Exercise 5.2

Find the modulus and the arguments of each of the complex numbers in Exercises 1 to 2.

1. z = – 1 – i √3

Solution:

Given,

$z = -1 - i\sqrt{3}$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we get

$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$

$r^2 = 4 \quad [\cos^2\theta + \sin^2\theta = 1]$

$r = \sqrt{4} = 2 \quad [\text{Conventionally, } r > 0]$

Thus, modulus = 2

So, we have

$2\cos\theta = -1$ and $2\sin\theta = -\sqrt{3}$

$\cos\theta = \frac{-1}{2}$ and $\sin\theta = \frac{-\sqrt{3}}{2}$

As the values of both sin θ and cos θ are negative, θ lies in III Quadrant,

$\text{Argument} = -\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$

Therefore, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

2. z = -√3 + i

Solution:

Given,

$z = -\sqrt{3} + i$

Let $r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding , we get

$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\sqrt{3}\right)^2 + 1^2$

$r^2 = 3 + 1 = 4$ $\quad\left[\cos^2\theta + \sin^2\theta = 1\right]$

$r = \sqrt{4} = 2$ $\quad$[Conventionally, $r > 0$]

Thus, modulus = 2

So,

$2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\quad$[As θ lies in the II quadrant]

Therefore, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$ respectively.

Convert each of the complex numbers given in Exercises 3 to 8 in the polar form:

3. 1 – i

Solution:

Given complex number,

$1 - i$

Let $r\cos\theta = 1$ and $r\sin\theta = -1$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$

$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$

$r^2 = 2$

$r = \sqrt{2}$ = Modulus [Conventionally, $r > 0$]

So,

$\sqrt{2}\cos\theta = 1$ and $\sqrt{2}\sin\theta = -1$

$\cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta = -\frac{\pi}{4}$ $\quad$[As θ lies in the IV quadrant]

So,

$1 - i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

Hence, this is the required polar form.

4. – 1 + i
Solution:

Given complex number,
$-1 + i$
Let $r\cos\theta = -1$ and $r\sin\theta = 1$
On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$
$$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$
$$r^2 = 2$$
$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$$\sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$
$$\cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Hence, it can be written as

$$-1 + i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4}$$
$$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

5. – 1 – i
Solution:

Given complex number,
$-1 - i$
Let $r\cos\theta = -1$ and $r\sin\theta = -1$
On squaring and adding, we get

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$
$$r^2\left(\cos^2\theta + \sin^2\theta\right) = 1 + 1$$
$$r^2 = 2$$
$$r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$

So,

$\sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = -\dfrac{1}{\sqrt{2}}$ and $\sin\theta = -\dfrac{1}{\sqrt{2}}$

$\therefore \theta = -\left(\pi - \dfrac{\pi}{4}\right) = -\dfrac{3\pi}{4}$ [As θ lies in the III quadrant]

Hence, it can be written as

$$-1 - i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$$

$$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

6. – 3
Solution:

Given complex number,

–3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$

$r^2(\cos^2\theta + \sin^2\theta) = 9$

$r^2 = 9$

$r = \sqrt{9} = 3$ [Conventionally, $r > 0$]

So,

$3\cos\theta = -3$ and $3\sin\theta = 0$

$\Rightarrow \cos\theta = -1$ and $\sin\theta = 0$

$\therefore \theta = \pi$

Hence, it can be written as

$-3 = r\cos\theta + i r\sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$

This is the required polar form.

7. 3 + *i*
Solution:

Given complex number,

$\sqrt{3} + i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2\cos^2\theta + r^2\sin^2\theta = (\sqrt{3})^2 + 1^2$

$r^2(\cos^2\theta+\sin^2\theta)=3+1$

$r^2=4$

$r=\sqrt{4}=2$ [Conventionally, $r>0$]

So,

$2\cos\theta=\sqrt{3}$ and $2\sin\theta=1$

$\Rightarrow \cos\theta=\frac{\sqrt{3}}{2}$ and $\sin\theta=\frac{1}{2}$

$\therefore \theta=\frac{\pi}{6}$ [As θ lies in the I quadrant]

Hence, it can be written as

$\sqrt{3}+i=r\cos\theta+ir\sin\theta=2\cos\frac{\pi}{6}+i2\sin\frac{\pi}{6}$

$=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

This is the required polar form.

8. ***i***

Solution:

Given complex number, i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we get

$r^2\cos^2\theta+r^2\sin^2\theta=0^2+1^2$

$r^2(\cos^2\theta+\sin^2\theta)=1$

$r^2=1$

$r=\sqrt{1}=1$ [Conventionally, $r>0$]

So,

$\cos\theta=0$ and $\sin\theta=1$

$\therefore \theta=\frac{\pi}{2}$

Hence, it can be written as

$i=r\cos\theta+ir\sin\theta=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

This is the required polar form.