ax and bx in numerator and denominator, we get

$\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\frac{\sin ax}{ax} \times ax}{\frac{\sin bx}{bx} \times bx}$

Now, we get

$\frac{a}{b} \frac{\lim_{ax \to 0} \frac{\sin ax}{ax}}{\lim_{bx \to 0} \frac{\sin bx}{bx}}$

We know that,

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

Hence, a / b × 1

= a / b

15. Evaluate the given limit: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Solution:

$$\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$$

$$\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}=\lim_{\pi-x\to0}\frac{\sin(\pi-x)}{(\pi-x)}\times\frac{1}{\pi}$$

$$=\frac{1}{\pi}\lim_{\pi-x\to0}\frac{\sin(\pi-x)}{(\pi-x)}$$

We know that

$$\lim_{x\to0}\frac{\sin x}{x}=1$$

$$\frac{1}{\pi}\lim_{\pi-x\to0}\frac{\sin(\pi-x)}{(\pi-x)}=\frac{1}{\pi}\times1$$

$= 1/\pi$

16. Evaluate the given limit: $\lim_{x\to0}\frac{\cos x}{\pi-x}$

Solution:

$$\lim_{x\to0}\frac{\cos x}{\pi-x}=\frac{\cos 0}{\pi-0}$$

$= 1/\pi$

17. Evaluate the given limit: $\lim_{x\to0}\frac{\cos 2x-1}{\cos x-1}$

Solution:

$\lim_{x\to0}\frac{\cos 2x-1}{\cos x-1}=\frac{0}{0}$

Hence,

$\lim_{x\to0}\frac{\cos 2x-1}{\cos x-1}=\lim_{x\to0}\frac{1-2\sin^2 x-1}{1-2\sin^2\frac{x}{2}-1}$

($\cos 2x = 1 - 2\sin^2 x$)

$\lim_{x\to0}\frac{\sin^2 x}{\sin^2\frac{x}{2}}=\lim_{x\to0}\frac{\frac{\sin^2 x\times x^2}{x^2}}{\frac{\sin^2\frac{x}{2}\times\frac{x^2}{4}}{(\frac{x}{2})^2}}$

$= 4\,\frac{\lim_{x\to0}\frac{\sin^2 x}{x^2}}{\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}}$

$= 4\,\frac{\lim_{x\to0}\left(\frac{\sin^2 x}{x^2}\right)^2}{\lim_{x\to0}\left(\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}\right)^2}$

We know that,

$\lim_{x\to0}\frac{\sin x}{x}=1$

$= 4 \times 1^2/ 1^2$

$= 4$

18. Evaluate the given limit: $\lim_{x\to0}\frac{ax+x\cos x}{b\sin x}$

Solution:

$$\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}=\frac{0}{0}$$

Hence,

$$\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}=\frac{1}{b}\lim_{x\to 0}\frac{x(a+\cos x)}{\sin x}$$

$$=\frac{1}{b}\lim_{x\to 0}\times\lim_{x\to 0}(a+\cos x)$$

$$=\frac{1}{b}\times\frac{1}{\lim_{x\to 0}\frac{\sin x}{x}}\times\lim_{x\to 0}(a+\cos x)$$

We know that,

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$=\frac{1}{b}\times(a+\cos 0)$$

= (a + 1) / b

19. Evaluate the given limit: $\lim_{x\to 0} x\sec x$

Solution:

$$\lim_{x\to 0} x\sec x=\lim_{x\to 0}\frac{x}{\cos x}$$

$$=\lim_{x\to 0}\frac{0}{\cos 0}=\frac{0}{1}$$

= 0

20. Evaluate the given limit: $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}a,b,a+b\neq 0$

Solution:

$$\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}=\frac{0}{0}$$

Hence,

$$\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}=\lim_{x\to 0}\frac{(\sin\frac{ax}{ax})ax+bx}{ax+(\sin\frac{bx}{bx})}$$

$$=\frac{\left(\lim_{ax\to 0}\sin\frac{ax}{ax}\right)\times\lim_{x\to 0}ax+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx\times(\lim_{bx\to 0}\sin\frac{bx}{bx})}$$

We know that,

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$=\frac{\lim_{x\to 0}ax+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx}$$

We get,

$$=\frac{\lim_{x\to 0}(ax+bx)}{\lim_{x\to 0}(ax+bx)}$$

= 1

21. Evaluate the given limit: $\lim_{x\to 0}(\operatorname{cosec}x-\cot x)$

Solution:

$$\lim_{x \to 0}(\text{cosec } x - \cot x)$$

Applying the formulas for cosec x and cot x, we get

$$\text{cosec } x = \frac{1}{\sin x} \; and \; \cot x = \frac{\cos x}{\sin x}$$

$$\lim_{x \to 0}(\text{cosec } x - \cot x) = \lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$\lim_{x \to 0}(cosec\, x - cot\, x) = \lim_{x \to 0}\frac{1 - \cos x}{\sin x}$$

Now, by applying the formula we get,

$$1 - \cos x = 2\sin^2\frac{x}{2} \; and \; \sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\lim_{x \to 0}(\text{cosec } x - \cot x) = \lim_{x \to 0}\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$\lim_{x \to 0}(cosec\, x - cot\, x) = \lim_{x \to 0}\tan\frac{x}{2}$$

= 0

22. Evaluate the given limit: $\lim_{x \to \frac{\pi}{2}}\frac{\tan 2x}{x - \frac{\pi}{2}}$

Solution:

$\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}} = \frac{0}{0}$

Let x – (π / 2) = y

Then, x→ (π/2) = y→ 0

Now, we get

$\lim_{x\to\frac{\pi}{2}} \frac{\tan 2x}{x-\frac{\pi}{2}} = \lim_{y\to 0} \frac{\tan 2(y+\frac{\pi}{2})}{y}$

$= \lim_{y\to 0} \frac{\tan(2y+\pi)}{y}$

$= \lim_{y\to 0} \frac{\tan(2y)}{y}$

We know that,

tan x = sin x / cos x

$= \lim_{y\to 0} \frac{\sin 2y}{y \cos 2y}$

By multiplying and dividing by 2, we get

$= \lim_{y\to 0} \frac{\sin 2y}{2y} \times \frac{2}{\cos 2y}$

$= \lim_{2y\to 0} \frac{\sin 2y}{2y} \times \lim_{y\to 0} \frac{2}{\cos 2y}$

= 1 × 2 / cos 0

= 1 × 2 / 1

= 2

23. Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x > 0 \end{cases}$

Solution:

Given function is $f(x) = \begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x > 0 \end{cases}$

$\lim_{x\to 0} f(x)$:

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(2x+3)$

$= 2(0)+3$

$= 0+3$

$= 3$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} 3(x+1)$

$=3(0+1)$

$= 3(1)$

$= 3$

Hence, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 3$

Now, for $\lim_{x\to 1} f(x)$:

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 3(x+1)$

$= 3\ (1 + 1)$

$= 3\ (2)$

$= 6$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 3(x+1)$

$= 3\ (1 + 1)$

$= 3\ (2)$

$= 6$

Hence, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x) = 6$

$\lim_{x\to 0} f(x) = 3$ and $\lim_{x\to 1} f(x) = 6$

24. Find $\lim_{x\to 1} f(x)$, where

$$f(x) = \begin{cases} x^2 - 1 & x \le 1 \\ -x^2 - 1 & x > 1 \end{cases}$$

Solution:

Given function is:

$$f(x) = \begin{cases} x^2 - 1 & x \le 1 \\ -x^2 - 1 & x > 1 \end{cases}$$

$\lim_{x \to 1} f(x)$:

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} x^2 - 1$

$= 1^2 - 1$

$= 1 - 1$

$= 0$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (-x^2 - 1)$

$= (-1^2 - 1)$

$= -1 - 1$

$= -2$

We find,

$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

Hence, $\lim_{x \to 1} f(x)$ does not exist

25. Evaluate $\lim_{x \to 0} f(x)$**, where f(x) =** $\begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Solution:

Given function is $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

We know that,

$\lim_{x \to a} f(x)$ exists only when $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

Now, we need to prove that: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

We know,

|x| = x, if x > = -x, if x < 0

Hence,

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{|x|}{x}$

$= \lim_{x \to 0} \frac{-x}{x} = \lim_{x \to 0} (-1)$

= -1

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{|x|}{x}$

$= \lim_{x \to 0} \frac{x}{x} = \lim_{x \to 0} (1)$

= 1

We find here,

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Hence, $\lim_{x \to 0} f(x)$ does not exist.

26. Find $\lim_{x \to 0} f(x)$**, where f (x) =** $\begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Solution:

Given function is:

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\lim_{x \to 0} f(x)$:

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x}{|x|}$$

$$= \lim_{x \to 0} \frac{x}{-x} = \lim_{x \to 0} \frac{1}{-1}$$

$= -1$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{|x|}$$

$= \lim_{x \to 0} \frac{x}{x} = \lim_{x \to 0} (1)$

$= 1$

We find here,

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Hence, $\lim_{x \to 0} f(x)$ does not exist.

27. Find $\lim_{x \to 5} f(x)$, where $f(x) = |x| - 5$

Solution:

Given function is:

$f(x) = |x| - 5$

$\lim_{x \to 5} f(x)$:

$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} |x| - 5$

$= \lim_{x \to 5} (x - 5) = 5 - 5$

$= 0$

$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} |x| - 5$

$= \lim_{x \to 5} (x - 5)$

$= 5 - 5$

$= 0$

Hence, $\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = \lim_{x \to 5} f(x) = 0$

28. Suppose $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax & x > 1 \end{cases}$ **and if** $\lim_{x \to 1} f(x) = f(1)$ **what are possible values of a and b**

Solution:

Given function is:

$f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$ and

$\lim_{x \to 1} f(x) = f(1)$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} a + bx$

$= a + b\,(1)$

$= a + b$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} b - ax$

$= b - a\,(1)$

$= b - a$

Here,

$f(1) = 4$

Hence, $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = \lim_{x\to1} f(x) = f(1)$

Then, $a + b = 4$ and $b - a = 4$

By solving the above two equations, we get,

$a = 0$ and $b = 4$

Therefore, the possible values of a and b is 0 and 4 respectively

29. Let $a_1, a_2, \ldots\ldots a_n$ be fixed real numbers and define a function $f(x) = (x - a_1)(x - a_2) \ldots\ldots (x - a_n)$.

What is $\lim_{x\to a_1} f(x)$? For some $a \neq a_1, a_2, \ldots\ldots a_n$, compute $\lim_{x\to a} f(x)$

Solution:

Given function is:

$f(x) = (x - a_1)(x - a_2) \ldots (x - a_n)$

$\lim_{x \to a_1} f(x)$:

$$\lim_{x \to a_1} f(x) = \lim_{x \to a_1} [(x - a_1)(x - a_2) \ldots (x - a_n)]$$

$$= [\lim_{x \to a_1}(x - a_1)] \left[\lim_{x \to a_1}(x - a_2)\right] \ldots \left[\lim_{x \to a_1}(x - a_n)\right]$$

We get,

$$= (a_1 - a_1)(a_1 - a_2) \ldots (a_1 - a_n) = 0$$

Hence, $\lim_{x \to a_1} f(x) = 0$

$\lim_{x \to a} f(x)$:

$$\lim_{x \to a} f(x) = \lim_{x \to a}[(x - a_1)(x - a_2) \ldots (x - a_n)]$$

$$= [\lim_{x \to a}(x - a_1)] \left[\lim_{x \to a}(x - a_2)\right] \ldots \left[\lim_{x \to a}(x - a_n)\right]$$

We get,

$$= (a - a_1)(a - a_2) \ldots\ldots (a - a_n)$$

Hence, $\lim_{x \to a} f(x) = (a - a_1)(a - a_2) \ldots (a - a_n)$

Therefore, $\lim_{x \to a_1} f(x) = 0$ and $\lim_{x \to a} f(x) = (a - a_1)(a - a_2) \ldots (a - a_n)$

30. If $f(x)=\begin{cases}|x|+1, x<0\\ 0, \quad x=0\\ |x|-1, x>0\end{cases}$ **For what value (s) of a does** $\lim_{x\to a} f(x)$ **exists?**

Solution:

Given function is:

$$f(x)=\begin{cases}|x|+1, x<0\\ 0, \quad x=0\\ |x|-1, x>0\end{cases}$$

There are three cases.

Case 1:

When $a=0$

$\lim_{x\to 0} f(x)$:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(|x|+1)$$

$$= \lim_{x\to 0}(-x+1) = -0+1$$

$$= 1$$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(|x| - 1)$

$= \lim_{x\to 0}(x - 1) = 0 - 1$

= -1

Here, we find

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Hence, $\lim_{x\to 0} f(x)$ does not exit.

Case 2:

When a < 0

$\lim_{x\to a} f(x)$:

$\lim_{x\to a^-} f(x) = \lim_{x\to a^-}(|x| + 1)$

$= \lim_{x\to a}(-x + 1) = -a + 1$

$\lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x| + 1)$

$= \lim_{x\to a}(-x + 1) = -a + 1$

Hence, $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x) = -a + 1$

Therefore, lim (f(x)) exists at x = a and a < 0

Case 3:

When a > 0

$\lim_{x \to a} f(x)$:

$\lim_{x \to a^-} f(x) = \lim_{x \to a^-} (|x| - 1)$

$= \lim_{x \to a} (x - 1) = a - 1$

$\lim_{x \to a^+} f(x) = \lim_{x \to a^+} (|x| - 1)$

$= \lim_{x \to a} (x - 1) = a - 1$

Hence, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a} f(x) = a - 1$

Therefore, lim (f (x)) exists at x = a when a > 0

31. If the function f(x) satisfies $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \to 1} f(x)$

Solution:

Given function that f (x) satisfies $\lim_{x\to 1}\frac{f(x)-2}{x^2-1}=\pi$

$$\frac{\lim_{x\to 1} f(x)-2}{\lim_{x\to 1} x^2-1}=\pi$$

$$\lim_{x\to 1}(f(x)-2)=\pi(\lim_{x\to 1}(x^2-1))$$

Substituting x = 1, we get,

$$\lim_{x\to 1}(f(x)-2)=\pi(1^2-1)$$

$$\lim_{x\to 1}(f(x)-2)=\pi(1-1)$$

$$\lim_{x\to 1}(f(x)-2)=0$$

$$\lim_{x\to 1} f(x)-\lim_{x\to 1} 2=0$$

$$\lim_{x\to 1} f(x)-2=0$$

$$=2$$

32. If $f(x)=\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m. & x>1 \end{cases}$ **For what integers m and n does both** $\lim_{x\to 0} f(x)$ **and** $\lim_{x\to 1} f(x)$ **exist?**

Solution:

Given function is $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1 \\ nx^3 + m. & x > 1 \end{cases}$

$\lim_{x\to 0} f(x)$:

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(mx^2 + n)$

$= m(0) + n$

$= 0 + n$

$= n$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(nx + m)$

$= n(0) + m$

$= 0 + m$

$= m$

Hence,

$\lim_{x \to 0} f(x)$ exists if n = m.

Now,

$\lim_{x \to 1} f(x)$:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1}(nx + m)$$

$= n(1) + m$

$= n + m$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1}(nx^3 + m)$$

$= n(1)^3 + m$

$= n(1) + m$

$= n + m$

Therefore $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x)$

Hence, for any integral value of m and n $\lim_{x \to 1} f(x)$ exists.