Exercise 2.1

1. If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of *x* and *y*.
Solution:

Given, $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

As the ordered pairs are equal, the corresponding elements should also be equal.
Thus,
x/3 + 1 = 5/3 and y – 2/3 = 1/3
Solving, we get
x + 3 = 5 and 3y – 2 = 1 [Taking L.C.M and adding]
x = 2 and 3y = 3
Therefore,
x = 2 and y = 1

2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?
Solution:

Given, set A has 3 elements and the elements of set B are {3, 4, and 5}.
So, the number of elements in set B = 3
Then, the number of elements in (A × B) = (Number of elements in A) × (Number of elements in B)
= 3 × 3 = 9
Therefore, the number of elements in (A × B) will be 9.

3. If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.
Solution:

Given, G = {7, 8} and H = {5, 4, 2}
We know that,
The Cartesian product of two non-empty sets P and Q is given as
P × Q = {(*p*, *q*): *p* ∈ P, *q* ∈ Q}
So,
G × H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}
H × G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

4. State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.
(i) If P = {*m*, *n*} and Q = {*n*, *m*}, then P × Q = {(*m*, *n*), (*n*, *m*)}.
(ii) If A and B are non-empty sets, then A × B is a non-empty set of ordered pairs (*x*, *y*) such that *x* ∈ A and *y* ∈ B.
(iii) If A = {1, 2}, B = {3, 4}, then A × (B ∩ Φ) = Φ.
Solution:

(i) The statement is False. The correct statement is:
If P = {m, n} and Q = {n, m}, then
P × Q = {(m, m), (m, n), (n, m), (n, n)}
(ii) True
(iii) True

5. If A = {–1, 1}, find A × A × A.
Solution:

The A × A × A for a non-empty set A is given by
A × A × A = {(a, b, c): a, b, c ∈ A}
Here, given A = {–1, 1}
So,
A × A × A = {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

6. If A × B = {(a, x), (a, y), (b, x), (b, y)}. Find A and B.
Solution:

Given,
A × B = {(a, x), (a, y), (b, x), (b, y)}
We know that the Cartesian product of two non-empty sets P and Q is given by:
P × Q = {(p, q): p ∈ P, q ∈ Q}
Hence, A is the set of all first elements and B is the set of all second elements.
Therefore, A = {a, b} and B = {x, y}

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that
(i) A × (B ∩ C) = (A × B) ∩ (A × C)
(ii) A × C is a subset of B × D
Solution:

Given,
A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}
(i) To verify: A × (B ∩ C) = (A × B) ∩ (A × C)
Now, B ∩ C = {1, 2, 3, 4} ∩ {5, 6} = Φ
Thus,
L.H.S. = A × (B ∩ C) = A × Φ = Φ
Next,
A × B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}
A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
Thus,
R.H.S. = (A × B) ∩ (A × C) = Φ
Therefore, L.H.S. = R.H.S
- Hence verified

(ii) To verify: A × C is a subset of B × D
First,

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}
And,
B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}
Now, it's clearly seen that all the elements of set A × C are the elements of set B × D.
Thus, A × C is a subset of B × D.
- Hence verified

8. Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have? List them.
Solution:

Given,
A = {1, 2} and B = {3, 4}
So,
A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}
Number of elements in A × B is n(A × B) = 4
We know that,
If C is a set with n(C) = m, then n[P(C)] = 2^m.
Thus, the set A × B has 2^4 = 16 subsets.
And, these subsets are as below:
Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

9. Let A and B be two sets such that n(A) = 3 and n (B) = 2. If (x, 1), (y, 2), (z, 1) are in A × B, find A and B, where x, y and z are distinct elements.
Solution:

Given,
n(A) = 3 and n(B) = 2; and (x, 1), (y, 2), (z, 1) are in A × B.
We know that,
A = Set of first elements of the ordered pair elements of A × B
B = Set of second elements of the ordered pair elements of A × B.
So, clearly x, y, and z are the elements of A; and
1 and 2 are the elements of B.
As n(A) = 3 and n(B) = 2, it is clear that set A = {x, y, z} and set B = {1, 2}.

10. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.
Solution:

We know that,
If n(A) = p and n(B) = q, then n(A × B) = pq.
Also, n(A × A) = n(A) × n(A)
Given,
n(A × A) = 9

So, $n(A) \times n(A) = 9$
Thus, $n(A) = 3$
Also given that, the ordered pairs (–1, 0) and (0, 1) are two of the nine elements of A × A.
And, we know in $A \times A = \{(a, a): a \in A\}$.
Thus, –1, 0, and 1 has to be the elements of A.
As $n(A) = 3$, clearly $A = \{–1, 0, 1\}$.
Hence, the remaining elements of set A × A are as follows:
(–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0), and (1, 1)