MISCELLANEOUS EXERCISE   **PAGE NO: 345**

1. Write the negation of the following statements:
(i) p: For every positive real number x, the number $x – 1$ is also positive.
(ii) q: All cats scratch.
(iii) r: For every real number x, either $x > 1$ or $x < 1$.
(iv) s: There exists a number x such that $0 < x < 1$.
Solution:
(i) The negation of statement p is given below
There exists a positive real number x, such that x – 1 is not positive
(ii) The negation of statement q is given below
There exists a cat which does not scratch.
(iii) The negation of statement r is given below
There exists a real number x, such that neither x > 1 nor x < 1
(iv) The negation of statement s is given below
There does not exist a number x, such that 0 < x < 1

2. State the converse and contrapositive of each of the following statements:
(i) p: A positive integer is prime only if it has no divisors other than 1 and itself.
(ii) q: I go to a beach whenever it is a sunny day.
(iii) r: If it is hot outside, then you feel thirsty.
Solution:
(i) Statement p can be written in the form 'if then' is as follows
If a positive integer is prime, then it has no divisors other than 1 and itself
The converse of the statement is given below
If a positive integer has no divisors other than 1 and itself, then it is prime.
The contrapositive of the statement is given below
If a positive integer has divisors other than 1 and itself, then it is not prime.
(ii) The given statement can be written as follows
If it is a sunny day, then I go to a beach.
The converse of the statement is given below
If I go to a beach, then it is a sunny day.
The contrapositive of the statement is given below
If I do not go to a beach, then it is not a sunny day.
(iii) The converse of statement r is given below
If you feel thirsty, then it is hot outside.
The contrapositive of statement r is given below
If you do not feel thirsty, then it is not hot outside.

3. Write each of the statements in the form "if p, then q".

(i) *p*: It is necessary to have a password to log on to the server.
(ii) *q*: There is traffic jam whenever it rains.
(iii) *r*: You can access the website only if you pay a subscription fee.
Solution:
(i) The statement p in the form 'if then' is as follows

If you log on to the server, then you have a password.
(ii) The statement q in the form 'if then' is as follows

If it rains, then there is a traffic jam.
(iii) The statement r in the form 'if then' is as follows

If you can access the website, then you pay a subscription fee.

4. Re write each of the following statements in the form "*p* if and only if *q*".
(i) *p*: If you watch television, then your mind is free and if your mind is free, then you watch television.
(ii) *q*: For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.
(iii) *r*: If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular.
Solution:
(i) You watch television if and only if your mind is free
(ii) You get an A grade if and only if you do all the homework regularly
(iii) A quadrilateral is equiangular if only if it is a rectangle

5. Given below are two statements
***p*: 25 is a multiple of 5.**
q: 25 is a multiple of 8.
Write the compound statements connecting these two statements with "And" and "Or". In both cases check the validity of the compound statement.
Solution:
The compound statement with 'And' is as follows

25 is a multiple of 5 and 8
This is false statement because 25 is not a multiple of 8
The compound statement with 'Or' is as follows

25 is a multiple of 5 or 8
This is true statement because 25 is not a multiple of 8 but it is a multiple of 5

6. Check the validity of the statements given below by the method given against it.
(i) *p*: The sum of an irrational number and a rational number is irrational (by contradiction method).

(ii) q: If n is a real number with $n > 3$, then $n^2 > 9$ (by contradiction method).
Solution:
(i) The given statement is as follows
p: The sum of an irrational number and a rational number is irrational.
Let us assume that the statement p is false. That is,
The sum of an irrational number and a rational number is rational.

Hence, $\sqrt{a} + \dfrac{b}{c} = \dfrac{d}{e}$ where √a is irrational and b, c, d, e are integers.

∴ d / e – b / c = √a
But here, d / e – b / c is a rational number and √a is an irrational number
This is a contradiction. Hence, our assumption is false.
∴ The sum of an irrational number and a rational number is rational.
Hence, the given statement is true.
(ii) The given statement q is as follows
If n is a real number with n > 3, then $n^2 > 9$
Let us assume that n is a real number with n > 3, but $n^2 > 9$ is not true
i.e. $n^2 < 9$
So, n > 3 and n is a real number
By squaring both sides, we get
$n^2 > (3)^2$
This implies that $n^2 > 9$ which is a contradiction, since we have assumed that $n^2 < 9$
Therefore, the given statement is true i.e., if n is a real number with n > 3, then $n^2 > 9$

7. Write the following statement in five different ways, conveying the same meaning.
p: If triangle is equiangular, then it is an obtuse angled triangle.
Solution:
The given statement can be written in five different ways is given below
(i) A triangle is equiangular implies that it is an obtuse angled triangle
(ii) A triangle is equiangular only if the triangle is an obtuse angled triangle
(iii) For a triangle to be equiangular, it is necessary that the triangle is an obtuse angled triangle
(iv) For a triangle to be an obtuse angled triangle, it is sufficient that the triangle is equiangular.
(v) If a triangle is not an obtuse angled triangle, then the triangle is not equiangular.