Polynomials

Polynomials in One Variable

Polynomials

An expression of two or more than two algebraic terms that contain variable(s) that are raised to non-negative integral powers are called polynomials.

Types of Polynomials

Based on the number of terms a polynomial can be classified into monomial, binomial, trinomial, etc.

- An algebraic expression having only **one term** is called a **monomial**. $P(x) = x$ is a monomial.
- Polynomials having **two terms** are called **binomials**. $P(x) = x^2 + 2x$ has two terms, x^2 and $2x$.So, it is a binomial.
- Polynomials having **three terms** are called **trinomials**. $P(x) = x^4 + 3x^2 - 4$ has three terms, x^4, $3x^2$ and -4.So, it is a trinomial.
- An algebraic expression of the form $P(x) = c$, where c is a constant is called **constant polynomial.**
- The constant polynomial **0** is called the **zero polynomial**.

Degree of a Polynomial

The **degree** of a **polynomial** is the **highest degree** of its individual terms with non-zero coefficients. The **degree** of a **term** is the **sum of the exponents** of the variables that appear in it.
For a polynomial in **one variable**, the **highest power of the variable** in the polynomial is the **degree** of the polynomial.
$f(x) = x^2 - 9x^3 + 2x^8 - 6$ is a polynomial with degree 8 as the highest power to which x is raised is 8.
Note:

- The degree of a **non-zero constant polynomial** is **zero**.
- The degree of the **zero polynomial** is **not defined**.

Classification of Polynomials according to their Degree

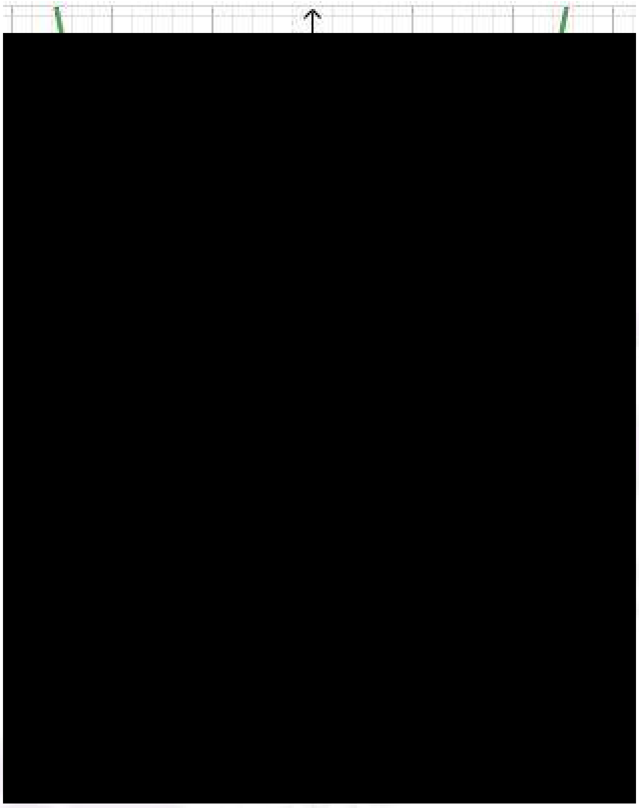
Polynomials can be classified on the basis of their degree as follows:

- A polynomial of degree **one** is called a **linear polynomial**. $P(x) = x - 2$ is a linear polynomial.
- A polynomial of degree **two** is called a **quadratic polynomial**. $P(x) = x^2 - 3x + 4$ is a quadratic polynomial.

- A polynomial of degree **three** a cubic **polynomial**. $P(x) = x^3 + 3x - 2$ is a cubic polynomial.

Representing equations on a graph

All polynomials can be represented on the graph to understand the nature of the polynomial, its zeroes etc.
For example, Geometrically **zeros** of a polynomial are the points where its graph **cuts the x-axis**.



Graph of a quadratic polynomial.

Zeroes of a Polynomial

Zeroes of a Polynomial

A **zero** of a polynomial $P(x)$ is a number c such that $P(c) = 0$.
The zero's of the polynomial $P(x) = x^2 - 4$ are 2 and (-2) since $P(2) = (2)^2 - 4 = 0$ and $P(-2) = (-2)^2 - 4 = 0$.
Note:

- A **non-zero constant** polynomial has no **zero**.
- Every **real number** is a zero of the **zero polynomial**.

Number of zeroes

In general, a polynomial of degree *n* has at most *n* zeros.

- A **linear** polynomial has **one** zero.
- A **quadratic** polynomial has at most **two** zeros.
- A **cubic** polynomial has at most **three** zeros.

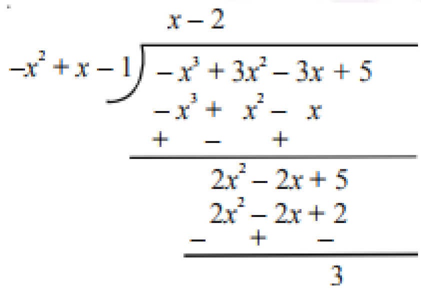
Remainder Theorem

Long Division method to divide two polynomials

To divide one polynomial by another, follow the steps given below.

- arrange the terms of the dividend and the divisor in the decreasing order of their degrees.
- To obtain the first term of the quotient, divide the highest degree term of the dividend by the highest degree term of the divisor. Then carry out the division process.
- The remainder of the previous division becomes the dividend for the next step. Repeat this process until the degree of the remainder is less than the degree of the divisor.

An example of the use of long division method to divide two polynomials is given below.



Dividing one polynomial by another polynomial.

Remainder Theorem

When a polynomial $f(x)$ of degree **greater than or equal to one** is divided by a linear polynomial $x - a$ the remainder is equal to the value of $f(a)$.
If $f(a) = 0$ then $x - a$ is a factor of the polynomial $f(x)$.

Factor Theorem

If $P(x)$ is a polynomial of degree *greater than or equal to one* and *a* is any real number then $x - a$ is a factor of $P(x)$ if $P(a) = 0$.

Factorization of Polynomials

Factorisation of Quadratic Polynomials- Splitting the middle term

Factorisation of the polynomial $ax^2 + bx + c$ by splitting the middle term is as follows:

Step 1: We split the middle term by finding two numbers such that their sum is equal to the coefficient of x and their product is equal to the product of the constant term and the coefficient of x^2.
For example for the quadratic polynomial $(x^2 + 5x + 6)$ the middle term can be split as,
$x^2 + 2x + 3x + 6$
Here, $2 + 3 = 5$ and $2 \times 3 = 6$.

Step 2: Now, we factorise by pairing the terms and taking the common factors.
$x^2 + 2x + 3x + 6$
$= x(x + 2) + 3(x + 2)$
$= (x + 2)(x + 3)$
Thus, $x + 2$ and $x + 3$ are factors of $x^2 + 5x + 6$.

Factorisation of Quadratic Polynomials - Factor theorem

To factorise a quadratic polynomial $f(x) = ax^2 + bx + c$, find two numbers p and q such that $f(p) = f(q) = 0$. Let us factorise the quadratic polynomial $f(x) = x^2 - 3x + 2$.

(i) $f(2) = 2^2 - 3(2) + 2 = 4 - 6 + 2 = 0$
Hence, $x - 2$ is a factor of $x^2 - 3x + 2$.

(ii) $f(3) = 3^2 - 3 \times 3 + 2 = 9 - 9 + 2 = 2 \neq 0$
Hence, $x - 3$ is not a factor of $x^2 - 3x + 2$.

(iii) $f(1) = 1^2 - 3 \times 1 + 2 = 0$
Hence, $x - 1$ is a factor of $x^2 - 3x + 2$.
So, $x - 1$ and $x - 2$ are the factors of the quadratic polynomial $x^2 - 3x + 2$.
$x^2 - 3x + 2 = (x - 2)(x - 1)$

Algebraic Identities

Algebraic Identities

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $a^2 - b^2 = (a - b)(a + b)$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$