## Exercise 2.2



**1. Let A = {1, 2, 3, … , 14}. Define a relation R from A to A by R = {($x$, $y$): $3x - y = 0$, where $x, y \in$ A}. Write down its domain, codomain and range.**
**Solution:**

The relation R from A to A is given as:
R = {($x$, $y$): $3x - y = 0$, where $x, y \in$ A}
= {($x$, $y$): $3x = y$, where $x, y \in$ A}
So,
R = {(1, 3), (2, 6), (3, 9), (4, 12)}
Now,
The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of R = {1, 2, 3, 4}
The whole set A is the codomain of the relation R.
Hence, Codomain of R = A = {1, 2, 3, …, 14}
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of R = {3, 6, 9, 12}

**2. Define a relation R on the set N of natural numbers by R = {($x$, $y$): $y = x + 5$, $x$ is a natural number less than 4; $x, y \in$ N}. Depict this relationship using roster form. Write down the domain and the range.**
**Solution:**

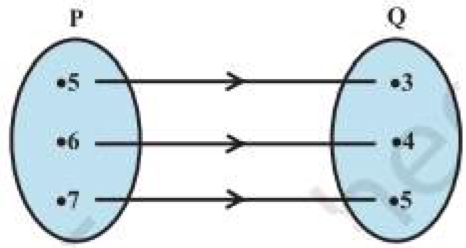
**The relation R is given by:**
R = {($x$, $y$): $y = x + 5$, $x$ is a natural number less than 4, $x, y \in$ **N**}
The natural numbers less than 4 are 1, 2, and 3.
So,
R = {(1, 6), (2, 7), (3, 8)}
Now,
The domain of R is the set of all first elements of the ordered pairs in the relation.
Hence, Domain of R = {1, 2, 3}
The range of R is the set of all second elements of the ordered pairs in the relation.
Hence, Range of R = {6, 7, 8}

**3. A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {($x$, $y$): the difference between $x$ and $y$ is odd; $x \in$ A, $y \in$ B}. Write R in roster form.**
**Solution:**

Given,
A = {1, 2, 3, 5} and B = {4, 6, 9}
The relation from A to B is given as:
R = {($x$, $y$): the difference between $x$ and $y$ is odd; $x \in$ A, $y \in$ B}
Thus,
R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

**4. The figure shows a relationship between the sets P and Q. write this relation**
**(i) in set-builder form (ii) in roster form.**

**What is its domain and range?**
**Solution:**




From the given figure, it's seen that
P = {5, 6, 7}, Q = {3, 4, 5}
The relation between P and Q:
Set-builder form
(i) R = {($x$, $y$): $y = x - 2$; $x \in$ P} or R = {($x$, $y$): $y = x - 2$ for $x$ = 5, 6, 7}
Roster form
(ii) R = {(5, 3), (6, 4), (7, 5)}
Domain of R = {5, 6, 7}
Range of R = {3, 4, 5}

**5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by**
**{($a$, $b$): $a, b \in$ A, $b$ is exactly divisible by $a$}.**
**(i) Write R in roster form**
**(ii) Find the domain of R**
**(iii) Find the range of R.**
**Solution:**

Given,
A = {1, 2, 3, 4, 6} and relation R = {($a$, $b$): $a, b \in$ A, $b$ is exactly divisible by $a$}
Hence,
(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}
(ii) Domain of R = {1, 2, 3, 4, 6}
(iii) Range of R = {1, 2, 3, 4, 6}

**6. Determine the domain and range of the relation R defined by R = {($x$, $x + 5$): $x \in$ {0, 1, 2, 3, 4, 5}}.**
**Solution:**

Given,
Relation R = {($x$, $x + 5$): $x \in$ {0, 1, 2, 3, 4, 5}}
Thus,
R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
So,
Domain of R = {0, 1, 2, 3, 4, 5} and,
Range of R = {5, 6, 7, 8, 9, 10}

**7. Write the relation R = {($x$, $x^3$): $x$ is a prime number less than 10} in roster form.**
**Solution:**

Given,
Relation R = {($x$, $x^3$): $x$ is a prime number less than 10}
The prime numbers less than 10 are 2, 3, 5, and 7.
Therefore,

R = {(2, 8), (3, 27), (5, 125), (7, 343)}

**8. Let A = {*x*, *y*, z} and B = {1, 2}. Find the number of relations from A to B.**
**Solution:**

Given, A = {*x*, *y*, z} and B = {1, 2}.
Now,
A × B = {(*x*, 1), (*x*, 2), (*y*, 1), (*y*, 2), (*z*, 1), (*z*, 2)}
As *n*(A × B) = 6, the number of subsets of A × B will be $2^6$.
Thus, the number of relations from A to B is $2^6$.

**9. Let R be the relation on Z defined by R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}. Find the domain and range of R.**
**Solution:**

Given,
Relation R = {(*a*, *b*): *a*, *b* ∈ Z, *a* – *b* is an integer}
We know that the difference between any two integers is always an integer.
Therefore,
Domain of R = Z and Range of R = Z