EXERCISE 15.2 PAGE: 371

Find the mean and variance for each of the data in Exercise 1 to 5.

1. 6, 7, 10, 12, 13, 4, 8, 12

Solution:-

We have,

Mean = $\bar{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\bar{x}$ = (6 + 7 + 10 + 12 + 13 + 4 + 8 + 12)/8

= 72/8

= 9

Let us make the table of the given data and append other columns after calculations.

X_i	Deviations from mean $(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	6 – 9 = -3	9
7	7 – 9 = -2	4
10	10 – 9 = 1	1
12	12 – 9 = 3	9
13	13 – 9 = 4	16
4	4 – 9 = - 5	25
8	8 – 9 = - 1	1
12	12 – 9 = 3	9
		74

We know that Variance,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$

σ^2 = (1/8) × 74

= 9.2

∴Mean = 9 and Variance = 9.25

2. First n natural numbers

Solution:-

We know that Mean = Sum of all observations/Number of observations

∴Mean, $\bar{x}$ = ((n(n + 1))2)/n

= (n + 1)/2

and also WKT Variance,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \bar{x})^2$

By substitute that value of $\bar{x}$ we get,

$$= \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \frac{n+1}{2}\right)^2$$

We know that $(a - b)^2 = a^2 - 2ab + b^2$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i)^2 - \frac{1}{n}\sum_{i=1}^{n}2x_i\left(\frac{n+1}{2}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2$$

Substituting the summation values

$$= \frac{1}{n}\frac{n(n+1)(2n+1)}{6} - \frac{n+1}{n}\left[\frac{n(n+1)}{2}\right] + \frac{(n+1)^2}{4n} \times n$$

Multiplying and Computing

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{2} + \frac{(n+1)^2}{4}$$

By taking LCM and simplifying, we get

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

By taking (n + 1) common from each term, we get

$$= (n+1)\left[\frac{4n + 2 - 3n - 3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

WKT, $(a + b)(a - b) = a^2 - b^2$

$\sigma^2 = (n^2 - 1)/12$

$\therefore$Mean = $(n + 1)/2$ and Variance = $(n^2 - 1)/12$

3. First 10 multiples of 3

Solution:-

First we have to write the first 10 multiples of 3,
3, 6, 9, 12, 15, 18, 21, 24, 27, 30

We have,

Mean = $\overline{x} = \frac{\sum_{i=1}^{a} x_i}{n}$

Where, n = number of observation

$\sum_{i=1}^{a} x_i$ = sum of total observation

So, $\overline{x}$ = (3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30)/10

= 165/10

= 16.5

Let us make the table of the data and append other columns after calculations.

X_i	Deviations from mean $(x_i - \overline{x})$	$(x_i - \overline{x})^2$
3	3 – 16.5 = -13.5	182.25
6	6 – 16.5 = -10.5	110.25
9	9 – 16.5 = -7.5	56.25
12	12 – 16.5 = -4.5	20.25
15	15 – 16.5 = -1.5	2.25
18	18 – 16.5 = 1.5	2.25
21	21 – 16.5 = - 4.5	20.25
24	24 – 16.5 = 7.5	56.25
27	27 – 16.5 = 10.5	110.25
30	30 – 16.5 = 13.5	182.25
		742.5

Then, Variance

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{a}(x_i - \overline{x})^2$

= (1/10) × 742.5

= 74.25

∴Mean = 16.5 and Variance = 74.25

4.

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	f_ix_i	**Deviations from mean** $(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	6 – 19 = 13	169	338
10	4	40	10 – 19 = -9	81	324
14	7	98	14 – 19 = -5	25	175
18	12	216	18 – 19 = -1	1	12
24	8	192	24 – 19 = 5	25	200
28	4	112	28 – 19 = 9	81	324
30	3	90	30 – 19 = 11	121	363
	N = 40	760			1736

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 760/40

= 19

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/40) × 1736

= 43.4

∴Mean = 19 and Variance = 43.4

5.

x_i	92	93	97	98	102	104	109
f_i	3	2	3	2	6	3	3

Solution:-

Let us make the table of the given data and append other columns after calculations.

X_i	f_i	f_ix_i	**Deviations from mean** $(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	92 – 100 = -8	64	192
93	2	186	93 – 100 = -7	49	98
97	3	291	97 – 100 = -3	9	27

98	2	196	98 – 100 = -2	4	8
102	6	612	102 – 100 = 2	4	24
104	3	312	104 – 100 = 4	16	48
109	3	327	109 – 100 = 9	81	243
	N = 22	2200			640

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 2200/22

= 100

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/22) × 640

= 29.09

∴Mean = 100 and Variance = 29.09

6. Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Solution:-

Let the assumed mean A = 64. Here h = 1

We obtain the following table from the given data.

X_i	Frequency f_i	$Y_i = (x_i – A)/h$	Y_i^2	f_iy_i	$f_iy_i^2$
60	2	-4	16	-8	32
61	1	-3	9	-3	9
62	12	-2	4	-24	48
63	29	-1	1	-29	29
64	25	0	0	0	0
65	12	1	1	12	12
66	10	2	4	20	40
67	4	3	9	12	36
68	5	4	16	20	80

				0	286

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where A = 64, h = 1

So, $\bar{x}$ = 64 + ((0/100) × 1)

= 64 + 0

= 64

Then, variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (1^2/100^2)\ [100(286) - 0^2]$

= (1/10000) [28600 - 0]

= 28600/10000

= 2.86

Hence, standard deviation = σ = √2.886

= 1.691

∴ Mean = 64 and Standard Deviation = 1.691

Find the mean and variance for the following frequency distributions in Exercises 7 and 8.

7.

Classes	**0 – 30**	**30 - 60**	**60 – 90**	**90 - 120**	**120 – 150**	**150 – 180**	**180 – 210**
Frequencies	**2**	**3**	**5**	**10**	**3**	**5**	**2**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Classes	**Frequency f_i**	**Mid – points x_i**	**$f_i x_i$**	**$(x_i - \bar{x})$**	**$(x_i - \bar{x})^2$**	**$f_i(x_i - \bar{x})^2$**
0 – 30	2	15	30	-92	8464	16928
30 – 60	3	45	135	-62	3844	11532
60 – 90	5	75	375	-32	1024	5120
90 – 120	10	105	1050	-2	4	40
120 – 150	3	135	405	28	784	2352
150 – 180	5	165	825	58	3364	16820
180 - 210	2	195	390	88	7744	15488

	N = 30		3210			68280

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 3210/30

= 107

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/30) × 68280

= 2276

∴Mean = 107 and Variance = 2276

8.

Classes	**0 – 10**	**10 - 20**	**20 – 30**	**30 - 40**	**40 –50**
Frequencies	**5**	**8**	**15**	**16**	**6**

Solution:-

Let us make the table of the given data and append other columns after calculations.

Classes	**Frequency f_i**	**Mid – points x_i**	**$f_i x_i$**	**$(x_i - \bar{x})$**	**$(x_i - \bar{x})^2$**	**$f_i(x_i - \bar{x})^2$**
0 – 10	5	5	25	-22	484	2420
10 – 20	8	15	120	-12	144	1152
20 – 30	15	25	375	-2	4	60
30 – 40	16	35	560	8	64	1024
40 –50	6	45	270	18	324	1944
	N = 50		1350			6600

Then Mean, $\bar{x} = \frac{\sum_{i=1}^{a} f_i x_i}{N}$

Where N = $\sum_{i=1}^{n} f_i$

$\bar{x}$ = 1350/50

= 27

Now, Variance, $\sigma^2 = \frac{1}{N}\sum_{i=1}^{a} f_i(x_i - \bar{x})^2$

= (1/50) × 6600

= 132

∴Mean = 27 and Variance = 132

9. Find the mean, variance and standard deviation using short-cut method

Height in cms	70 – 75	75 - 80	80 – 85	85 - 90	90 – 95	95 – 100	100 – 105	105 - 110	110 - 115
Frequencies	**3**	**4**	**7**	**7**	**15**	**9**	**6**	**6**	**3**

Solution:-

Let the assumed mean, A = 92.5 and h = 5

Let us make the table of the given data and append other columns after calculations.

Height (class)	Number of children Frequency f_i	Midpoint X_i	$Y_i = (x_i - A)/h$	Y_i^2	f_iy_i	$f_iy_i^2$
70 – 75	2	72.5	-4	16	-12	48
75 - 80	1	77.5	-3	9	-12	36
80 – 85	12	82.5	-2	4	-14	28
85 – 90	29	87.5	-1	1	-7	7
90 – 95	25	92.5	0	0	0	0
95 – 100	12	97.5	1	1	9	9
100 – 105	10	102.5	2	4	12	24
105 – 110	4	107.5	3	9	18	54
110 - 115	5	112.5	4	16	12	48
	N = 60				6	254

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where, A = 92.5, h = 5

So, $\bar{x}$ = 92.5 + ((6/60) × 5)

= 92.5 + ½

= 92.5 + 0.5

= 93

Then, Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (5^2/60^2)\,[60(254) - 6^2]$

= (1/144) [15240 - 36]

= 15204/144

= 1267/12

= 105.583

Hence, standard deviation = σ = √105.583

= 10.275

∴ Mean = 93, variance = 105.583 and Standard Deviation = 10.275

10. The diameters of circles (in mm) drawn in a design are given below:

Diameters	33 – 36	37 - 40	41 – 44	45 - 48	49 – 52
No. of circles	15	17	21	22	25

Calculate the standard deviation and mean diameter of the circles.

[Hint first make the data continuous by making the classes as 32.5-36.5, 36.5-40.5, 40.5-44.5, 44.5 - 48.5, 48.5 - 52.5 and then proceed.]

Solution:-

Let the assumed mean, A = 42.5 and h = 4

Let us make the table of the given data and append other columns after calculations.

Height (class)	Number of children (Frequency f_i)	Midpoint X_i	$Y_i = (x_i - A)/h$	Y_i^2	$f_i y_i$	$f_i y_i^2$
32.5 – 36.5	15	34.5	-2	4	-30	60
36.5 – 40.5	17	38.5	-1	1	-17	17
40.5 – 44.5	21	42.5	0	0	0	0
44.5 – 48.5	22	46.5	1	1	22	22
48.5 – 52.5	25	50.5	2	4	50	100

	N = 100				25	199

Mean,

$$\bar{x} = A + \frac{\sum_{i=1}^{a} f_i y_i}{N} \times h$$

Where, A = 42.5, h = 4

So, $\bar{x}$ = 42.5 + (25/100) × 4

= 42.5 + 1

= 43.5

Then, Variance,

$$\sigma^2 = \frac{h^2}{N^2}[N\Sigma f_i y_i^2 - (\Sigma f_i y_i)^2]$$

$\sigma^2 = (4^2/100^2)[100(199) - 25^2]$

= (1/625) [19900 - 625]

= 19275/625

= 771/25

= 30.84

Hence, standard deviation = σ = √30.84

= 5.553

∴ Mean = 43.5, variance = 30.84 and Standard Deviation = 5.553.