MISCELLANEOUS EXERCISE PAGE: 409

1. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that
(i) all will be blue? (ii) at least one will be green?
Solution:-
From the question it is given that,
Number of red marbles in the box = 10
Number of blue marbles in the box = 20
Number of green marbles in the box = 30
So, Total number of marbles in the box = 10 + 20 + 30 = 60
Number of ways of drawing 5 marbles from 60 marbles = $^{60}C_5$
(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.
We have,
Number of ways of drawing 5 blue marbles from 20 blue marbles = $^{20}C_5$
Then,
Probability that all marbles will be blue = $^{20}C_5$ / $^{60}C_5$
(ii) Number of ways in which the drawn marble is not green = $^{(20+10)}C_5$
We have,
Probability that no marble is green = $^{30}C_5$ / $^{60}C_5$
Then,
Probability that at least one marble is green = 1 - $^{30}C_5$ / $^{60}C_5$

2. 4 cards are drawn from a well – shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?
Solution:-
From the question it is given that,
4 cards are drawn from a well – shuffled deck of 52 cards
Number of ways of drawing 4 cards from 52 cards = $^{52}C_4$
In a deck of 52 cards, there are 13 diamonds and 13 spades.
Number of ways of drawing 3 diamonds and one spade = $^{13}C_3 \times {}^{13}C_1$
Therefore, the probability of obtaining 3 diamonds and one spade
= $(^{13}C_3 \times {}^{13}C_1)/ {}^{52}C_4$

3. A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine
(i) P(2) (ii) P(1 or 3) (iii) P(not 3)
Solution:-

From the question it is given that,
Die has two faces each with number '1',
Three faces each with number '2'
And one face with number '3'
Then, Total number of faces of die = 6
(i) P(2)
Number faces with number '2' = 3 … [given]
So, P(2) = 3/6
= ½
(ii) P(1 or 3)
We know that, P (1 or 3) = P (not 2) = 1 − P (2)
So, P(1 or 3) = 1 – ½
= (2 -1)/2
= ½
(iii) P(not 3)
Number of faces with number '3' = 1
P(3) = 1/6
P(not 3) = 1 – P(3)
= 1 – 1/6
= (6 - 1)/6
= 5/6

4. In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets.

Solution:-

From the question it is given that,
Number of lottery tickets sold = 10,000
Number prizes awarded = 10
(a) The probability of not getting a prize if we buy one ticket,
P (getting a prize) = 10/10000
= 1/1000
Then,
P (not getting a prize) = 1 – (1/1000)
= (1000 – 1)/1000
= 999/1000
(b) The probability of not getting a prize if we buy two tickets,

Then,
Number of tickets not awarded = 10,000 − 10 = 9990
Therefore, P (not getting a prize) = $^{9990}C_2 / ^{10000}C_2$
(iii) The probability of not getting a prize if we buy 10 tickets, then
Number of tickets not awarded = 10,000 − 10 = 9990
Therefore, P (not getting a prize) = $^{9990}C_{10} / ^{10000}C_{10}$

5. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that
(a) you both enter the same section?
(b) you both enter the different sections?
Solution:-
From the question,
Total number of students = 100
I and my friend are among the 100 students.
Then, two sections of 40 and 60 are formed.
Total number of ways of selecting 2 students out of 100 students =
(a) Let S = the two of us will enter the same section if both of us are among 40 students or among 60 students.
Number of ways in which both of us enter the same section = P(S)
P(S) = $(^{40}C_2 + ^{60}C_2) / ^{100}C_2$

$$P(S) = \frac{\frac{40!}{2!\times 38!}+\frac{60!}{2!\times 58!}}{\frac{100!}{2!\times 98!}} = \frac{39\times 40+59\times 60}{99\times 100} = \frac{5100}{9900} = \frac{17}{33}$$

(b) P(we enter different sections)
= 1 − P(we enter the same section)
P(we enter different sections) = 1 – (17/33)
= (33 - 17)/33
= 16/33

6. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.
Solution:-
Let us assume L1, L2, L3 be three letters and E1, E2, and E3 be their corresponding envelops respectively.

Then, Sample space is
L_1E_1, L_2E_3, L_3E_2,
L_2E_2, L_1E_3, L_3E_1,
L_3E_3, L_1E_2, L_2E_1,
L_1E_1, L_2E_2, L_3E_3,
L_1E_2, L_2E_3, L_3E_1,
L_1E_3, L_2E_1, L_3E_2,
Hence, there are 6 ways of inserting 3 letters in 3 envelops.
And there are 4 ways in which at least one letter is inserted in proper envelope. (first 4 rows of sample space)
Probability that at least one letter is inserted in proper envelope,
= 4/6
= 2/3

7. A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) = 0.35. Find (i) P(A ∪ B) (ii) P(A' ∩ B') (iii) P(A ∩ B') (iv) P(B ∩ A')
Solution:-
From the question it is given that, A and B are two events such that,
P(A) = 0.54, P(B) = 0.69, P(A ∩ B) = 0.35
(i) P (A ∪ B)
We know that, P (A ∪ B) = P(A) + P(B) − P(A ∩ B)
⇒ P (A ∪ B) = 0.54 + 0.69 − 0.35 = 0.88
∴ P (A ∪ B) = 0.88

(ii) P(A' ∩ B')
We know that, A' ∩ B' = (A ∪ B)' [by De Morgan's law]
So, P (A' ∩ B') = P(A ∪ B)'
= 1 − P(A ∪ B)
= 1 − 0.88 = 0.12
∴ P (A' ∩ B') = 0.12

(iii) P(A ∩ B')
We have,
P (A ∩ B') = P(A) − P(A ∩ B)
= 0.54 − 0.35
= 0.19
Therefore, P (A ∩ B') = 0.19

(iv) $P(B \cap A^{I})$

We know that: $P(B \cap A^{I}) = P(B) - P(A \cap B)$

$\Rightarrow P(B \cap A^{I}) = 0.69 - 0.35$

Hence, $P(B \cap A^{I}) = 0.34$

8. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S.No.	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Solution:-

From the given table,

The number of person = 5

Out of 5 persons 3 are Male

Out of 5 persons 2 are 35 years of age.

Let us assume 'A' be the event in which the spokesperson will be a male and B be the event in which the spokesperson will be over 35 years of age.

Accordingly, $P(A) = 3/5$ and $P(B) = 2/5$

Since there is only one male who is over 35 years of age,

$P(A \cap B) = 1/5$

We know that: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= (3/5) + (2/5) - (1/5)$

$= 4/5$

Hence, the probability that the spokesperson will either be a male or over 35 years of age is 4/5.

9. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) the repetition of digits is not allowed?

Solution:-

(i) When the digits are repeated

Since four-digit numbers greater than 5000 are formed,
The thousand's place digit is either 7 or 5.
Total number of 4-digit numbers greater than 5000 = 2 × 5 × 5 × 5 − 1
= 250 − 1
= 249
A number is divisible by 5 if the digit at its unit's place is either 0 or 5.
∴ Total number of 4-digit numbers greater than 5000 that are divisible by 5
= 2 × 5 × 5 × 2 − 1
= 100 − 1 = 99
Hence, the probability of forming a number divisible by 5 when the digits are repeated is
= P (number divisible by 5 when digits repeated)
P(number divisible by 5 when digits repeated) = 99/249
= 33/83

(ii) When repetition of digits is not allowed
The thousands place can be filled with either of the two digits 5 or 7 i.e. by 2 ways.
The remaining 3 places can be filled with any of the remaining 4 digits.
Total number of 4-digit numbers greater than 5000 = 2 × 4 × 3 × 2 = 48
Here, number of 4-digit numbers starting with 5 and divisible by 5
= 1 × 3 × 2 × 1 = 6
Here, number of 4-digit numbers starting with 7 and divisible by 5
= 1 × 2 × 3 × 2 = 12
Total number of 4-digit numbers greater than 5000 that are divisible by 5
= 6 + 12 = 18
Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed
= P (number divisible by 5 when digits are not repeated)
P (number divisible by 5 when digits are not repeated) = 18/48
= 3/8

10. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?
Solution:-
From the question it is given that,
The number lock has 4 wheels, each labelled with ten digits i.e., from 0 to 9.

Then,
The Number of ways of selecting 4 different digits out of the 10 digits = ${}^{10}C_4$
Now, each combination of 4 different digits can be arranged in 4! Ways.
We have,
Number of four digits with no repetitions = $4! \times {}^{10}C_4 = 5040$
There is only one number that can open the suitcase.
Hence, the required probability is 1/5040