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In each of the following Exercises 1 to 7, describe the sample space for the indicated experiment.

1. A coin is tossed three times.

Solution:-

Since either coin can turn up Head (H) or Tail (T), the possible outcomes may be
So, when 1 coin is tossed once the sample space = 2
Then,
Coin is tossed 3 times the sample space = $2^3 = 8$
Thus, the sample space is S = {HHH, THH, HTH, HHT, TTT, HTT, THT, TTH}

2. A die is thrown two times.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
Then, the total number of sample space = (6 × 6)
= 36
Thus, the sample space is
S={(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3)(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

3. A coin is tossed four times.

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
So, when 1 coin is tossed once the sample space = 2
Then,
Coin is tossed 3 times the sample space = $2^4 = 16$
Thus, the sample space is S = {HHHH, THHH, HTHH, HHTH, HHHT, TTTT, HTTT, THTT, TTHT, TTTH, TTHH, HHTT, THTH, HTHT, THHT, HTTH}

4. A coin is tossed and a die is thrown.

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers comes when the die is thrown.
Then, total number of space = (2 × 6) = 12
Thus, the sample space is,

S={(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}

5. A coin is tossed and then a die is rolled only in case a head is shown on the coin.
Solution:-
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers comes when the die is thrown.
When head in encountered,
Then, number of space = (1 × 6) = 6
Sample Space S_H= {H1, H2, H3, H4, H5, H6}
Now, tail is encountered, Sample space S_T= {T}
Therefore the total sample space S = {H1, H2, H3, H4, H5, H6, T}

6. 2 boys and 2 girls are in Room X, and 1 boy and 3 girls in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.
Solution:-
From the question it is given that,
2 boys and 2 girls are in Room X
1 boy and 3 girls in Room Y
Let us assume b1, b2 and g1, g2 be 2 boys and 2 girls are in Room X.
And also assume b3 and g3, g4, g5 be 1 boy and 3 girls in Room Y.
The problem is solved by dividing into two cases
Case 1: Room X is selected
Sample Space S_x = {(X,b1),(X,b2),(X,g1),(X,g2)}
Case 2: Room Y is selected
Sample Space S_y ={(Y,b3),(Y,g3),(Y,g4),(Y,g5)}
The overall sample space
S={(X,b1),(X,b2),(X,g1),(X,g2),(Y,b3),(Y,g3),(Y,g4),(Y,g5)}

7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.
Solution:-
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers comes when the die is thrown.
And also assume die of red colour be ‘R’, die of white colour be ‘W’, die of blue colour be ‘B’.

So, the total number of sample space = (6 × 3) = 18
The sample space of the event is
S={(R,1),(R,2),(R,3),(R,4),(R,5),(R,6),(W,1),(W,2),(W,3),(W,4),(W,5),(W,6)
(B,1),(B,2),(B,3),(B,4),(B,5),(B,6)}

8. An experiment consists of recording boy–girl composition of families with 2 children.
(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?
(ii) What is the sample space if we are interested in the number of girls in the family?
Solution:-
Let us assume boy be 'B' and girl be 'G'
(i) The sample space if we are interested in knowing whether it is a boy or girl in the order of their births. S = {GG, BB, GB, BG}
(ii) The sample space if we are interested in the number of girls in the family when there are two child in the family then,
Sample space S = {2, 1, 0}

9. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.
Solution:-
From the question it is given that, a box contains 1 red and 3 identical white balls.
Let us assume 'R' be the event of red ball is drawn and 'W' be the event of white ball is drawn.
Given in the question that white balls are identical, therefore the event of drawing any one of the three white ball is same.
Then, total number of sample space = $(2^2 - 1) = 3$
∴Sample space S = {WW, WR, RW}

10. An experiment consists of tossing a coin and then throwing it second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.
Solution:-
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us take,
Case 1: Head is encountered
Sample space S_1 = {HT, HH}

Case 2: Tail is encountered
Sample Space S_2 = {(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}
Then,
The Overall Sample space
S = {(HT), (HH), (T1), (T2), (T3), (T4), (T5), (T6)}

11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non – defective (N). Write the sample space of this experiment.
Solution:-
From the question,
'D' denotes the event of bulb is defective and 'N' denotes event of non-defective bulbs.
Then,
Total number of Sample space = 2 × 2 × 2 = 8
Thus, Sample space S = {DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

12. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?
Solution:-
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
The problem can be solved by dividing it into 3 cases
Case 1: The outcome is Head and the corresponding number on the die shows Odd number
Total number of sample space = (1 × 3) = 3
Sample space S_{HO} = {(H,1), (H,3), (H,5)}
Case 2: The outcome is Head and the corresponding number on the die shows Even number
Total number of sample space = (1 × 3 × 6) = 18
S_{HE}={(H,2,1),(H,2,2),(H,2,3),(H,2,4),(H,2,5),(H,2,6),(H,4,1),(H,4,2),(H,4,3),(H,2,4),(H,4,5),(H,4,6), (H,6,1),(H,6,2),(H,6,3),(H,6,4),(H,6,5),(H,6,6)}
Case 3: The outcome is Tail
Total number of sample space=1
Sample space S_T = {(T)}
The overall sample spaces
S={(H,1),(H,3),(H,5),
(H,2,1),(H,2,2),(H,2,3),(H,2,4),(H,2,5),(H,2,6),(H,4,1),(H,4,2),(H,4,3),(H,2,4),(H,4,5),

(H,4,6), (H,6,1),(H,6,2),(H,6,3),(H,6,4),(H,6,5),(H,6,6),(T)}

13. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment

Solution:-

From the question it is given that, 1, 2, 3, 4 are the numbers written on the four slip.
When two slips are drawn without replacement the first event has 4 possible outcomes and the second event has 3 possible outcomes because 1 slip is already picked.
Therefore, the total number of possible outcomes = (4 × 3) = 12
Thus sample space,
S={(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)}

14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
The following problem can be divided in two cases
(i) The number on the die is even.

The sample space S_E={(2,H),(4,H),(6,H),(2,T),(4,T),(6,T)}

(ii) The number on the die is odd and the coin is tossed twice.
The sample space

S_o = {(1,H,H), (3,H,H), (5,H,H), (1,H,T), (3,H,T), (5,H,T), (1,T,H), (3,T,H), (5,T,H), (1,T,T), (3,T,T), (5,T,T)}

Hence, the overall sample space for the problem= $S_E + S_o$

S = {(2,H), (4,H), (6,H), (2,T), (4,T), (6,T), (1,H,H), (3,H,H), (5,H,H), (1,H,T), (3,H,T), (5,H,T), (1,T,H), (3,T,H), (5,T,H), (1,T,T), (3,T,T), (5,T,T)}

15. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
Let us assume R_1, R_2 denote the event the red balls are drawn and B_1, B_2, B_3 denote the events black ball are drawn.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
(i) Coin shows Tail.

So, the sample space $S_T = \{(TR_1), (TR_2), (TB_1), (TB_2), (TB_3)\}$

(ii) Coin shows head.

So, the sample space $S_H = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)\}$

Hence, the overall sample space for the problem= $S_T + S_H$

$S = \{(T,R_1), (T,R_2), (T,B_1), (T,B_2), (T,B_3), (H,1), (H,2), (H,3), (H,4), (H,5), (H,6)\}$

16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.
As per the condition given in the question, a die is thrown repeatedly until a six comes up.
If six may come up for the first throw or six may come up on second throw this process will goes continuously until the six comes.
The sample space when 6 comes on very first throw $S_1 = \{6\}$
The sample space when 6 comes on second throw $S_2 = \{(1,6), (2,6), (3,6), (4,6), (5,6)\}$
This event can go on for infinite times.
So, the sample space is infinitely defined

$S = \{(6), (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6).....\}$