EXERCISE 15.1 PAGE: 360

Find the mean deviation about the mean for the data in Exercises 1 and 2.

1. 4, 7, 8, 9, 10, 12, 13, 17

Solution:-

First we have to find ($\bar{x}$) of the given data

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{80}{8} = 10$$

So, the respective values of the deviations from mean,

i.e., $x_i - \bar{x}$ are, 10 – 4 = 6, 10 – 7 = 3, 10 – 8 = 2, 10 – 10 = 0,

10 – 12 = - 2, 10 – 13 = - 3, 10 – 17 = - 7

6, 3, 2, 1, 0, -2, -3, -7

Now absolute values of the deviations,

6, 3, 2, 1, 0, 2, 3, 7

$\therefore \sum_{i=1}^{8} |x_i - \bar{x}| = 24$

MD = sum of deviations/ number of observations

= 24/8

= 3

So, the mean deviation for the given data is 3.

2. 38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution:-

First we have to find ($\bar{x}$) of the given data

$$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = \frac{500}{10} = 50$$

So, the respective values of the deviations from mean,

i.e., $x_i - \bar{x}$ are, 50 – 38 = -12, 50 -70 = -20, 50 – 48 = 2, 50 – 40 = 10, 50 – 42 = 8,

50 – 55 = - 5, 50 – 63 = - 13, 50 – 46 = 4, 50 – 54 = -4, 50 – 44 = 6

-12, 20, -2, -10, -8, 5, 13, -4, 4, -6

Now absolute values of the deviations,

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

$\therefore \sum_{i=1}^{10} |x_i - \bar{x}| = 84$

MD = sum of deviations/ number of observations

= 84/10

= 8.4

So, the mean deviation for the given data is 8.4.

Find the mean deviation about the median for the data in Exercises 3 and 4.

3. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution:-

First we have to arrange the given observations into ascending order,
10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18.
The number of observations is 12
Then,
Median = ((12/2)th observation + ((12/2)+ 1)th observation)/2
(12/2)th observation = 6th = 13
(12/2)+ 1)th observation = 6 + 1
= 7th = 14
Median = (13 + 14)/2
= 27/2
= 13.5
So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are
3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5
$\therefore \sum_{i=1}^{12}|x_i - M| = 28$
Mean Deviation,
$\text{M.D.}(M) = \frac{1}{12}\sum_{i=1}^{12}|x_i - M|$
= (1/12) × 28
= 2.33
So, the mean deviation about the median for the given data is 2.33.

4. 36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution:-

First we have to arrange the given observations into ascending order,
36, 42, 45, 46, 46, 49, 51, 53, 60, 72.
The number of observations is 10
Then,
Median = ((10/2)th observation + ((10/2)+ 1)th observation)/2
(10/2)th observation = 5th = 46
(10/2)+ 1)th observation = 5 + 1
= 6th = 49
Median = (46 + 49)/2

= 95
= 47.5

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are 11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

$\therefore \sum_{i=1}^{10} |x_i - M| = 70$

Mean Deviation,

$\text{M.D.}(M) = \frac{1}{10}\sum_{i=1}^{10} |x_i - M|$

= (1/10) × 70
= 7

So, the mean deviation about the median for the given data is 7.

Find the mean deviation about the mean for the data in Exercises 5 and 6.

5.

x_i	5	10	15	20	25
f_i	7	4	6	3	5

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 25, \sum_{i=1}^{5} f_i x_i = 350$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_i x_i = \frac{1}{25} \times 350 = 14$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i|x_i - \bar{x}| = 158$

Therefore $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i|x_i - \bar{x}|$

= (1/25) × 158

= 6.32

So, the mean deviation about the mean for the given data is 6.32.

6.

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

The sum of calculated data,

$$N = \sum_{i=1}^{5} f_i = 80, \sum_{i=1}^{5} f_ix_i = 4000$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{5} f_ix_i = \frac{1}{80} \times 4000 = 50$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

From the table, $\sum_{i=1}^{5} f_i|x_i - \bar{x}| = 1280$

Therefore $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{5} f_i|x_i - \bar{x}|$

= (1/80) × 1280

= 16

So, the mean deviation about the mean for the given data is 16.

Find the mean deviation about the median for the data in Exercises 7 and 8.

7.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	c.f.	$\|x_i - M\|$	$f_i \|x_i - M\|$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48

Now, N = 26, which is even.

Median is the mean of the 13^{th} and 14^{th} observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.

Then,

Median = (13^{th} observation + 14^{th} observation)/2

= (7 + 7)/2

= 14/2

= 7

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{6} f_i = 26$ and $\sum_{i=1}^{6} f_i|x_i - M| = 84$

And $M.D.(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - M|$

= (1/26) × 84

= 3.23

Hence, the mean deviation about the median for the given data is 3.23.

8.

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	c.f.	$\|x_i - M\|$	$f_i \|x_i - M\|$
15	3	3	13.5	40.5
21	5	8	7.5	37.5
27	6	14	1.5	9
30	7	21	1.5	10.5
35	8	29	6.5	52

Now, N = 30, which is even.

Median is the mean of the 15^{th} and 16^{th} observations. Both of these observations lie in the cumulative frequency 21, for which the corresponding observation is 30.

Then,

Median = (15^{th} observation + 16^{th} observation)/2

= (30 + 30)/2

= 60/2

= 30

So, the absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are shown in the table.

Therefore $\sum_{i=1}^{5} f_i = 29$ and $\sum_{i=1}^{5} f_i|x_i - M| = 149.5$

And $M.D.(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - M|$

= (1/29) × 149.5

= 5.1

Hence, the mean deviation about the median for the given data is 5.1.

Find the mean deviation about the mean for the data in Exercises 9 and 10.

9.

Income per day in ₹	**0 - 100**	**100 - 200**	**200 - 300**	**300 - 400**	**400 - 500**	**500 - 600**	**600 – 700**	**700 - 800**
Number of persons	**4**	**8**	**9**	**10**	**7**	**5**	**4**	**3**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Income per day in ₹	**Number of persons f_i**	**Mid – points x_i**	**f_ix_i**	**$\lvert x_i - \bar{x}\rvert$**	**$f_i\lvert x_i - \bar{x}\rvert$**
0 – 100	4	50	200	308	1232
100 – 200	8	150	1200	208	1664
200 – 300	9	250	2250	108	972
300 – 400	10	350	3500	8	80
400 – 500	7	450	3150	92	644
500 – 600	5	550	2750	192	960
600 – 700	4	650	2600	292	1160
700 - 800	3	750	2250	392	1176
	50		17900		7896

The sum of calculated data,

$$N = \sum_{i=1}^{8} f_i = 50, \sum_{i=1}^{8} f_i x_i = 17900$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

So, $\sum_{i=1}^{8} f_i |x_i - \bar{x}| = 7896$

And $M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i |x_i - \bar{x}|$

$= (1/50) \times 7896$

$= 157.92$

Hence, the mean deviation about the mean for the given data is 157.92.

10.

Height in cms	**95 – 105**	**105 – 115**	**115 – 125**	**125 - 135**	**135 - 145**	**145 - 155**
Number of boys	**9**	**13**	**26**	**30**	**12**	**10**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Height in cms	**Number of boys f_i**	**Mid – points x_i**	**$f_i x_i$**	**$\lvert x_i - \bar{x}\rvert$**	**$f_i \lvert x_i - \bar{x}\rvert$**
95 – 105	9	100	900	25.3	227.7
105 – 115	13	110	1430	15.3	198.9
115 – 125	26	120	3120	5.3	137.8
125 – 135	30	130	3900	4.7	141
135 – 145	12	140	1680	14.7	176.4
145 - 155	10	150	1500	24.7	247

	100		12530		1128.8

The sum of calculated data,

$$N = \sum_{i=1}^{6} f_i = 100, \sum_{i=1}^{6} f_i x_i = 12530$$

Now, we have to find $(\bar{x})$ by using the formula

$$\Rightarrow \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$$

The absolute values of the deviations from the mean, i.e., $|x_i - \bar{x}|$, as shown in the table.

So $\sum_{i=1}^{6} f_i |x_i - \bar{x}| = 1128.8$

And $\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}|$

= (1/100) ×1128.8

= 11.28

Hence, the mean deviation about the mean for the given data is 11.28.

11. Find the mean deviation about median for the following data:

Marks	**0 -10**	**10 -20**	**20 – 30**	**30 - 40**	**40 - 50**	**50 - 60**
Number of girls	**6**	**8**	**14**	**16**	**4**	**2**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Marks	Number of Girls f_i	Cumulative frequency (c.f.)	Mid – points x_i	$\lvert x_i - \text{Med}\rvert$	$f_i\lvert x_i - \text{Med}\rvert$
0 – 10	6	6	5	22.85	137.1
10 – 20	8	14	15	12.85	102.8
20 – 30	14	28	25	2.85	39.9

30 – 40	16	44	35	7.15	114.4
40 – 50	4	48	45	17.15	68.6
50 - 60	2	50	55	27.15	54.3
	50				517.1

The class interval containing $N^{th}/2$ or 25^{th} item is 20-30

So, 20-30 is the median class.

Then,

Median = l + (((N/2) - c)/f) × h

Where, l = 20, c = 14, f = 14, h = 10 and n = 50

Median = 20 + (((25 - 14))/14) × 10

= 20 + 7.85

= 27.85

The absolute values of the deviations from the median, i.e., $|x_i - \text{Med}|$, as shown in the table.

So $\sum_{i=1}^{6} f_i|x_i - \text{Med.}| = 517.1$

And $\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - \text{Med.}|$

= (1/50) × 517.1

= 10.34

Hence, the mean deviation about the median for the given data is 10.34.

12. Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age (in years)	16 - 20	21 - 25	26 – 30	31 - 35	36 - 40	41 - 45	46 – 50	51 – 55
Number	5	6	12	14	26	12	16	9

[Hint Convert the given data into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval]

Solution:-

The given data is converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding the 0.5 to the upper limit of each class intervals and append other columns after calculations.

Age	Number f_i	Cumulative frequency (c.f.)	Mid – points x_i	$\|x_i - Med\|$	$f_i\|x_i - Med\|$
15.5 – 20.5	5	5	18	20	100
20.5 – 25.5	6	11	23	15	90
25.5 – 30.5	12	23	28	10	120
30.5 – 35.5	14	37	33	5	70
35.5 – 40.5	26	63	38	0	0
40.5 – 45.5	12	75	43	5	60
45.5 – 50.5	16	91	48	10	160
50.5 – 55.5	9	100	53	15	135
	100				735

The class interval containing $N^{th}/2$ or 50^{th} item is 35.5 – 40.5

So, 35.5 – 40.5 is the median class.

Then,

Median = l + (((N/2) - c)/f) × h

Where, l = 35.5, c = 37, f = 26, h = 5 and N = 100

Median = 35.5 + (((50 - 37))/26) × 5

= 35.5 + 2.5

= 38

The absolute values of the deviations from the median, i.e., $|x_i - Med|$, as shown in the table.

So $\sum_{i=1}^{8} f_i|x_i - Med.| = 735$

And $M.D.(M) = \frac{1}{N}\sum_{i=1}^{6} f_i|x_i - Med.|$

= (1/100) × 735

= 7.35

Hence, the mean deviation about the median for the given data is 7.35.