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1. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Solution:-

Form the question it is given that,

Variance of eight observations are 9 and 9.25.

There are six observations given 6, 7, 10, 12, 12, and 13

Let us assume the remaining two observations to be x and y respectively such that,

Observations: 6, 7, 10, 12, 12, 13, x, y.

We have to calculate the mean of given observations,

$\therefore$ Mean, $\bar{x} = \frac{6+7+10+12+12+13+x+y}{8}$

$$9 = \frac{6+7+10+12+12+13+x+y}{8}$$

60 + x + y = 72

x + y = 12 ... [we call it as equation (i)]

$$\text{Now, Variance } = \frac{1}{n}\sum_{i=1}^{8}(x_i - x)^2$$

$$9.25 = \frac{1}{8}[(-3)^2 + (-2)^2 + 1^2 + 3^2 + 4^2 + x^2 + y^2 - 18(x+y) + 2 \times 9^2]$$

By using equation (i) substitute 12 instead of (x + y)

$$9.25 = \frac{1}{8}[9 + 4 + 1 + 9 + 9 + 16 + x^2 + y^2 - 18 \times 12 + 162]$$

$$9.25 = \frac{1}{8}[48 + x^2 + y^2 - 216 + 162]$$

$$9.25 = \frac{1}{8}[x^2 + y^2 - 6]$$

$x^2 + y^2 = 80$ ... [we call it as equation (ii)]

So, from equation (i) we have:

$$x^2 + y^2 + 2xy = 144 \text{ (iii)}$$

Thus, from (ii) and (iii), we have

2xy = 64 (iv)

Now by subtracting (iv) from (ii), we get:

$x^2 + y^2 - 2xy = 80 - 64$

$x - y = \pm 4$ (v)

Hence, from equation (i) and (v) we have:

When x – y = 4

Then, x = 8 and y = 4

And, when x – y = - 4

Then, x = 4 and y = 8

∴ The remaining observations are 4 and 8

2. The mean and variance of 7 observations are 8 and 16, respectively. If five of the observations are 2, 4, 10, 12, 14. Find the remaining two observations.

Solution:-

Form the question it is given that,

Variance of seven observations are 8 and 16.

There are six observations given 2, 4, 10, 12, and 14

Let us assume the remaining two observations to be x and y respectively such that,

Observations: 2, 4, 10, 12, 14, x, y.

We have to calculate the mean of given observations,

$$\therefore \text{Mean}, \bar{x} = \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$$

$x + y = 14$ ... [we call it as equation (i)]

In the question it is given that,

Variance = 16

We know that,

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n}(x_1 - \bar{x})^2$$

$$16 = \frac{1}{7}[(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + x^2 + y^2 - 2 \times 8\,(x + y) + 2 \times (8)^2]$$

By using equation (i) substitute 14 instead of (x + y)

$$16 = \frac{1}{7}\,[36 + 16 + 4 + 16 + 36 + x^2 + y^2 - 16\,(14) + 2\,(64)]$$

$$16 = \frac{1}{7}\,[12 + x^2 + y^2]$$

$x^2 + y^2 = 112 - 12$

$x^2 + y^2 = 100$ ... [we call it as equation (ii)]

So, from equation (i) we have:

$x^2 + y^2 + 2xy = 196$ ... [we call it as equation (iii)]

Thus, from equation (ii) and (iii) we have:

$2xy = 196 – 100$

$2xy = 96$ (iv)

Now subtracting equation (iv) from (ii),

We get:

$x^2 + y^2 – 2xy = 100 – 96$

$(x – y)^2 = 4$

$x – y = \pm 2$ (v)

Hence, from equation (i) and (v) we have:
When $x - y = 2$ then $x = 8$ and $y = 6$
And, when $x - y = -2$ then $x = 6$ and $y = 8$
∴ the remaining observations are 6 and 8

3. The mean and standard deviation of six observations are 8 and 4, respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.
Solution:-

From the question it is given that,

Mean of six observations = 8

Standard deviation of six observations = 4

Let us assume the observations be x_1, x_2, x_3, x_4, x_5 and x_6

So, mean of assumed observations,

$$\therefore \text{Mean } \bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 8$$

Then, as per the question if each observation is multiplied by 3 and the resulting observations are y_i then, we have:

$y_i = 3x_i$

Hence, $x_i = \frac{1}{3} y_i$ (For $i = 1$ to 6)

$$\therefore \text{New mean, } \bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6}{6}$$

$$= \frac{3(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)}{6}$$

$= 3 \times 8$

$= 24$

We know that,

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{6}(x_i - \bar{x})^2}$$

By squaring on both the sides

$$\therefore (4)^2 = \frac{1}{6}\sum_{i=1}^{6}(x_i - \bar{x})^2$$

$$\sum_{i=1}^{6}(x_i - \bar{x})^2 = 96 \text{ (ii)}$$

Hence, from (i) and (ii) we have:

$$\bar{y} = 3\bar{x}$$

$$\bar{x} = \frac{1}{3}\bar{y}$$

Now, by substituting the values of x_i and $\bar{x}$ in (ii) we have:

$$\sum_{i=1}^{6}\left(\frac{1}{3}y_i - \frac{1}{3}\bar{y}\right)^2 = 96$$

Thus, $\sum_{i=1}^{6}(y_i - \bar{y})^2 = 864$

So, the variance of new observation = (1/6) × 864

= 144

Therefore, standard deviation of new observation = $\sqrt{144}$

= 12

4. Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, \ldots, x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, \ldots, ax_n$ are $a\bar{x}$ and $a^2\sigma^2$, respectively, (a ≠ 0).

Solution:-

From the question it is given that, n observations are $x_1, x_2, \ldots x_n$

Mean of the n observation = $\bar{x}$

Variance of the n observation = σ^2

As we know that,

Variance, $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} y_1(x_i - \bar{x})^2$ … [equation (i)]

As per the condition given in the question, if each of the observation is being multiplied by 'a' and the new observation are y_i the, we have:

$y_i = ax_i$

$$\text{Thus}, x_i = \frac{1}{a} y_i$$

$$\therefore \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} ax_i$$

$$\bar{y} = \frac{a}{n}\sum_{i=1}^{n} x_i$$

$$\bar{y} = a\bar{x}$$

Therefore, mean of the observations $ax_1, ax_2, \ldots ax_n$ is a $\bar{x}$

Now, by substituting the values of x_i and $\bar{x}$ in equation(i), we get:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{a}y_1 - \frac{1}{a}\bar{y}\right)^2$$

$$a^2\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(y_1 - \bar{y})^2$$

∴ the variance of the given observations $ax_1, ax_2, \ldots ax_n$ is $a^2\sigma^2$

5. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases: (i) If wrong item is omitted. (ii) If it is replaced by 12
Solution:-

(i) If wrong item is omitted,

From the question it is given that,
The number of observations i.e. n = 20
The incorrect mean = 20
The incorrect standard deviation = 2

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{20} X_1$$

$$10 = \frac{1}{20}\sum_{i=1}^{20} X_1$$

$$\sum_{i=1}^{20} X_1 = 200$$

By the calculation the incorrect sum of observations = 200

Hence, correct sum of observations = 200 – 8

= 192

Therefore the correct mean = correct sum/19

= 192/19

= 10.1

We know that, Standard deviation $(\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_{\ 1}\right)^2}$

$$2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - (\bar{X})^2}$$

$$4 = \frac{1}{20} \text{ Incorrect} \sum_{i=1}^{n} X_1^2 - 100$$

$$\text{Incorrect} \sum_{i=1}^{n} X_1^2 = 2080$$

Therefore, correct $\sum_{i=1}^{n} X_1^2 = \text{ Incorrect } \sum_{i=1}^{n} X_1^2 - (8)^2$

= 2080 – 64

= 2016

Finally we came to calculate correct standard deviation,

$$\text{Hence, Correct standard deviation} = \sqrt{\frac{\text{Correct } \sum X_1^2}{n} - (\text{Correct Mean})^2}$$

$$= \sqrt{\frac{2016}{19} - (10.1)^2}$$

$$= \sqrt{1061.1 - 102.1}$$

= 2.02

(ii) If it is replaced by 12,

From the question it is given that,
The number of incorrect sum observations i.e. n = 200
The correct sum of observations n = 200 – 8 + 12
n = 204
Then, correct mean = correct sum/20
= 204/20
= 10.2

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X_{\ 1}\right)^2}$$

$$\therefore 2 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - (\overline{X})^2}$$

$$4 = \frac{1}{20}\text{ Incorrect }\sum_{i=1}^{n} X_1^2 - 100$$

$$\text{Incorrect }\sum_{i=1}^{n} X_1^2 = 2080$$

Thus, $\text{correct } \sum_{i=1}^{n} X_1^2 = \text{ Incorrect } \sum_{i=1}^{n} X_1^2 - (8)^2 + (12)^2$

= 2080 – 64 + 144

= 2160

$$\text{Hence, Correct standard deviation} = \sqrt{\frac{\text{Correct } \sum X_1^2}{n} - (\text{Correct Mean})^2}$$

$$= \sqrt{\frac{2160}{20} - (10.2)^2}$$

$$= \sqrt{108 - 104.04}$$

$$= \sqrt{3.96}$$

= 1.98

6. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of the three subjects shows the highest variability in marks and which shows the lowest?

Solution:-

From the question it is given that,

Mean of Mathematics = 42

Standard deviation of Mathematics = 12

Mean of Physics = 32

Standard deviation of physics = 15

Mean of Chemistry = 40.9

Standard deviation of chemistry = 20

As we know that,

$$\text{Coefficient of variation (C.V)} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

Then,

C.V. in Mathematics = (12/42) × 100

= 28.57

C.V. in Mathematics = (15/32) × 100

= 46.87

C.V. in Mathematics = (20/40.9) × 100

= 48.89

Hence, subject with highest variability in marks is chemistry as subject with the greater C.V is more variable than others

7. The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Solution:-

From the question it is given that,

The total number of observations (n) = 100
Incorrect mean, ($\bar{x}$) = 20
And, Incorrect standard deviation (σ) = 3

$$\therefore 20 = \frac{1}{100}\sum_{i=1}^{100} X_1$$

By cross multiplication, we get

$$\sum_{i=1}^{100} X_1 = 20 \times 100$$

$$\sum_{i=1}^{100} X_1 = 2000$$

Hence, incorrect sum of observations is 2000

Now, correct sum of observations = 2000 – 21 – 21 – 18

= 2000 – 60

= 1940

Therefore correct Mean = Correct sum/(100 – 3)

= 1940/97

= 20

We know that, Standard deviation (σ) = $\sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1 - \frac{1}{n^2}\left(\sum_{i=1}^{n} X\right)^2}$

$$3 = \sqrt{\frac{1}{n}\sum_{i=1}^{n} X_1^2 - (\bar{X})^2}$$

$$3 = \sqrt{\frac{1}{100} \times \text{Incorrect} \sum X_1^2 - (20)^2}$$

$$\text{Incorrect} \sum X_1^2 = 100\,(9 + 400)$$

$$\text{Incorrect} \sum X_1^2 = 40900$$

$$\text{Correct} \sum_{i=1}^{n} X_1^2 = \text{Incorrect} \sum_{i=1}^{n} X_1^2 - (21)^2 - (21)^2 - (18)^2$$

$= 40900 - 441 - 441 - 324$

$= 40900 - 1206$

$= 39694$

$$\text{Hence, correct standard deviation} = \sqrt{\frac{\text{Correct} \sum X_1^2}{n} - (\text{Correct mean})^2}$$

$$= \sqrt{\frac{39694}{97} - (20)^2}$$

$$= \sqrt{409.216 - 400}$$

$= 3.036$