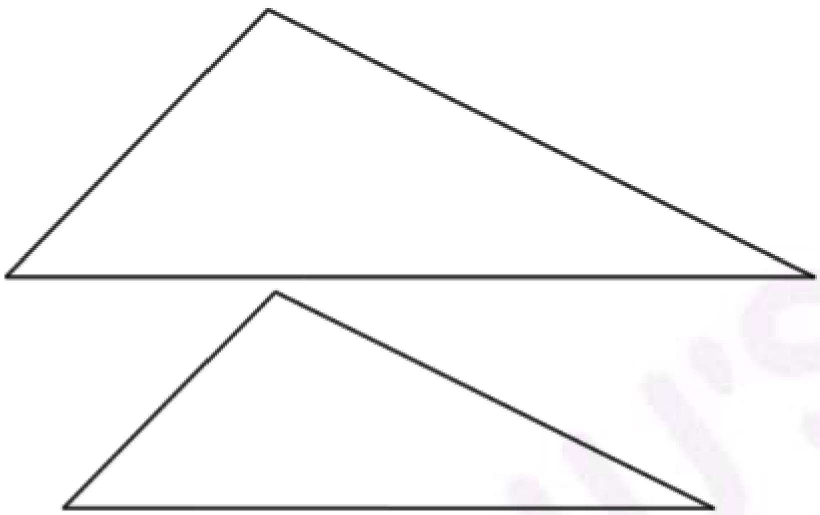
Triangles

Similar Triangles

Similar Figures

Similar figures are the figures having the same shape but not necessarily the same size. For example in the figure given below, the two triangles have the same shape, but their perimeter and area are different.



Similar Triangles

Congruent Figures

Two figures are said to be **congruent** if they have the same shape as well as the same size. Congruent figures are exactly the same. They have the same perimeter, area and can be **superposed** on each other.

Difference between Congruency and Similarity

Congruent figures are exactly the same, with the same size, shape and dimensions.

Similar figures are **scaled up or scaled down** versions of each other. They have the same shape but their sizes need not be the same.

Similar Polygons

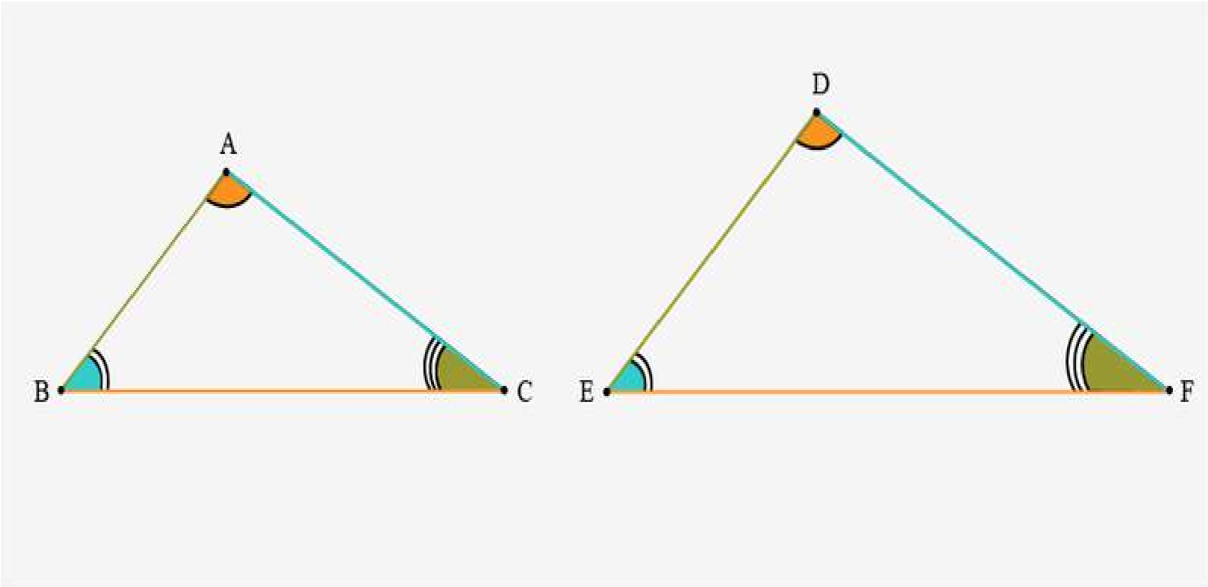
Two **polygons** are **similar** if their **corresponding angles** are equal and their **corresponding sides** are in the same ratio.

Two polygons cannot be similar if their number of sides are different.

Similar Triangles

Two triangles are similar if their

- corresponding angles are equal
- corresponding sides are in the same ratio.



$\triangle ABC$ & $\triangle DEF$ are similar triangles

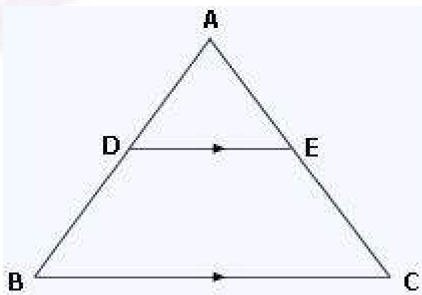
$\Delta ABC \sim \Delta DEF$ if and only if

- $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$
- $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

Basic Proportionality Theorem

Basic Proportionality Theorem

Basic Proportionality Theorem (BPT) states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

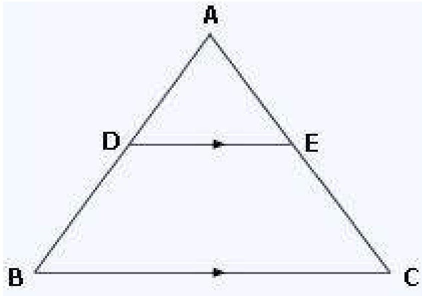


In the above figure DE| |BC. Then, BPT says that,

$\frac{AD}{DB} = \frac{AE}{EC}$

Converse of Basic Proportionality Theorem

The converse of **Basic Proportionality Theorem** is true. It states that if a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.



In the above figure, if $\frac{AD}{DB} = \frac{AE}{EC}$,
then, the converse of BPT states that DE| |BC.

Criteria for Similarity of Triangles

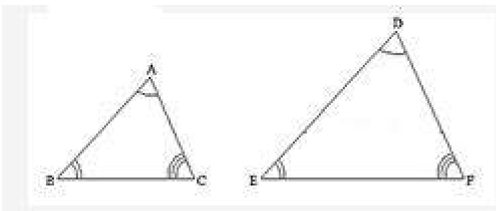
Criteria for Similarity of Triangles

Two triangles are said to be **similar** if their **corresponding angles** are equal and their **corresponding sides** are in the same ratio. However, we need not check for all angles and sides to ensure similarity. There are certain criteria to confirm the similarity of two triangles by comparing a lesser number of corresponding parts of a triangle. These are

- **AAA similarity**
- **SSS similarity**
- **SAS similarity**

AAA Similarity

According to **AAA similarity criterion**, if the corresponding angles of two triangles are equal, then the corresponding sides are in the same ratio and the triangles are similar.

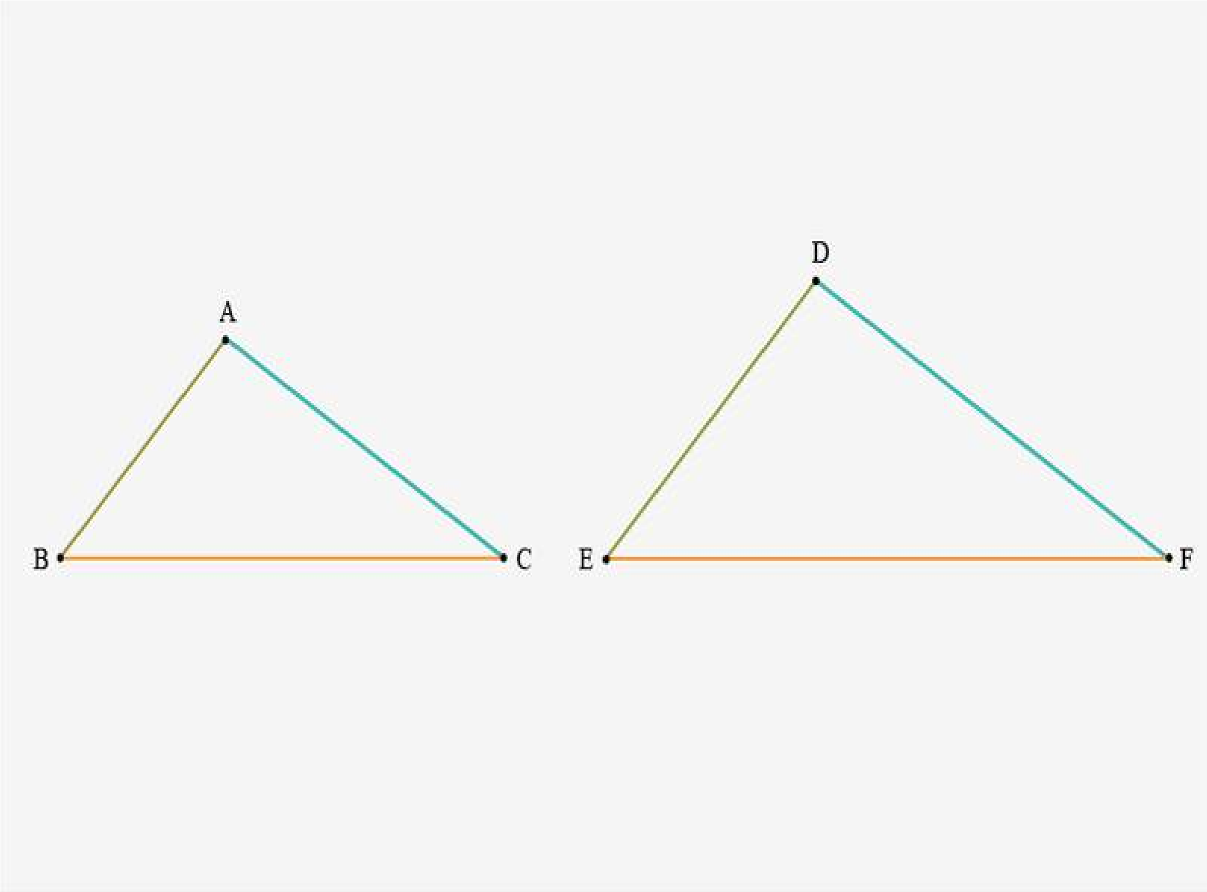


If $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$, then $\Delta ABC \sim \Delta DEF$

If two angles of a triangle are respectively equal to two angles of another triangle, then by **angle sum property**, the third angle of the triangles are equal and the triangles are similar. This is called **AA similarity criterion**.

SSS Similarity

According to **SSS similarity criterion**, if the sides of one triangle are **proportional to the corresponding sides** of another triangle, then their corresponding angles are equal and the triangles are similar.

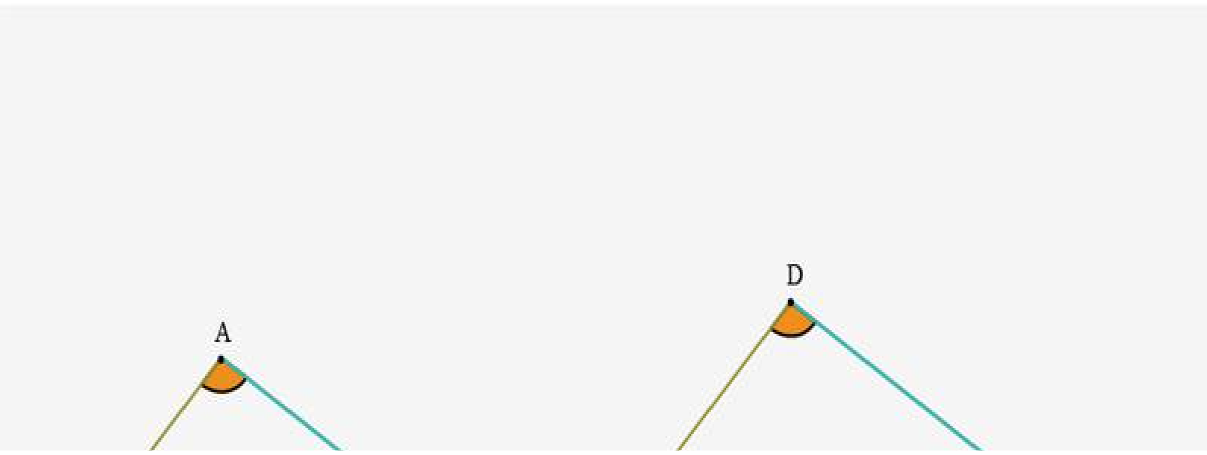


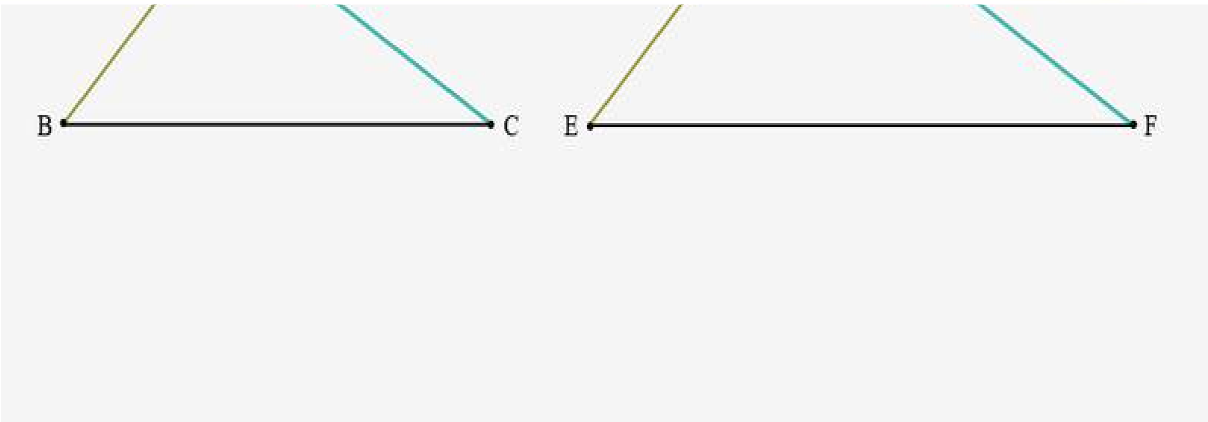
$\triangle ABC$ & $\triangle DEF$ are similar by SSS criterion

If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\Delta ABC \sim \Delta DEF$

SAS Similarity

According to SAS similarity criterion, if two sides of one triangle are proportional to two sides of another triangle and the corresponding included angles are equal, then, the triangles are similar.



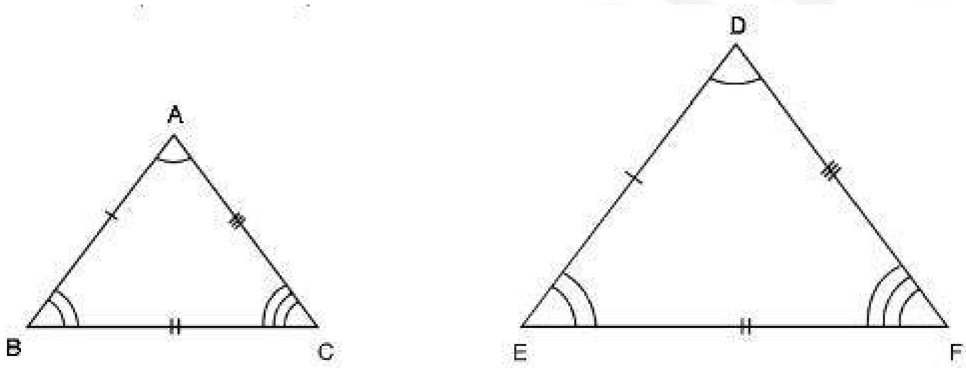


$\triangle ABC$ & $\triangle DEF$ are similar by SAS criterion

If $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$

Areas of Similar Triangles

Relation between Areas and Sides of Similar Triangles

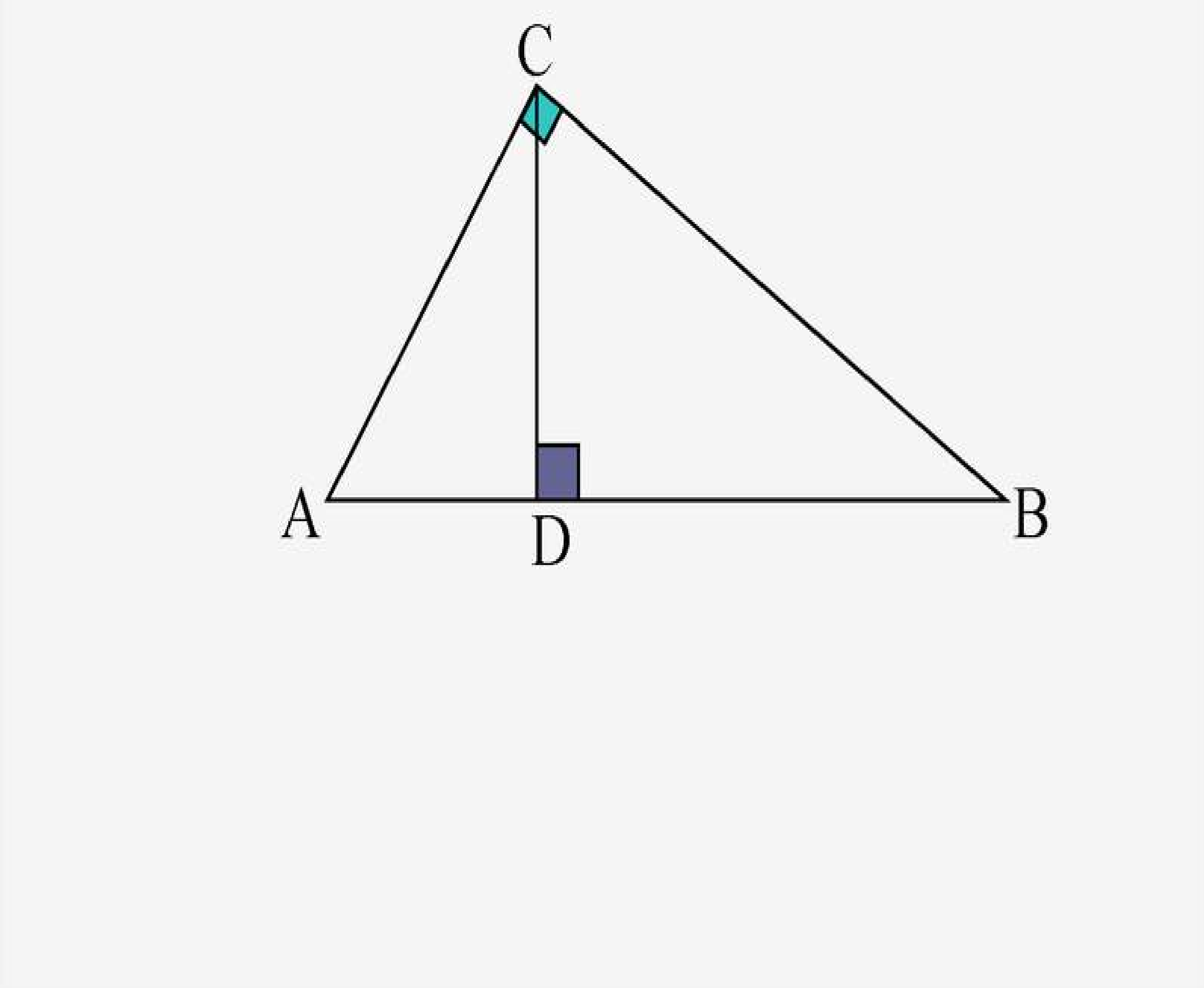


In the given figure, ΔABC is similar to ΔDEF, the ratio of their areas is given by,
$\frac{area(\Delta ABC)}{area(\Delta DEF)} = (\frac{AB}{DE})^2 = (\frac{BC}{EF})^2 = (\frac{AC}{DF})^2$

Pythagoras Theorem

Perpendicular from Right Angle to Hypotenuse Divides the Triangle into Two Similar Triangles

In a right-angled triangle, if a perpendicular is drawn from the vertex of the right angle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.



CD is the perpendicular from the vertex to hypotenuse AB

In the above figure, CD is the perpendicular drawn from the vertex C on the hypotenuse AB of ΔABC.
So, $\Delta ABC \sim \Delta CBD \sim \Delta ACD$

Pythagoras Theorem

Pythagoras Theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Conversely, In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.