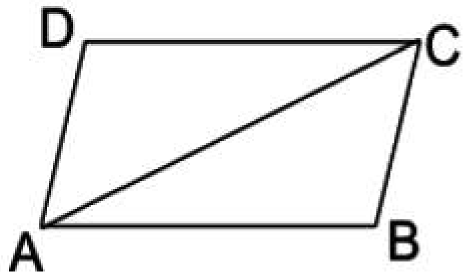
Quadrilaterals

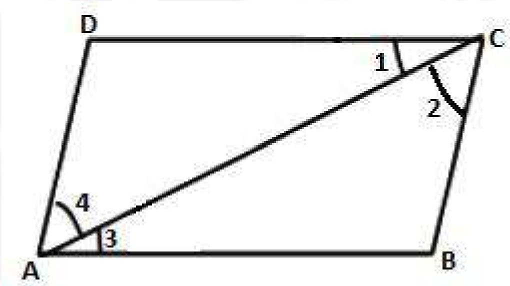
Properties of Parallelogram

Opposite sides of a parallelogram are equal



in $\Delta ABC \; and \; \Delta CDA$
$AC = AC$ [Common / transversal]
$\angle BCA = \angle DAC$ [alternate angles]
$\angle BAC = \angle DCA$ [alternate angles]
$\Delta ABC \cong \Delta CDA$ [ASA rule]
Hence,
$AB = DC$ and $AD = BC$ [C.P.C.T.C]

Opposite angles in a parallelogram are equal



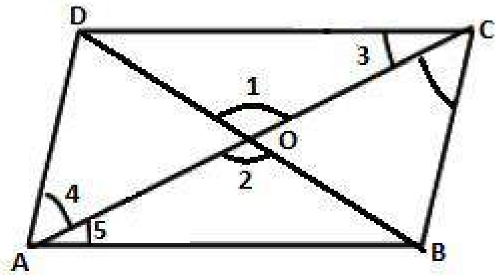
In parallelogram $ABCD$
$AB \parallel CD$; and AC is the transversal
Hence, $\angle 1 = \angle 3$....(1) (alternate interior angles)

$BC \parallel DA$; and AC is the transversal
Hence, $\angle 2 = \angle 4$....(2) (alternate interior angles)

Adding (1) and (2)
$\angle 1 + \angle 2 = \angle 3 + \angle 4$
$\angle BAD = \angle BCD$
Similarly,
$\angle ADC = \angle ABC$

Properties of diagonal of a parallelogram

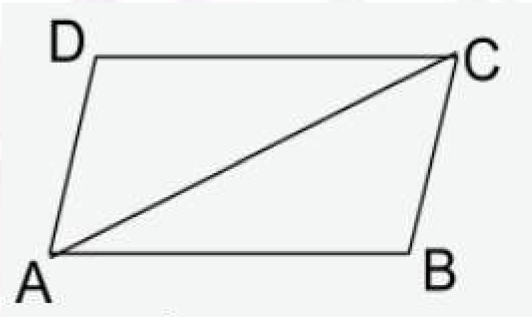
- Diagonals of a parallelogram bisect each other.



In $\Delta AOB\ and\ \Delta COD$,
$\angle 3 = \angle 5$ [alternate interior angles]
$\angle 1 = \angle 2$ [vertically opposite angles]
$AB = CD$ [opp. Sides of parallelogram]
$\Delta AOB \cong \Delta COD$ [AAS rule]
$OB = OD$ and $OA = OC$ [C.P.C.T]
Hence, proved

Conversly,
- If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

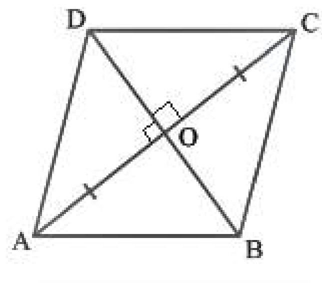
- Diagonal of a parallelogram divides it into two congruent triangles.



In ΔABC and ΔCDA,
$AB = CD$ [Opposite sides of parallelogram]
$BC = AD$ [Opposite sides of parallelogram]
$AC = AC$ [Common side]
$\Delta ABC \cong \Delta CDA$ [by SSS rule]
Hence, proved

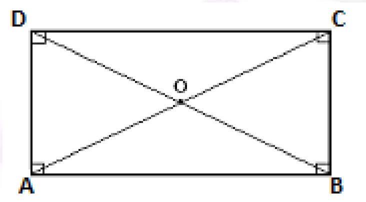
Diagonals of a rhombus bisect each-other at right angles

Diagonals of a rhombus bisect each - other at right angles



In ΔAOD and ΔCOD,
$OA = OC$ [Diagonals of parallelogram bisect each other]
$OD = OD$ [Common side]
$AD = CD$ [Adjacent sides of a rhombus]
$\Delta AOD \cong \Delta COD$ [SSS rule]
$\angle AOD = \angle DOC$ [C.P.C.T]
$\angle AOD + \angle DOC = 180$ [∵ AOC is a straight line]
Hence, $\angle AOD = \angle DOC = 90$
Hence proved

Diagonals of a rectangle bisect each-other and are equal

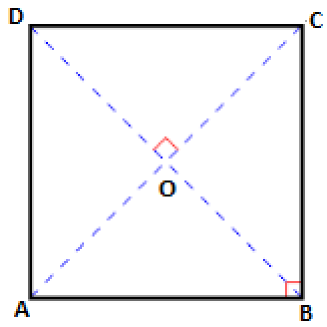


Rectangle ABCD

In ΔABC and ΔBAD,
$AB = BA$ [Common side]
$BC = AD$ [Opposite sides of a rectangle]
$\angle ABC = \angle BAD$ [Each = 90^0 ∵ ABCD is a Rectangle]
$\Delta ABC \cong \Delta BAD$ [SAS rule]
$\therefore AC = BD$ [C.P.C.T]

Consider ΔOAD and ΔOCB,
$AD = CB$ [Opposite sides of a rectangle]
$\angle OAD = \angle OCB$ [∵ AD||BC and transversal AC intersects them]
$\angle ODA = \angle OBC$ [∵ AD||BC and transversal BD intersects them]
$\Delta OAD \cong \Delta OCB$ [ASA rule]
$\therefore OA = OC$ [C.P.C.T]
Similarly we can prove $OB = OD$

Diagonals of a square bisect each-other at right angles and are equal



Square ABCD

In $\Delta ABC \text{ and } \Delta BAD$,

$AB = BA$ [Common side]

$BC = AD$ [Opposite sides of a Square]

$\angle ABC = \angle BAD$ [Each = 90^0 $\because$ ABCD is a Square]

$\Delta ABC \cong \Delta BAD$ [SAS rule]

$\therefore AC = BD$ [C.P.C.T]

Consider $\Delta OAD \text{ and } \Delta OCB$,

$AD = CB$ [Opposite sides of a Square]

$\angle OAD = \angle OCB$ [$\because$ AD||BC and transversal AC intersects them]

$\angle ODA = \angle OBC$ [$\because$ AD||BC and transversal BD intersects them]

$\Delta OAD \cong \Delta OCB$ [ASA rule]

$\therefore OA = OC$ [C.P.C.T]

Similarly we can prove $OB = OD$

In $\Delta OBA \text{ and } \Delta ODA$,

$OB = OD$ [proved above]

$BA = DA$ [Sides of a Square]

$OA = OA$ [Common side]

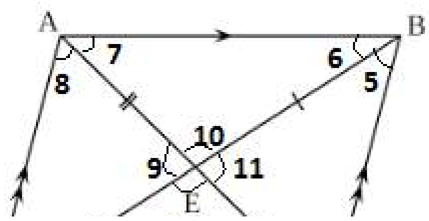
$\Delta OBA \cong \Delta ODA$, [SSS rule]

$\therefore \angle AOB = \angle AOD$ [C.P.C.T]

But, $\angle AOB + \angle AOD = 180^0$ [Linear pair]

$\therefore \angle AOB = \angle AOD = 90^0$

Important results related to parallelograms



Parallelogram ABCD

Opposite **sides** of a parallelogram are **parallel** and **equal**.
$AB||CD, AD||BC, AB = CD, AD = BC$

Opposite **angles** of a parallelogram are **equal** adjacent angels are **supplementary**.
$\angle A = \angle C, \angle B = \angle D,$
$\angle A + \angle B = 180^0, \angle B + \angle C = 180^0, \angle C + \angle D = 180^0, \angle D + \angle A = 180^0$

A **diagonal** of parallelogram divides it into **two congruent triangles**.
$\Delta ABC \cong \Delta CDA$ [With respect to AC as diagonal]
$\Delta ADB \cong \Delta CBD$ [With respect to BD as diagonal]

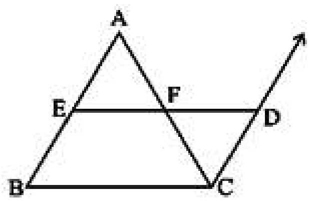
The diagonals of a parallelogram **bisect** each other.
$AE = CE, BE = DE$

$\angle 1 = \angle 5$ (alternate interior angles)
$\angle 2 = \angle 6$ (alternate interior angles)
$\angle 3 = \angle 7$ (alternate interior angles)
$\angle 4 = \angle 8$ (alternate interior angles)
$\angle 9 = \angle 11$ (vertically opp. angles)
$\angle 10 = \angle 12$ (vertically opp. angles)

The Mid-Point Theorem

The mid-point theorem

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half of the third side



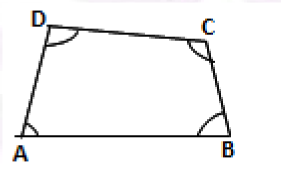
In $\Delta ABC, E$ - midpoint of $AB; F$ - midpoint of AC
Construction: Produce EF to D such that $EF = DF$.
In ΔAEF and ΔCDF,
$AF = CF$ [F is midpoint of AC]

$\angle AFE = \angle CFD$ [V.O.A]
$EF = DF$ [Construction]
$\therefore \Delta AEF \cong \Delta CDF$ [SAS rule]
Hence,
$\angle EAF = \angle DCF$....(1)
$DC = EA = EB$ [E is the midpoint of AB]
$DC \parallel EA \parallel AB$ [Since, (1), alternate interior angles]
$DC \parallel EB$
So $EBCD$ is a parallelogram
Therefore, $BC = ED$ and $BC \parallel ED$
Since, $ED = EF + FD = 2EF = BC$ [$\because$ EF=FD]
We have,$EF = \frac{1}{2}BC$ and $EF||BC$
Hence proved

Introduction to Quadrilaterals

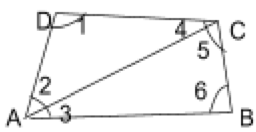
Quadrilaterals

Any four points in a plane, of which three are non collinear are joined in order results in to a four sided closed figure called **'quadrilateral'**



Quadrilateral

Angle sum property of a quadrilateral



Angle sum property - Sum of angles in a quadrilateral is 360

In $\triangle ADC$,

$\angle 1 + \angle 2 + \angle 4 = 180$ (Angle sum property of triangle)................(1)

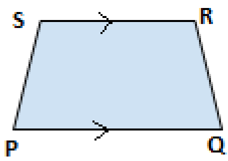
In $\triangle ABC$,
$\angle 3 + \angle 5 + \angle 6 = 180$ (Angle sum property of triangle)..................(2)

(1) + (2):
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360$
I.e, $\angle A + \angle B + \angle C + \angle D = 360$
Hence proved

Types of Quadrilaterals

Trapezium

A **trapezium** is a quadrilateral with any **one pair of opposite sides parallel**.

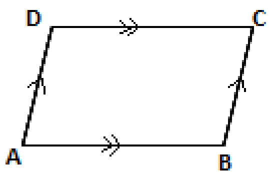


Trapezium

$PQRS$ is trapezium in which $PQ||RS$

Parallelogram

A **parallelogram** is a quadrilateral, with both pair of **opposite sides parallel and equal**. In parallelogram, diagonals bisect each other.

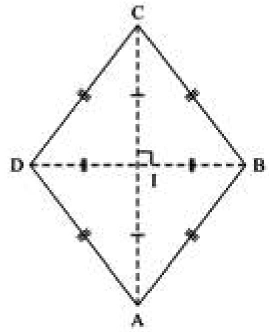


Parallelogram ABCD

Parallelogarm ABCD in which $AB||CD, BC||AD$ and $AB = CD, BC = AD$

Rhombus

A **rhombus** is a parallelogram with **all sides equal.** In rhombus, diagonals bisect each other perpendicularly

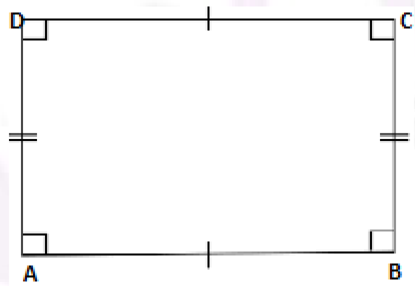


Rhombus ABCD

A rhombus $ABCD$ in which $AB = BC = CD = AD$ and $AC \perp BD$

Rectangle

A **rectangle** is a parallelogram with **all angles as right angles**.

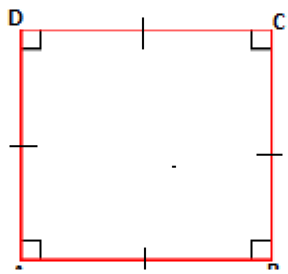


Rectangle ABCD

A rectangle $ABCD$ in which, $\angle A = \angle B = \angle C = \angle D = 90^0$

Square

A **square** is a special case of parallelogram with **all angles as right angles and all sides equal.**

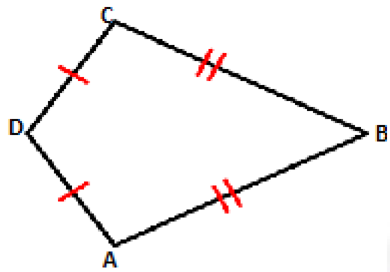


Square ABCD

A square $ABCD$ in which $\angle A = \angle B = \angle C = \angle D = 90^0$ and $AB = BC = CD = AD$

Kite

A **kite** is a quadrilateral with **adjacent sides equal**.



Kite ABCD

A kite $ABCD$ in which $AB = BC$ and $AD = CD$

Venn diagram for different types of quadrilaterals