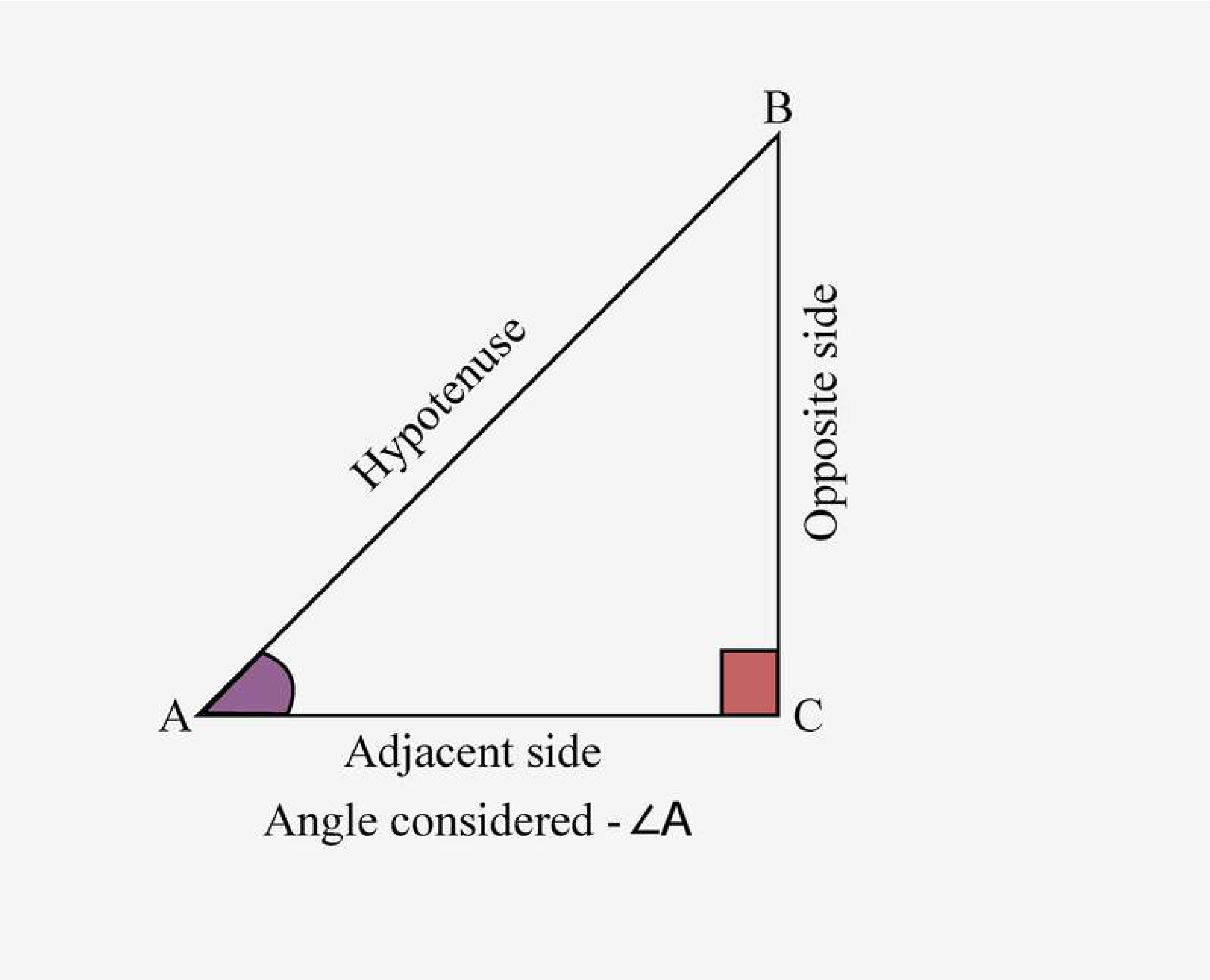
Introduction To Trigonometry

Trigonometric Ratios

Opposite & Adjacent Sides in a Right Angled Triangle

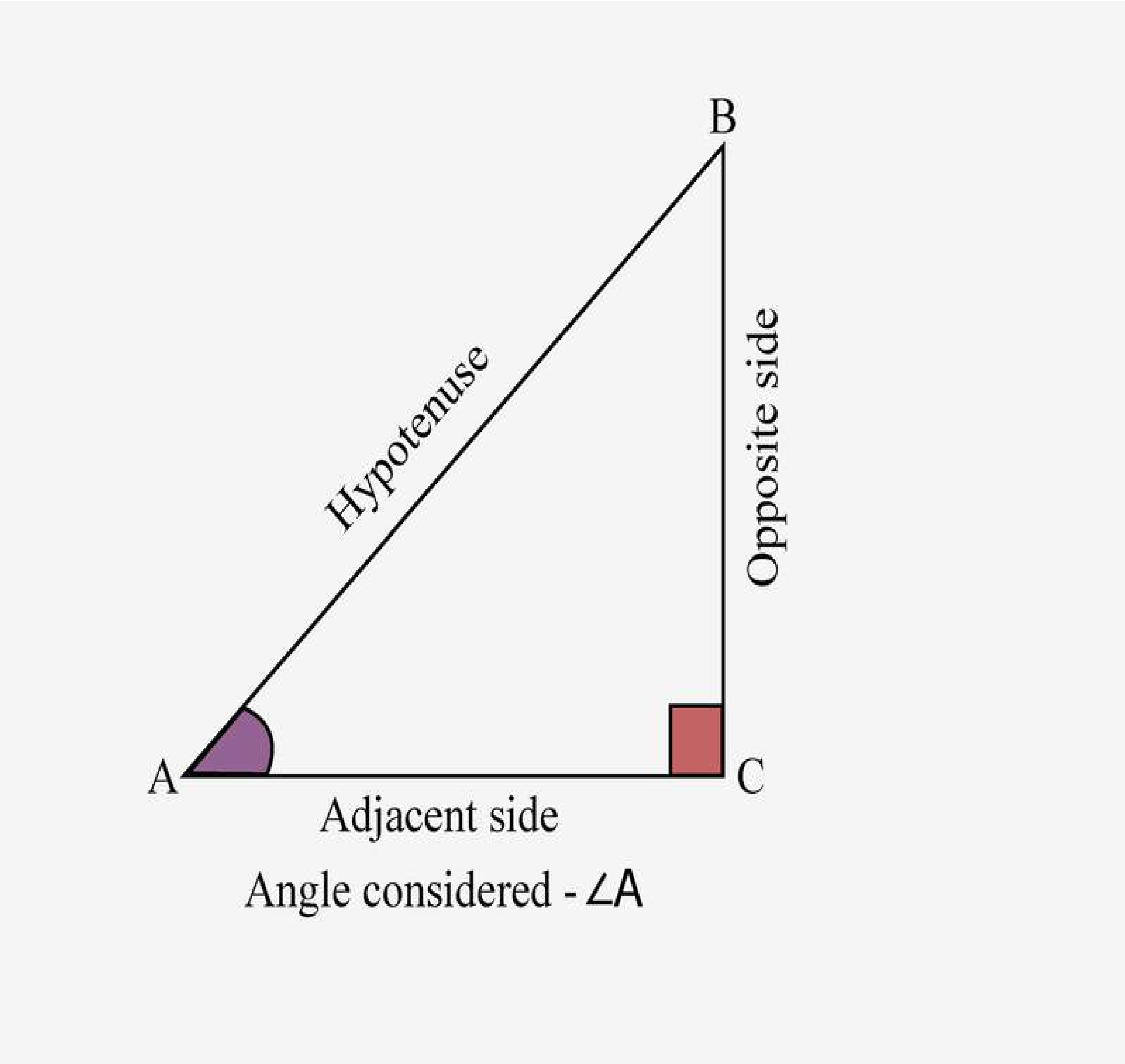
In the ΔABC right-angled at B, BC is the side opposite to $\angle A$, AC is the hypotenuse and AB is the side adjacent to $\angle A$.



Trigonometric Ratios

For the right ΔABC, right angled at $\angle B$, the trigonometric ratios of the $\angle A$ are as follows:

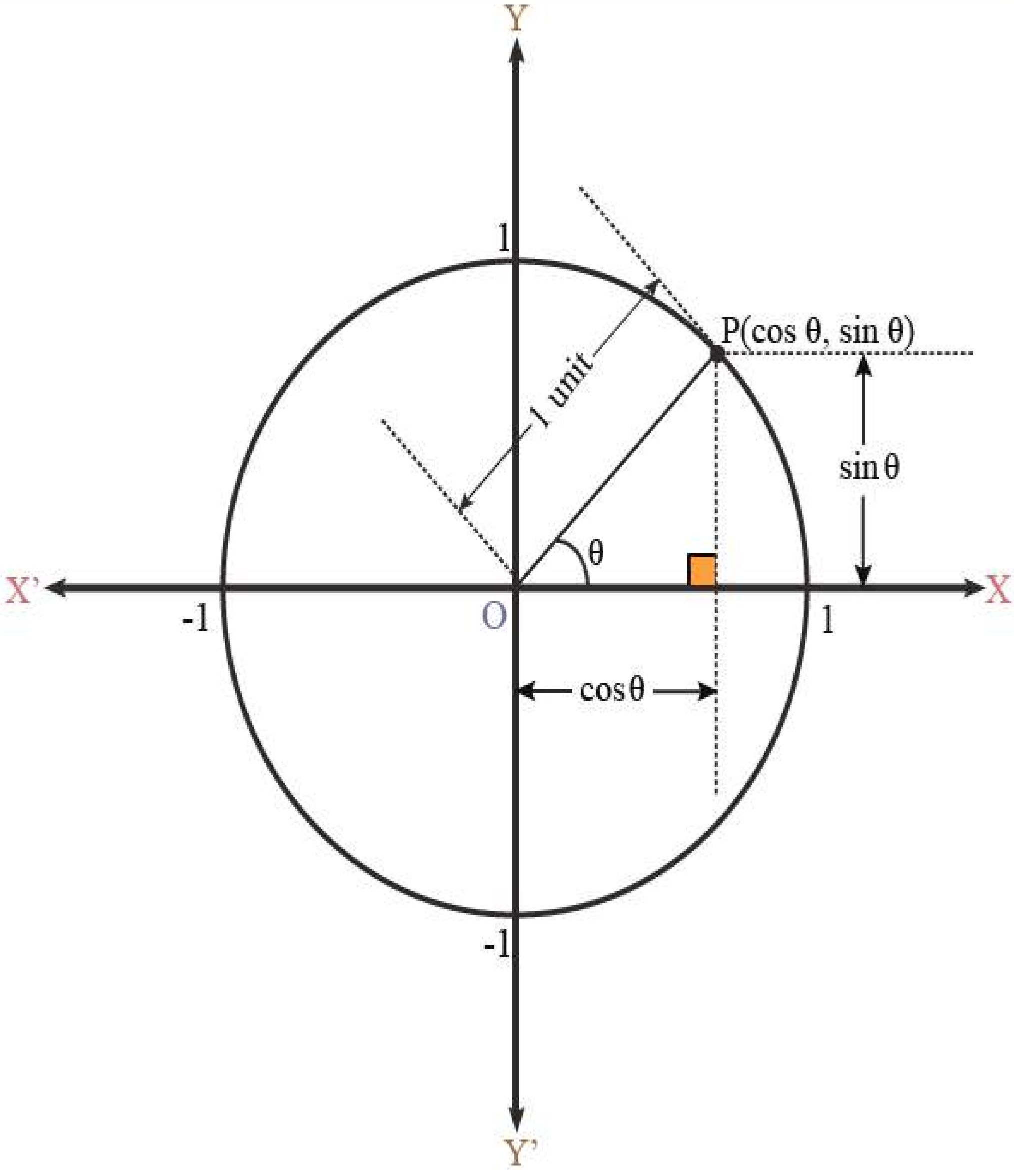
- $sinA = \frac{opposite\ side}{hypotenuse} = \frac{BC}{AC}$
- $cosA = \frac{adjacent\ side}{hypotenuse} = \frac{AB}{AC}$
- $tanA = \frac{opposite\ side}{adjacent\ side} = \frac{BC}{AB}$
- $cosecA = \frac{hypotenuse}{opposite\ side} = \frac{AC}{BC}$
- $secA = \frac{hypotenuse}{adjacent\ side} = \frac{AC}{AB}$
- $cotA = \frac{adjacent\ side}{opposite\ side} = \frac{AB}{BC}$



Visualisation of Trigonometric Ratios Using a Unit Circle

Draw a circle of unit radius with the origin as the centre. Consider a line segment OP joining a point P on the circle to the centre which makes an angle θ with the x-axis. Draw a perpendicular from P to the x-axis to cut it at Q.

- $sin\theta = \frac{PQ}{OP} = \frac{PQ}{1} = PQ$
- $cos\theta = \frac{OQ}{OP} = \frac{OQ}{1} = OQ$
- $tan\theta = \frac{PQ}{OQ} = \frac{sin\theta}{cos\theta}$
- $cosec\theta = \frac{OP}{PQ} = \frac{1}{PQ}$
- $sec\theta = \frac{OP}{OQ} = \frac{1}{OQ}$
- $cot\theta = \frac{OQ}{PQ} = \frac{cos\theta}{sin\theta}$



Visualisation of Trigonometric Ratios Using a Unit Circle

Relation between Trigonometric Ratios

- $cosec\theta = \frac{1}{sin\theta}$
- $sec\theta = \frac{1}{cos\theta}$
- $tan\theta = \frac{sin\theta}{cos\theta}$
- $cot\theta = \frac{cos\theta}{sin\theta} = \frac{1}{tan\theta}$

Trigonometric Ratios of Specific Angles

Range of Trigonometric Ratios from 0 to 90 degrees

For $0^\circ \leq \theta \leq 90^\circ$,

- $0 \leq sin\theta \leq 1$
- $0 \leq cos\theta \leq 1$
- $0 \leq tan\theta < \infty$

- $1 \leq sec\theta < \infty$
- $0 \leq cot\theta < \infty$
- $1 \leq cosec\theta < \infty$

$tan\theta$ and $sec\theta$ are not defined at 90°.
$cot\theta$ and $cosec\theta$ are not defined at 0°.

Variation of trigonometric ratios from 0 to 90 degrees

As θ increases from 0° to 90°

- $sin\theta$ increases from 0 to 1.
- $cos\theta$ decreases from 1 to 0.
- $tan\theta$ increases from 0 to ∞.
- $cosec\theta$ decreases from ∞ to 1.
- $sec\theta$ increases from 1 to ∞.
- $cot\theta$ decreases from ∞ to 0.

Standard values of Trigonometric ratios

$\angle A$	0°	30°	45°	60°	90°
$sin\ A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$cos\ A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$tan\ A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$Not\ defined$
$cosec\ A$	$Not\ defined$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$sec\ A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$Not\ defined$
$cot\ A$	$Not\ defined$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Ratios of Complementary Angles

Complementary Trigonometric ratios

If θ is an acute angle, its complementary angle is $90^\circ - \theta$. The following relations hold true for trigonometric ratios of complementary angles.

- $sin(90^\circ - \theta) = cos\theta$
- $cos(90^\circ - \theta) = sin\theta$
- $tan(90^\circ - \theta) = cot\theta$
- $cot(90^\circ - \theta) = tan\theta$
- $cosec(90^\circ - \theta) = sec\theta$

- $sec(90^\circ - \theta) = cosec\theta$

Trigonometric Identities

Trigonometric Identities

- $sin^2\theta + cos^2\theta = 1$
- $1 + cot^2\theta = coesc^2\theta$
- $1 + tan^2\theta = sec^2\theta$