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1.Show that the statement
***p*: "If *x* is a real number such that $x^3 + 4x = 0$, then *x* is 0" is true by**
(i) direct method
(ii) method of contradiction
(iii) method of contrapositive
Solution:
Let p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0'
q: x is a real number such that $x^3 + 4x = 0$
r: x is 0
(i) We assume that q is true to show that statement p is true and then show that r is true
Therefore, let statement q be true
Hence, $x^3 + 4x = 0$
$x (x^2 + 4) = 0$
$x = 0$ or $x^2 + 4 = 0$
Since x is real, it is 0.
So, statement r is true.
Hence, the given statement is true.
(ii) By contradiction, to show statement p to be true, we assume that p is not true.
Let x be a real number such that $x^3 + 4x = 0$ and let $x \neq 0$
Hence, $x^3 + 4x = 0$
$x (x^2 + 4) = 0$
$x = 0$ or $x^2 + 4 = 0$
$x = 0$ or $x^2 = -4$
However x is real. Hence, $x = 0$, which is a contradiction since we have assumed that $x \neq 0$
Therefore, the given statement p is true.
(iii) By contrapositive method, to prove statement p to be true, we assume that r is false and prove that q must be false
$\sim r$: $x \neq 0$
Clearly, it can be seen that
$(x^2 + 4)$ will always be positive
$x \neq 0$ implies that the product of any positive real number with x is not zero.
Now, consider the product of x with $(x^2 + 4)$
$\therefore x (x^2 + 4) \neq 0$
$x^3 + 4x \neq 0$
This shows that statement q is not true.
Hence, proved that
$\sim r \Rightarrow \sim q$

Hence, the given statement p is true.

2. Show that the statement "For any real numbers *a* and *b*, $a^2 = b^2$ implies that *a* = *b*" is not true by giving a counter-example.

Solution:

The given statement can be written in the form of 'if then' is given below

If a and b are real numbers such that $a^2 = b^2$, then a = b

Let p: a and b are real numbers such that $a^2 = b^2$

q: a = b

The given statement has to be proved false. To show this, two real numbers, a and b, with $a^2 = b^2$ are required such that $a \neq b$

Let us consider a = 1 and b = - 1

$a^2 = (1)^2$

= 1 and

$b^2 = (-1)^2$

= 1

Hence, $a^2 = b^2$

However, $a \neq b$

Therefore, it can be concluded that the given statement is false.

3. Show that the following statement is true by the method of contrapositive.

p*: *If x is an integer and x^2 is even, then x is also even.

Solution:

Let p: If x is an integer and x^2 is even, then x is also even

Let q: x is an integer and x^2 is even

r: x is even

By contrapositive method, to prove that p is true, we assume that r is false and prove that q is also false

Let x is not even

To prove that q is false, it has to be proved that x is not an integer or x^2 is not even

x is not even indicates that x^2 is also not even.

Hence, statement q is false.

Therefore, the given statement p is true

4. By giving a counter example, show that the following statements are not true.

(i) *p*: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

(ii) *q*: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Solution:

(i) Let q: All the angles of a triangle are equal
r: The triangle is an obtuse angled triangle
The given statement p has to be proved false.
To show this, required angles of a triangle should not be an obtuse angle
We know that, sum of all the angles of a triangle is 180^0. Therefore, if all the three angles are equal then each angle measures 60^0, which is not an obtuse.
In an equilateral triangle, all angles are equal. However, the triangle is not an obtuse angled triangle.
Hence, it can be concluded that the given statement p is false
(ii) The given statement is
q: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.
This statement has to be proved false
To show this, let us consider,
$x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$
One root of the equation $x^2 - 1 = 0$, i.e. the root x = 1, lies between 0 and 2
Therefore, the given statement is false.

5. Which of the following statements are true and which are false? In each case give a valid reason for saying so.
(i) *p*: Each radius of a circle is a chord of the circle.
(ii) *q*: The centre of a circle bisects each chord of the circle.
(iii) *r*: Circle is a particular case of an ellipse.
(iv) *s*: If *x* and *y* are integers such that *x* > *y*, then –*x* < –*y*.
(v) *t*: $\sqrt{11}$ is a rational number.
Solution:
(i) The given statement p is false.
By the definition of chord, it should intersect the circle at two distinct points
(ii) The given statement q is false.
The centre will not bisect that chord which is not the diameter of the circle.
In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.
(iii) The equation of an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If we put a = b = 1, then we get
$x^2 + y^2 = 1$, which is an equation of a circle

Hence, circle is a particular case of an ellipse.
Therefore, statement r is true
(iv) $x > y$
By a rule of inequality
$-x < -y$
Hence, the given statement s is true
(v) 11 is a prime number
We know that, the square root of any prime number is an irrational number.
Therefore $\sqrt{11}$ is an irrational number
Hence, the given statement t is false.