

EXERCISE 3.2

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Find the values of other five trigonometric functions in Exercises 1 to 5.

1. $\cos x = -1/2$, x lies in third quadrant.

Solution:

It is given that

$$\cos x = -1/2$$

$$\sec x = 1/\cos x$$

Substituting the values

$$= \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

Consider

$$\sin^2 x + \cos^2 x = 1$$

We can write it as

$$\sin^2 x = 1 - \cos^2 x$$

Substituting the values

$$\sin^2 x = 1 - (-1/2)^2$$

$$\sin^2 x = 1 - 1/4 = 3/4$$

$$\sin^2 x = \pm \sqrt{3}/2$$

Here x lies in the third quadrant so the value of $\sin x$ will be negative

$$\sin x = -\sqrt{3}/2$$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

2. $\sin x = 3/5$, x lies in second quadrant.

Solution:

It is given that

$$\sin x = 3/5$$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

We know that

$$\sin^2 x + \cos^2 x = 1$$

We can write it as

$$\cos^2 x = 1 - \sin^2 x$$

Substituting the values

$$\cos^2 x = 1 - (3/5)^2$$

$$\cos^2 x = 1 - 9/25$$

$$\cos^2 x = 16/25$$

$$\cos x = \pm 4/5$$

Here x lies in the second quadrant so the value of $\cos x$ will be negative

$$\cos x = -4/5$$

We can write it as

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$$

Here

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

3. $\cot x = 3/4$, x lies in third quadrant.

Solution:

It is given that

$$\cot x = 3/4$$

We can write it as

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

We know that

$$1 + \tan^2 x = \sec^2 x$$

We can write it as

$$1 + (4/3)^2 = \sec^2 x$$

Substituting the values

$$1 + 16/9 = \sec^2 x$$

$$\sec^2 x = 25/9$$

$$\sec x = \pm 5/3$$

Here x lies in the third quadrant so the value of $\sec x$ will be negative

$$\sec x = -5/3$$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{4}{3} = \frac{\sin x}{\left(-\frac{3}{5}\right)}$$

By further calculation

$$\sin x = \left(\frac{4}{3}\right) \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

4. sec x = 13/5, x lies in fourth quadrant.

Solution:

It is given that

$$\sec x = 13/5$$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

We know that

$$\sin^2 x + \cos^2 x = 1$$

We can write it as

$$\sin^2 x = 1 - \cos^2 x$$

Substituting the values

$$\sin^2 x = 1 - (5/13)^2$$

$$\sin^2 x = 1 - 25/169 = 144/169$$

$$\sin^2 x = \pm 12/13$$

Here x lies in the fourth quadrant so the value of sin x will be negative

$$\sin x = -12/13$$

We can write it as

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

So we get

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$$

Here

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

5. $\tan x = -5/12$, x lies in second quadrant.

Solution:

It is given that

$$\tan x = -5/12$$

We can write it as

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

We know that

$$1 + \tan^2 x = \sec^2 x$$

We can write it as

$$1 + (-5/12)^2 = \sec^2 x$$

Substituting the values

$$1 + 25/144 = \sec^2 x$$

$$\sec^2 x = 169/144$$

$$\sec x = \pm 13/12$$

Here x lies in the second quadrant so the value of $\sec x$ will be negative

$$\sec x = -13/12$$

We can write it as

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{13}{12}\right)} = -\frac{12}{13}$$

So we get

$$\tan x = \frac{\sin x}{\cos x}$$

$$-\frac{5}{12} = \frac{\sin x}{\left(-\frac{12}{13}\right)}$$

By further calculation

$$\sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

Here

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

Find the values of the trigonometric functions in Exercises 6 to 10.

6. $\sin 765^\circ$

Solution:

We know that values of $\sin x$ repeat after an interval of 2π or 360°

So we get

$$\sin 765^\circ = \sin(2 \times 360^\circ + 45^\circ)$$

By further calculation

$$= \sin 45^\circ$$

$$= 1/\sqrt{2}$$

7. cosec (-1410°)

Solution:

We know that values of cosec x repeat after an interval of 2π or 360°

So we get

$$\text{cosec } (-1410^\circ) = \text{cosec}(-1410^\circ + 4 \times 360^\circ)$$

By further calculation

$$= \text{cosec } (-1410^\circ + 1440^\circ)$$

$$= \text{cosec } 30^\circ = 2$$

8. $\tan \frac{19\pi}{3}$

Solution:

We know that values of $\tan x$ repeat after an interval of π or 180°

So we get

$$\tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$

By further calculation

$$= \tan \left(6\pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3}$$

We get

$$= \tan 60^\circ$$

$$= \sqrt{3}$$

9. $\sin \left(-\frac{11\pi}{3} \right)$

Solution:

We know that values of $\sin x$ repeat after an interval of 2π or 360°

So we get

$$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right)$$

By further calculation

$$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

10. $\cot\left(-\frac{15\pi}{4}\right)$

Solution:

We know that values of $\tan x$ repeat after an interval of π or 180°

So we get

$$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$$

By further calculation

$$= \cot\frac{\pi}{4} = 1$$

