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1. Which of the following cannot be valid assignment of probabilities for outcomes of sample Space S = {ω_1, ω_2, ω_3, ω_4, ω_5, ω_6, ω_7}

Assignment

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	1/7	1/7	1/7	1/7	1/7	1/7	1/7
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	1/14	2/14	3/14	4/14	5/14	6/14	15/14

Solution:-

(a) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$

Therefore, the given assignment is valid.

b) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$= (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7)$

$= 7/7$

$= 1$

Therefore, the given assignment is valid.

c) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$

$= 2.8 > 1$

Therefore, the 2nd condition is not satisfied

Which states that $p(w_i) \leq 1$

So, the given assignment is not valid.

d) The conditions of axiomatic approach don't hold true in the given assignment, that is

1) Each of the number $p(w_i)$ is less than zero but also negative

To be true each of the number $p(w_i)$ should be less than zero and positive

So, the assignment is not valid

e) Condition (i): Each of the number $p(\omega_i)$ is positive and less than zero. Condition (ii):

Sum of probabilities

= (1/14) + (2/14) + (3/14) + (4/14) + (5/14) + (6/14) + (7/14)

= (28/14) ≥ 1

The second condition doesn't hold true so the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

Here coin is tossed twice, then sample space is S = (TT, HH, TH, HT)

∴ Number of possible outcomes n (S) = 4

Let A be the event of getting at least one tail

∴ n (A) = 3

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

P(A) = n(A)/n(S)

= ¾

3. A die is thrown, find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.

Here S = {1, 2, 3, 4, 5, 6}

∴n(S) = 6

(i) A prime number will appear,

Let us assume 'A' be the event of getting a prime number,

A = {2, 3, 5}

Then, n(A) = 3

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

∴P(A) = n(A)/n(S)

= 3/6

= ½

(ii) A number greater than or equal to 3 will appear,

Let us assume 'B' be the event of getting a number greater than or equal to 3,

B = {3, 4, 5, 6}

Then, n(B) = 4

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

∴P(B) = n(B)/n(S)

= 4/6

= 2/3

(iii) A number less than or equal to one will appear,

Let us assume 'C' be the event of getting a number less than or equal to 1,

C = {1}

Then, n (C) = 1

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

∴P(C) = n(C)/n(S)

= 1/6

(iv) A number more than 6 will appear,

Let us assume 'D' be the event of getting a number more than 6, then

D = {0)}

Then, n (D) = 0

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

∴P(D) = n(D)/n(S)

= 0/6

= 0

(v) A number less than 6 will appear.

Let us assume 'E' be the event of getting a number less than 6, then

E= (1, 2, 3, 4, 5)

Then, n (E) = 5

P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes

∴P(E) = n(E)/n(S)

= 5/6

4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card

Solution:-

From the question it is given that, there are 52 cards in the deck.

(a) Number of points in the sample space = 52 (given)

∴n(S) = 52

(b) Let us assume 'A' be the event of drawing an ace of spades.

A= 1
Then, n (A) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(A) = n(A)/n(S)
= 1/52
(c) Let us assume 'B' be the event of drawing an ace. There are four aces.
Then, n (B)= 4
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(B) = n(B)/n(S)
= 4/52
= 1/13
(d) Let us assume 'C' be the event of drawing a black card. There are 26 black cards.
Then, n (C) = 26
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(C) = n(C)/n(S)
= 26/52
= ½

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12
Solution:-
Let us assume that 1, 2, 3, 4, 5 and 6 are the possible out comes when the die is thrown.
So, the sample space S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)}
Then, n(S) = 12
(i) Let us assume 'P' be the event having sum of numbers as 3.
P = {(1, 2)},
Then, n (P) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(P) = n(P)/n(S)
= 1/12
(ii) Let us assume 'Q' be the event having sum of number as 12.
Then Q = {(6, 6)}, n (Q) = 1
P(Event) = Number of outcomes favorable to event/ Total number of possible outcomes
∴P(Q) = n(Q)/n(S)
= 1/12

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Solution:-

From the question it is given that, there are four men and six women on the city council.
Here total members in the council = 4 + 6 = 10,
Hence, the sample space has 10 points
∴ n (S) = 10
Number of women are 6 … [given]
Let us assume 'A' be the event of selecting a woman
Then n (A) = 6
P(Event) = Number of outcomes favourable to event/Total number of possible outcomes
∴P(A) = n(A)/n(S)
= 6/10 … [divide both numerator and denominators by 2]
= 3/5

7. A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs 1.50 for each tail that turns up.
From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.
But, now coin is tossed four times so the possible sample space contains,
S = (HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, TTTH, TTHT, THTT, HTTT, TTTT)
As per the condition given the question, a person will win or lose money depending up on the face of the coin so,
(i) For 4 heads = 1 + 1 + 1 + 1 = ₹ 4
So, he wins ₹ 4
(ii) For 3 heads and 1 tail = 1 + 1 + 1 – 1.50
= 3 – 1.50
= ₹ 1.50
So, he will be winning ₹ 1.50
(iii) For 2 heads and 2 tails = 1 + 1 – 1.50 – 1.50
= 2 - 3
= - ₹ 1
So, he will be losing ₹ 1
(iv)For 1 head and 3 tails = 1 – 1.50 – 1.50 – 1.50

= 1 – 4.50

= - ₹ 3.50

So, he will be losing Rs. 3.50

(v) For 4 tails = – 1.50 – 1.50 – 1.50 – 1.50

= - ₹ 6

So, he will be losing Rs. 6

Now the sample space of amounts is

S= {4, 1.50, 1.50, 1.50, 1.50, – 1, – 1, – 1, – 1, – 1, – 1, – 3.50, – 3.50, – 3.50, – 3.50, – 6}

Then, n (S) = 16

P (winning ₹ 4) = 1/16

P (winning ₹ 1.50) = 4/16 … [divide both numerator and denominator by 4]

= ¼

P (winning ₹ 1) = 6/16 … [divide both numerator and denominator by 2]

= 3/8

P (winning ₹ 3.50) = 4/16 … [divide both numerator and denominator by 4]

= ¼

P (winning ₹ 6) = 1/16

= 3/8

8. Three coins are tossed once. Find the probability of getting

(i) 3 heads (ii) 2 heads (iii) at least 2 heads

(iv) at most 2 heads (v) no head (vi) 3 tails

(vii) Exactly two tails (viii) no tail (ix) at most two tails

Solution:-

Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

But, now three coin is tossed so the possible sample space contains,

S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where s is sample space and here n(S) = 8

(i) 3 heads

Let us assume 'A' be the event of getting 3 heads

n(A)= 1

∴P(A) = n(A)/n(S)

= 1/8

(ii) 2 heads

Let us assume 'B' be the event of getting 2 heads

n (A) = 3

∴P(B) = n(B)/n(S)

= 3/8

(iii) at least 2 heads

Let us assume 'C' be the event of getting at least 2 head

n(C) = 4

∴P(C) = n(C)/n(S)

= 4/8

= ½

(iv) at most 2 heads

Let us assume 'D' be the event of getting at most 2 heads

n(D) = 7

∴P(D) = n(D)/n(S)

= 7/8

(v) no head

Let us assume 'E' be the event of getting no heads

n(E) = 1

∴P(E) = n(E)/n(S)

= 1/8

(vi) 3 tails

Let us assume 'F' be the event of getting 3 tails

n(F) = 1

∴P(F) = n(F)/n(S)

= 1/8

(vii) Exactly two tails

Let us assume 'G' be the event of getting exactly 2 tails

n(G) = 3

∴P(G) = n(G)/n(S)

= 3/8

(viii) no tail

Let us assume 'H' be the event of getting no tails

n(H) = 1

∴P(H) = n(H)/n(S)

= 1/8

(ix) at most two tails

Let us assume 'I' be the event of getting at most 2 tails

n(I) = 7

∴P(I) = n(I)/n(S)
= 7/8

9. If 2/11 is the probability of an event, what is the probability of the event 'not A'.
Solution:-
From the question it is given that, 2/11 is the probability of an event A,
i.e. P (A) = 2/11
Then,
P (not A) = 1 – P (A)
= 1 – (2/11)
= (11 - 2)/11
= 9/11

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is (i) a vowel (ii) a consonant
Solution:-
The word given in the question is 'ASSASSINATION'.
Total letters in the given word = 13
Number of vowels in the given word = 6
Number of consonants in the given word = 7
Then, the sample space n(S) = 13
(i) a vowel
Let us assume 'A' be the event of selecting a vowel
n(A) = 6
∴P(A) = n(A)/n(S)
= 6/13
(ii) Let us assume 'B' be the event of selecting the consonant
n(B)= 7
∴P(B) = n(B)/n(S)
= 7/13

11. In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint order of the numbers is not important.]
Solution:-
From the question it is given that,

Total numbers of numbers in the draw = 20

Numbers to be selected = 6

∴ n (S) = $^{20}C_6$

Let us assume 'A' be the event that six numbers match with the six numbers already fixed by the lottery committee.

∴ n(A) = $^6C_6$ = 1

Probability of winning the prize

$$P(A) = \frac{n(A)}{n(S)} = \frac{^6C_6}{^{20}C_6} = \frac{6!\,14!}{20!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 14!}{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!}$$

$$= \frac{1}{38760}$$

12. Check whether the following probabilities P(A) and P(B) are consistently defined

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Solution:-

(i) P(A) = 0.5, P(B) = 0.7, P(A ∩ B) = 0.6

P(A ∩ B) > P(A)

Therefore, the given probabilities are not consistently defined.

(ii) P(A) = 0.5, P(B) = 0.4, P(A ∪ B) = 0.8

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.8 = 0.5 + 0.4 - P(A ∩ B)

Transposing - P(A ∩ B) to LHS and it becomes P(A ∩ B) and 0.8 to RHS and it becomes – 0.8.

P(A ∩ B) = 0.9 – 0.8

= 0.1

Therefore, P(A ∩ B) < P(A) and P(A ∩ B) < P(B)

So, the given probabilities are consistently defined.

13. Fill in the blanks in following table:

	P(A)	P(B)	P(A ∩ B)	P(A ∪ B)
(i)	**1/3**	**1/5**	**1/15**	**….**
(ii)	**0.35**	**…..**	**0.25**	**0.6**
(iii)	**0.5**	**0.35**	**….**	**0.7**

Solution:-

From the given table,

(i) P(A) = 1/3, P(B) = 1/5, P(A ∩ B) = 1/15, P(A ∪ B) = ?

We know that,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= (1/3) + (1/5) – (1/15)
= ((5 + 3)/15) – (1/15)
= (8/15) – (1/15)
= (8 - 1)/15
= 7/15

(ii) P(A) = 0.35, P(B) = ?, P(A ∩ B) = 0.25, P(A ∪ B) = 0.6

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.6 = 0.35 + P(B) - 0.25

Transposing – 0.25, 0.35 to LHS and it becomes 0.25and – 0.35.

P(B) = 0.6 + 0.25 – 0.35
= 0.5

(iii) P(A) = 0.5, P(B) = 0.35, P(A ∪ B) = 0.7, P(A ∩ B) = ?

Then,

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

0.7 = 0.5 + 0.35 - P(A ∩ B)

Transposing - P(A ∩ B) to LHS and it becomes P(A ∩ B) and 0.7 to RHS and it becomes – 0.7.

P(A ∩ B) = 0.85 – 0.7
= 0.15

14. Given P(A) = 5/3 and P(B) = 5/1 . Find P(A or B), if A and B are mutually exclusive events.

Solution:-

From the question it is given that,

P(A) = 5/3 and P(B) = 5/1

Then, P(A or B), if A and B are mutually exclusive

P(A∪B) or P(A or B) = P(A) + P(B)

= (3/5) + (1/5)
= 4/5

15. If E and F are events such that P(E) = ¼ , P(F) = ½ and P(E and F) = 1/8 , find (i) P(E or F), (ii) P(not E and not F)

Solution:-

From the question, we have P(E) = ¼ , P(F) = ½ and P(E ∩ F) = 1/8

(i) P(E or F) i.e. P(E∪F) = P(E) + P(F) – P(E ∩ F)
= ¼ + ½ - (1/8)
= 5/8

(ii) $P(\text{not E and not F}) = P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$
= 1 – (5/8)
= (8 - 5)/8
= 3/8

16. Events E and F are such that P(not E or not F) = 0.25, State whether E and F are mutually exclusive.

Solution:-

From the question it is given that, P(not E and not F) = 0.25

So, $P(\bar{E} \cup \bar{F}) = 0.25$

Then we have,

$\Rightarrow P(\overline{E \cap F}) = 0.25$

⇒1-P(E∩F)=0.25

⇒P(E∩F)= 1 - 0.25

= 0.75

P(E∩F) ≠0

Hence, E and F are not mutually exclusive events.

17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine (i) P(not A), (ii) P(not B) and (iii) P(A or B)

Solution:-

From the question it is given that, P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16.

(i) P(not A) = 1 – P(A)
= 1 – 0.42
= 0.58
(ii) P(not B) = 1 – P(B)
= 1 – 0.48
= 0.52
(iii) P(A not B) = P(A ∪ B) = P(A) + P(B) – P(A ∩ B)
= 0.42 + 0.48 – 0.16
= 0.74

18. In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Solution:-

Let us assume 'A' be the event that the student is studying mathematics and 'B' be the event that the student is studying biology.

So, P(A) = 40/100
= 2/5

And, P(B) = 30/100
= 3/10

Then, P(A ∩ B) = (10/100)
= 1/10, P(A ∩ B) is probability of studying both mathematics and biology.

Here, Probability of studying mathematics or biology will be given by P (AUB)

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
= (2/5) + (3/10) – (1/10)
= 6/10
= 3/5

Hence, (3/5) is the probability that the student will studying mathematics or biology.

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Solution:-

Let us assume probability of a randomly chosen student passing the first examination is 0.8 be P(A).

And also assume the probability of passing the second examination is 0.7 be P(B)

Then,

$P(A \cup B)$ is probability of passing at least one of the examination

Now,

$P(A \cup B) = 0.95$, $P(A)=0.8$, $P(B)=0.7$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.95 = 0.8 + 0.7 - P(A \cap B)$

Transposing $- P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.95 to RHS and it becomes - 0.95

$P(A \cap B) = 1.5 - 0.95$

$= 0.55$

Hence, 0.55 is the probability that student will pass both the examinations.

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Solution:-

Let us assume probability of passing the English examination is 0.75 be P(A).

And also assume the probability of passing the Hindi examination is be P(B).

Here given, $P(A) = 0.75$, $P(A \cap B) – 0.5$, $P(A' \cap B') = 0.1$

We know that, $P(A' \cap B') = 1 – P(A \cup B)$

Then, $P(A \cup B) = 1 - P(A' \cap B')$

$= 1 – 0.1$

$= 0.9$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.9 = 0.75 + P(B) – 0.5$

Transposing 0.75, - 0.5 to LHS and it becomes - 0.75, 0.5.

$P(B) = 0.9 + 0.5 - 0.75$

$= 0.65$

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that
(i) The student opted for NCC or NSS.
(ii) The student has opted neither NCC nor NSS.
(iii) The student has opted NSS but not NCC.
Solution:-
From the question it is given that,
The total number of students in class = 60
Thus, the sample space consist of n(S) = 60
Let us assume that the students opted for NCC be 'A'
And also assume that the students opted for NSS be 'B'
So, $n(A) = 30$, $n(B) = 32$, $n(A \cap B) = 24$
We know that, $P(A) = n(A)/n(S)$
$= 30/60$
$= ½$
$P(B) = n(B)/n(S)$
$= 32/60$
$= 8/15$
$P(A \cap B) = n(A \cap B)/n(S)$
$= 24/60$
$= 2/5$
Therefore, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(i) The student opted for NCC or NSS.
P (A or B) = P(A) + P(B) –P(A and B)
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= ½ + (8/15) – (2/5)$
$= 19/30$
(ii) P(student opted neither NCC nor NSS)
P(not A and not B) = $P(A' \cap B')$
We know that, $P(A' \cap B') = 1 – P(A \cup B)$
$= 1 – (19/30)$
$= 11/30$
(iii) P(student opted NSS but not NCC)
$n(B - A) = n(B) – n(A \cap B)$
$\Rightarrow 32 – 24 = 8$
The probability that the selected student has opted for NSS and not NCC is
$= (8/60) = 2/15$