EXERCISE 16.2 PAGE: 393

1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, S = (1, 2, 3, 4, 5, 6)

As per the conditions given the question

E be the event "die shows 4"

E = (4)

F be the event "die shows even number"

F = (2, 4, 6)

$E \cap F = (4) \cap (2, 4, 6)$

$= 4$

$4 \neq \phi$ … [because there is a common element in E and F]

Therefore E and F are not mutually exclusive event.

2. A die is thrown. Describe the following events:

(i) A: a number less than 7 **(ii) B: a number greater than 7**

(iii) C: a multiple of 3 **(iv) D: a number less than 4**

(v) E: an even number greater than 4 **(vi) F: a number not less than 3**

Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F^{I}$, F^{I}

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.

So, S = (1, 2, 3, 4, 5, 6)

As per the conditions given in the question,

(i) A: a number less than 7

All the numbers in the die are less than 7,

A = (1, 2, 3, 4, 5, 6)

(ii) B: a number greater than 7

There is no number greater than 7 on the die

Then,

B= (ϕ)

(iii) C: a multiple of 3

There are only two numbers which are multiple of 3.

Then,

C= (3, 6)

(iv) D: a number less than 4
D= (1, 2, 3)

(v) E: an even number greater than 4
E = (6)

(vi) F: a number not less than 3
F= (3, 4, 5, 6)

Also we have to find, A U B, A ∩ B, B U C, E ∩ F, D ∩ E, D - E, A - C, E ∩ F', F'
So,
A ∩ B = (1, 2, 3, 4, 5, 6) ∩ (ϕ)
= (ϕ)

B U C = (ϕ) U (3, 6)
= (3, 6)

E ∩ F = (6) ∩ (3, 4, 5, 6)
= (6)

D ∩ E = (1, 2, 3) ∩ (6)
= (ϕ)

D - E = (1, 2, 3) - (6)
= (1, 2, 3)

A - C = (1, 2, 3, 4, 5, 6) - (3, 6)
= (1, 2, 4, 5)

F' = S – F
= (1, 2, 3, 4, 5, 6) – (3, 4, 5, 6)
= (1, 2)

E ∩ F' = (6) ∩ (1, 2)
= (ϕ)

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events: A: the sum is greater than 8, B: 2 occurs on either die C: the sum is at least 7 and a multiple of 3. Which pairs of these events are mutually exclusive?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
In the question is given that pair of die is thrown, so sample space will be,

$$S = \left\{\begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5). (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix}\right\}$$

A: the sum is greater than 8

$\therefore$ A= $\left\{\begin{matrix} (3,6), (4,5), (5,4), (6,3), (4,6), \\ (5,5), (6,4), (5,6), (6,5), (6,6) \end{matrix}\right\}$

Possible sum greater than 8 are 9, 10, 11 & 12

B: 2 occurs on either die

B= $\left\{\begin{matrix} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (1,2), (3,2), (4,2), (5,2), (6,2) \end{matrix}\right\}$

In this conditions possibilities are there that the number 2 will come on either first die or second die or both the die simultaneously

C: The sum is at least 7 and multiple of 3

C= $\{(3,6), (4,5), (5,4), (6,3), (6,6)\}$

So the sum can be only 9 or 12

Now, we shall find pairs of these events are mutually exclusive or not.

(i) A∩ B = ϕ

Since there is no common element in A and B

Therefore A & B are mutually exclusive

(ii) B ∩ C = ϕ

Since there is no common element between

Therefore B and C are mutually exclusive.

(iii) A ∩ C {(3,6), (4,5), (5,4), (6,3), (6,6)}

⇒ {(3,6), (4,5), (5,4), (6,3), (6,6)} ≠ ϕ

Since A and C has common elements.
Therefore A and C are mutually exclusive.

4. Three coins are tossed once. Let A denote the event 'three heads show", B denote the event "two heads and one tail show", C denote the event" three tails show and D denote the event 'a head shows on the first coin". Which events are
(i) Mutually exclusive? (ii) Simple? (iii) Compound?
Solution:-
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
But, now three coins are tossed once so the possible sample space contains,
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTH}
Now,
A: 'three heads'
A= (HHH)
B: "two heads and one tail"
B= (HHT, THH, HTH)
C: 'three tails'
C= (TTT)
D: a head shows on the first coin
D= (HHH, HHT, HTH, HTT)
(i) Mutually exclusive
$A \cap B = (HHH) \cap (HHT, THH, HTH)$
$= \phi$
Therefore, A and C are mutually exclusive.
$A \cap C = (HHH) \cap (TTT)$
$= \phi$
There, A and C are mutually exclusive.
$A \cap D = (HHH) \cap (HHH, HHT, HTH, HTT)$
$= (HHH)$
$A \cap D \neq \phi$
So they are not mutually exclusive
$B \cap C = (HHT, HTH, THH) \cap (TTT)$
$= \phi$
Since there is no common element in B & C, so they are mutually exclusive.
$B \cap D = (HHT, THH, HTH) \cap (HHH, HHT, HTH, HTT)$
$= (HHT, HTH)$
$B \cap D \neq \phi$

Since there are common elements in B & D,
So, they not mutually exclusive.
C ∩ D = (TTT) ∩ (HHH, HHT, HTH, HTT)
= ϕ
Since there is no common element in C & D,
So they are not mutually exclusive.
(ii) Simple event
If an event has only one sample point of a sample space, it is called a simple (or elementary) event.
A = (HHH)
C = (TTT)
Both A & C have only one element,
so they are simple events.
(iii) Compound events
If an event has more than one sample point, it is called a Compound event
B= (HHT, HTH, THH)
D= (HHH, HHT, HTH, HTT)
Both B & D have more than one element,
So, they are compound events.

5. Three coins are tossed. Describe
(i) Two events which are mutually exclusive.
(ii) Three events which are mutually exclusive and exhaustive.
(iii) Two events, which are not mutually exclusive.
(iv) Two events which are mutually exclusive but not exhaustive.
(v) Three events which are mutually exclusive but not exhaustive.
Solution:-
Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
But, now three coins are tossed once so the possible sample space contains,
S= (HHH, HHT, HTH, HTT, THH,THT, TTH, TTT)
(i) Two events which are mutually exclusive.
Let us assume A be the event of getting only head
A = (HHH)
And also let us assume B be the event of getting only Tail
B = (TTT)
So, A ∩ B = ϕ
Since there is no common element in A& B so these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive

Now,

Let us assume P be the event of getting exactly two tails

P = (HTT, TTH, THT)

Let us assume Q be the event of getting at least two heads

Q = (HHT, HTH, THH, HHH)

Let us assume R be the event of getting only one tail

C= (TTT)

P ∩ Q = (HTT, TTH, THT) ∩ (HHT, HTH, THH, HHH)

= ϕ

Since there is no common element in P and Q,

Therefore, they are mutually exclusive

Q ∩ R = (HHT, HTH, THH, HHH) ∩ (TTT)

= ϕ

Since there is no common element in Q and R

Hence, they are mutually exclusive.

P ∩ R = (HTT, TTH, THT) ∩ (TTT)

= ϕ

Since there is no common element in P and R,

So they are mutually exclusive.

Now, P and Q, Q and R, and P and R are mutually exclusive

∴ P, Q, and R are mutually exclusive.

And also,

P ∪ Q ∪ R = (HTT, TTH, THT, HHT, HTH, THH, HHH, TTT) = S

Hence P, Q and R are exhaustive events.

(iii) Two events, which are not mutually exclusive

Let us assume 'A' be the event of getting at least two heads

A = (HHH, HHT, THH, HTH)

Let us assume 'B' be the event of getting only head

B= (HHH)

Now A ∩ B = (HHH, HHT, THH, HTH) ∩ (HHH)

= (HHH)

A ∩ B ≠ ϕ

Since there is a common element in A and B,

So they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive

Let us assume 'P' be the event of getting only Head

$P = (HHH)$

Let us assume 'Q' be the event of getting only tail

$Q = (TTT)$

$P \cap Q = (HHH) \cap (TTT)$

$= \phi$

Since there is no common element in P and Q,

These are mutually exclusive events.

But,

$P \cup Q = (HHH) \cup (TTT)$

$= \{HHH, TTT\}$

$P \cup Q \neq S$

Since $P \cup Q \neq S$ these are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive

Let us assume 'X' be the event of getting only head

$X = (HHH)$

Let us assume 'Y' be the event of getting only tail

$Y = (TTT)$

Let us assume 'Z' be the event of getting exactly two heads

$Z= (HHT, THH, HTH)$

Now,

$X \cap Y = (HHH) \cap (TTT)$

$= \phi$

$X \cap Z = (HHH) \cap (HHT, THH, HTH)$

$= \phi$

$Y \cap Z = (TTT) \cap (HHT, THH, HTH)$

$= \phi$

Therefore, they are mutually exclusive

Also

$X \cup Y \cup Z = (HHH\ TTT, HHT, THH, HTH)$

$X \cup Y \cup Z \neq S$

So, X, Y and Z are not exhaustive.

Hence it is proved that X, Y and X are mutually exclusive but not exhaustive.

6. Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.
B: getting an odd number on the first die.
C: getting the sum of the numbers on the dice ≤ 5.
Describe the events

(i) A^l	**(ii) not B**	**(iii) A or B**
(iv) A and B	**(v) A but not C**	**(vi) B or C**
(vii) B and C	**(viii) $A \cap B^l \cap C^l$**	

Solution:-
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
In the question is given that pair of die is thrown, so sample space will be,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B: getting an odd number on the first die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$$

Then,

$$\text{(i) } A' = \left\{ \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix} \right\} = B$$

(ii) $B' = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = A$

(iii) $A \cup B\ (A\ or\ B) = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix} = S$

(iv) A and B (A ∩ B) = ϕ

(v) $A\ but\ not\ C = A - C = \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$

(vi) $B\ or\ C = B \cup C = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$

(vii) $B\ and\ C = B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii)

$$C' = \begin{Bmatrix} (1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2), \\ (4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \begin{Bmatrix} (2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5), \\ (4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

7. Refer to question 6 above, state true or false: (give reason for your answer)
(i) A and B are mutually exclusive
(ii) A and B are mutually exclusive and exhaustive
(iii) A = B^{I}
(iv) A and C are mutually exclusive
(v) A and B^{I} are mutually exclusive.

(vi) A', B', C are mutually exclusive and exhaustive.
Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
In the question is given that pair of die is thrown, so sample space will be,
By referring the question 6 above,

$$S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \begin{Bmatrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

B: getting an odd number on the first die.

$$B = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \end{Bmatrix}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$$

(i) A and B are mutually exclusive
So, (A ∩ B) = ϕ
So, A & B are mutually exclusive.
Hence, the given statement is true.

(ii) A and B are mutually exclusive and exhaustive

$$A \cup B = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\} = S$$

⇒ A ∪ B = S

From statement (i) we have A and B are mutually exclusive.

∴ A and B are mutually exclusive and exhaustive.

Hence, the statement is true.

(iii) A = B

$$B' = \left\{\begin{matrix} (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5).(4,6) \end{matrix}\right\} = A$$

Therefore, the statement is true.

(iv) A and C are mutually exclusive

We have,

A ∩ C = {(2, 1), (2, 2), (2, 3), (4, 1)}

A ∩ C ≠ ϕ

A and C are not mutually exclusive

Hence, the given statement is false

(v) A and B^l are mutually exclusive.

We have,

A ∩ B^l = A ∩ A = A

∴ A ∩ B^l ≠ ϕ

So, A and B^l not mutually exclusive.

Therefore, the given statement is false.

(vi) A^l, B^l, C are mutually exclusive and exhaustive.

Here $A^l = \left\{\begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \end{matrix}\right\}$

$$B=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\end{Bmatrix}$$

And $C=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(3,1),(3,2),(2,3),(4,1)\}$

$A' \cap B' = \phi$

Hence there is no common element in A' and B'

So they are mutually exclusive.

$B' \cap C = \{(2,1),(2,2),(2,3),(4,1)\}$

$B' \cap C \neq \phi$

They are not mutually exclusive.
Now, since B' and C are not mutually exclusive,
Therefore A', B' and C are not mutually exclusive and exhaustive.
So, the given statement is false.