Exercise 2.3

Short Answer (S.A)

1. Find the value of $\tan^{-1}[\tan(5\pi/6)] + \cos^{-1}[\cos(13\pi/6)]$

Solution:

We know that,

$\tan^{-1}\tan x = x,\ x \in (-\pi/2, \pi/2)$

And, here

$\tan^{-1}\tan(5\pi/6) \neq 5\pi/6$ as $5\pi/6 \notin (-\pi/2, \pi/2)$

Also,

$\cos^{-1}\cos x = x;\ x \in [0, \pi]$

So,

$\cos^{-1}\cos(13\pi/6) \neq 13\pi/6$ as $13\pi/6 \notin [0, \pi]$

Now,

$\tan^{-1}[\tan(5\pi/6)] + \cos^{-1}[\cos(13\pi/6)]$

$= \tan^{-1}[\tan(\pi - \pi/6)] + \cos^{-1}[\cos(2\pi + \pi/6)]$

$= \tan^{-1}[-\tan\pi/6] + \cos^{-1}[-\cos(7\pi/6)]$

$= -\tan^{-1}[\tan\pi/6] + \cos^{-1}[\cos(\pi/6)]$

$= -\pi/6 + \pi/6$

$= 0$

2. Evaluate $\cos[\cos^{-1}(-\sqrt{3}/2) + \pi/6]$

Solution:

$$\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$$

$$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right] \qquad \left(\because \cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}\right)$$

$$= \cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right) \qquad (\because \cos^{-1}\cos x = x;\ x \in [0, \pi])$$

$$= \cos(\pi) = -1$$

3. Prove that $\cot(\pi/4 - 2\cot^{-1}3) = 7$

Solution:

Re-writing the given,

$$\frac{\pi}{4} - 2\cot^{-1}3 = \cot^{-1}7$$

$$2\tan^{-1}\frac{1}{3} = \frac{\pi}{4} - \tan^{-1}\frac{1}{7}$$

$$2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

Now, $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$

$$=\tan^{-1}\frac{2/3}{1-(1/3)^2}+\tan^{-1}\frac{1}{7}\quad\left(\because 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right)$$

$$=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}$$

$$=\tan^{-1}\frac{(21+4)/28}{(28-3)/28}=\tan^{-1}\frac{25}{25}=\tan^{-1}1=\frac{\pi}{4}$$

L.H.S = R.H.S
- Hence Proved

4. Find the value of $\tan^{-1}(-1/\sqrt{3})+\cot^{-1}(1/\sqrt{3})+\tan^{-1}(\sin(-\pi/2))$
Solution:

Given,
$\tan^{-1}(-1/\sqrt{3})+\cot^{-1}(1/\sqrt{3})+\tan^{-1}(\sin(-\pi/2))$

$$=\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}(-1)$$

$$=-\frac{\pi}{6}+\frac{\pi}{3}+\left(-\frac{\pi}{4}\right)=-\frac{\pi}{12}$$

5. Find the value of $\tan^{-1}(\tan 2\pi/3)$.
Solution:

We know that,
$\tan^{-1}\tan x = x,\ x\in(-\pi/2,\pi/2)$

$$\tan^{-1}\left(\tan\frac{2\pi}{3}\right)=\tan^{-1}\tan\left(\pi-\frac{\pi}{3}\right)=\tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right)=-\frac{\pi}{3}$$

6. Show that $2\tan^{-1}(-3)=-\pi/2+\tan^{-1}(-4/3)$
Solution:

Taking L.H.S = $2\tan^{-1}(-3)=-2\tan^{-1}3$ ($\because \tan^{-1}(-x)=-\tan^{-1}x,\ x\in R$)

$$= -2\left[\frac{\pi}{2} - \cot^{-1} 3\right] \qquad \left(\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right)$$

$$= -2\left[\frac{\pi}{2} - \tan^{-1}\frac{1}{3}\right] \qquad \left(\because \tan^{-1} x = \cot^{-1}\frac{1}{x}, x > 0\right)$$

$$= -\pi + 2\tan^{-1}\frac{1}{3}$$

$$= -\pi + \tan^{-1}\frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2} \qquad \left(\because 2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\right)$$

$$= -\pi + \tan^{-1}\frac{2/3}{8/9} = -\pi + \tan^{-1}\frac{3}{4}$$

$$= -\pi + \frac{\pi}{2} - \cot^{-1}\frac{3}{4} \qquad \left(\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right)$$

$$= -\frac{\pi}{2} - \tan^{-1}\frac{4}{3} \qquad \left(\because \tan^{-1} x = \cot^{-1}\frac{1}{x}, x > 0\right)$$

$$= -\frac{\pi}{2} + \tan^{-1}\left(-\frac{4}{3}\right) \qquad (\because \tan^{-1}(-x) = -\tan^{-1} x, x \in R)$$

= R.H.S

- Hence Proved.

7. Find the real solution of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

Solution:

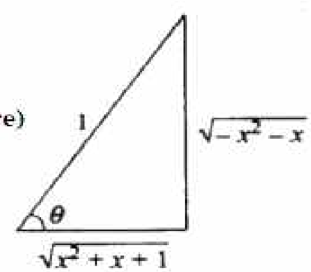
Given equation,

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

$$\tan^{-1}\sqrt{x(x+1)} = \frac{\pi}{2} - \sin^{-1}\sqrt{x^2+x+1}$$

$$= \cos^{-1}\sqrt{x^2+x+1}$$

$$= \tan^{-1}\frac{\sqrt{-x^2-x}}{\sqrt{x^2+x+1}} \quad \text{(From figure)}$$



$$\sqrt{x(x+1)} = \frac{\sqrt{-x^2-x}}{\sqrt{x^2+x+1}}$$

$$x^2 + x = 0$$

$$x = 0, -1$$

Hence, the real solutions of the given trigonometric equation are 0 and -1.

8. Find the value of the expression sin (2 tan^{-1} 1/3) + cos (tan^{-1} 2√2).
Solution:

Given expression, sin (2 tan^{-1} 1/3) + cos (tan^{-1} 2√2)

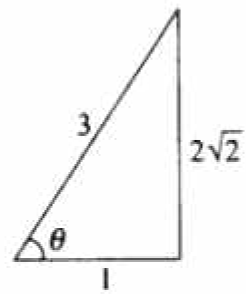
$$\sin\left(2\tan^{-1}\frac{1}{3}\right) = \sin\left(\sin^{-1}\frac{2\times\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}\right) \quad \left(\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right)$$

$$= \sin\left(\sin^{-1}\frac{2/3}{10/9}\right)$$

$$= \sin\left(\sin^{-1}\frac{3}{5}\right) = \frac{3}{5} \qquad (\because \sin(\sin^{-1}x) = x, x \in [-1, 1])$$

$$\cos(\tan^{-1}2\sqrt{2}) = \cos\left(\cos^{-1}\frac{1}{3}\right) = \frac{1}{3}$$

$$(\because \cos(\cos^{-1}x) = x, x \in [-1, 1])$$



Hence,

$$\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2}) = \frac{3}{5} + \frac{1}{3} = \frac{9+5}{15} = \frac{14}{15}$$

9. If 2 tan^{-1}(cos θ) = tan^{-1} (2 cosec θ), then show that θ = π/4.
Solution:

Given, 2 tan^{-1}(cos θ) = tan^{-1} (2 cosec θ)
So,

$$\tan^{-1}\left(\frac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}(2\operatorname{cosec}\theta) \quad \left(\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$$

$$\frac{2\cos\theta}{\sin^2\theta} = 2\operatorname{cosec}\theta$$

$$\frac{2\cos\theta}{\sin^2\theta} = \frac{2}{\sin\theta}$$

$$\frac{\cos\theta}{\sin\theta} = 1 \quad \Rightarrow \quad \cot\theta = 1 \quad \Rightarrow \quad \theta = \frac{\pi}{4}$$

10. Show that cos (2 tan^{-1} 1/7) = sin (4 tan^{-1} 1/3).
Solution:

Taking L.H.S, we have

$$\text{L.H.S.} = \cos\left(2\tan^{-1}\frac{1}{7}\right)$$

$$= \cos\left(\cos^{-1}\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}\right) \qquad \left(\because 2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}\right)$$

$$= \cos\left(\cos^{-1}\frac{48/49}{50/49}\right)$$

$$= \cos\left(\cos^{-1}\frac{24}{25}\right) = \frac{24}{25} \qquad (\because \cos(\cos^{-1}x) = x, x \in [-1, 1])$$

$$\text{R.H.S.} = \sin\left(4\tan^{-1}\frac{1}{3}\right)$$

$$= \sin\left(2\left(2\tan^{-1}\frac{1}{3}\right)\right)$$

$$= \sin\left(2\left(\tan^{-1}\frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right)\right) \qquad \left(\because 2\tan^{-1}x \quad \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)$$

$$= \sin\left(2\tan^{-1}\frac{2/3}{8/9}\right)$$

$$= \sin\left(2\tan^{-1}\frac{3}{4}\right)$$

$$= \sin\left(\sin^{-1}\frac{2\times\frac{3}{4}}{1+\left(\frac{3}{4}\right)^2}\right) \qquad \left(\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right)$$

$$= \sin\left(\sin^{-1}\frac{3/2}{25/16}\right)$$

$$= \sin\left(\sin^{-1}\frac{24}{25}\right) = \frac{24}{25} \qquad (\because \sin(\sin^{-1}x) = x, x \in [-1, 1])$$

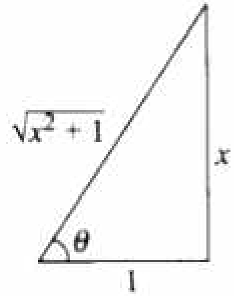
Thus, L.H.S = R.H.S

- Hence proved

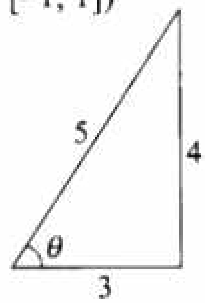
11. Solve the equation cos ($\tan^{-1}$ x) = sin ($\cot^{-1}$ 3/4).
Solution:

Given equation, cos ($\tan^{-1}$ x) = sin ($\cot^{-1}$ 3/4)
Taking L.H.S,

We have, $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$



$$\text{L.H.S.} = \cos(\tan^{-1} x)$$
$$= \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right)$$
$$= \frac{1}{\sqrt{x^2+1}} \qquad (\because \cos(\cos^{-1} x) = x, x \in [-1, 1])$$



$$\text{R.H.S.} = \sin\left(\cot^{-1}\frac{3}{4}\right)$$
$$= \sin\left(\sin^{-1}\frac{4}{5}\right)$$
$$= \frac{4}{5} \qquad (\because \sin(\sin^{-1} x) = x, x \in [-1, 1])$$

Hence, on equating the L.H.S and R.H.S, we get $\frac{1}{\sqrt{x^2+1}} = \frac{4}{5}$

On squaring on both sides,
$16(x^2 + 1) = 25$
$16x^2 + 16 = 25$
$16x^2 = 9$
$x^2 = 9/16$
Therefore, $x = \pm 3/4$

12. Prove that $\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$.

Solution:

Taking L.H.S, $\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$

Let $x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x^2$

So, L.H.S. $= \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right]$

$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right]$

$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right]$

$= \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right]$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right) = \frac{\pi}{4}+\theta = \frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$

= R.H.S

- Hence Proved

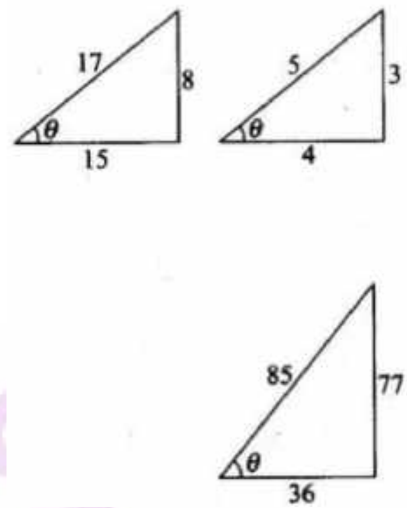
13. Find the simplified form of $\cos^{-1}$ [3/5 cos x + 4/5 sin x], x ∈ [-3π/4, π/4].
Solution:

We have,
$\cos^{-1}$ [3/5 cos x + 4/5 sin x], x ∈ [-3π/4, π/4]
Now, let cos α = 3/5
So, sin α = 4/5 and tan α = 4/3
$\cos^{-1}$ [3/5 cos x + 4/5 sin x]
⇒ $\cos^{-1}$ [3/5 cos x + 4/5 sin x] = $\cos^{-1}$ [cos α cos x + sin α sin x]
= $\cos^{-1}$ [cos (α - x)]
= α – x
= $\tan^{-1}$ 4/3 – x

14. Prove that $\sin^{-1}$ 8/17 + $\sin^{-1}$ 3/5 = $\sin^{-1}$ 77/85
Solution:

Taking the L.H.S,
= $\sin^{-1}$ 8/17 + $\sin^{-1}$ 3/5

$= \tan^{-1} 8/15 + \tan^{-1} 3/4$



$$= \tan^{-1} \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}}$$

$$= \tan^{-1} \frac{\frac{32 + 45}{60}}{\frac{60 - 24}{60}}$$

$$= \tan^{-1} \frac{77}{36}$$

$$= \sin^{-1} \frac{77}{\sqrt{5929 + 1296}}$$

$$= \sin^{-1} \frac{77}{85}$$

= R.H.S.

- Hence proved

15. Show that $\sin^{-1} 5/13 + \cos^{-1} 3/5 = \tan^{-1} 63/16$

Solution:

Here, $\sin^{-1} 5/13 = \tan^{-1} 5/12$

And,

$\cos^{-1} 3/5 = \tan^{-1} 4/3$

Taking the L.H.S, we have

$$\text{L.H.S.} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

$$= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{4}{3} = \tan^{-1} \frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \cdot \frac{4}{3}}$$

$$= \tan^{-1} \frac{\frac{15 + 48}{36}}{\frac{36 - 20}{36}} = \tan^{-1} \frac{63}{16}$$

Thus, L.H.S = R.H.S

- Hence Proved

16. Prove that $\tan^{-1} 1/4 + \tan^{-1} 2/9 = \sin^{-1} 1/\sqrt{5}$

Solution:

Taking the LHS,
$\tan^{-1} 1/4 + \tan^{-1} 2/9$

$$= \tan^{-1} \frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}} = \tan^{-1} \frac{9+8}{36-2} = \tan^{-1} \frac{1}{2} = \sin^{-1} \frac{1}{\sqrt{5}}$$

= RHS

- Hence Proved

17. Find the value of 4 $\tan^{-1}$ 1/5 – $\tan^{-1}$ 1/239
Solution:

$4 \tan^{-1} 1/5 - \tan^{-1} 1/239$
$= 2 (\tan^{-1} 1/5) - \tan^{-1} 1/239$

$$= 2 \tan^{-1} \frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2} - \tan^{-1} \frac{1}{239} \quad \left(\because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}\right)$$

$$= 2 \tan^{-1} \frac{2/5}{24/25} - \tan^{-1} \frac{1}{239}$$

$$= 2 \tan^{-1} \frac{5}{12} - \tan^{-1} \frac{1}{239}$$

$$= 2 \tan^{-1} \frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2} - \tan^{-1} \frac{1}{239} \quad \left(\because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}\right)$$

$$= 2 \tan^{-1} \frac{2/5}{24/25} - \tan^{-1} \frac{1}{239}$$

$= 2 \tan^{-1} 5/12 - \tan^{-1} 1/239$

$$= \tan^{-1} \frac{2\cdot\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} - \tan^{-1} \frac{1}{239} \quad \left(\because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}\right)$$

$$= \tan^{-1} \frac{144\times 5}{119\times 6} - \tan^{-1} \frac{1}{239}$$

$$= \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239}$$

$$= \tan^{-1} \frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\cdot\frac{1}{239}} \quad \left(\because \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}\right)$$

$$= \tan^{-1} \frac{120\times 239 - 119}{119\times 239 + 120} = \tan^{-1} \frac{28680-119}{28441+120}$$

$$= \tan^{-1} \frac{28561}{28561} = \tan^{-1} 1 = \frac{\pi}{4}$$

Thus,
$4 \tan^{-1} 1/5 - \tan^{-1} 1/239 = \pi/4$

18. Show that $\tan^{-1} (1/2 \sin^{-1} 3/4) = (4 - \sqrt{7})/ 3$ and justify why the other value $(4 + \sqrt{7})/ 3$ is ignored.
Solution:

We have, $\tan^{-1} (1/2 \sin^{-1} 3/4)$
Let $½ \sin^{-1} ¾ = \theta \Rightarrow \sin^{-1} ¾ = 2\theta \Rightarrow \sin 2\theta = ¾$
$2 \tan \theta/ 1 + \tan^2 \theta = ¾$
$3 \tan^2 \theta - 8 \tan \theta + 3 = 0$

$$\tan \theta = \frac{8 \pm \sqrt{64 - 36}}{6}$$

$$\tan \theta = \frac{8 \pm \sqrt{28}}{6} = \frac{8 \pm 2\sqrt{7}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

Now,

$$-\frac{\pi}{2} \le \sin^{-1}\frac{3}{4} \le \frac{\pi}{2}$$

$$\frac{-\pi}{4} \le \frac{1}{2}\sin^{-1}\frac{3}{4} \le \frac{\pi}{2}$$

$$\tan\left(\frac{-\pi}{4}\right) \le \tan\left(\frac{1}{2}\left(\sin^{-1}\frac{3}{4}\right)\right) \le \tan\frac{\pi}{4}$$

$$-1 \le \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) \le 1$$

Hence,

$$\tan \theta = \frac{4 - \sqrt{7}}{3} \left(\tan \theta = \frac{4 + \sqrt{7}}{3} > 1, \text{ which is not possible}\right)$$

19. If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d, then evaluate the following expression.

$$\tan\left[\tan^{-1}\left(\frac{d}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{d}{1 + a_2 a_3}\right) + \tan^{-1}\left(\frac{d}{1 + a_3 a_4}\right) + \cdots + \tan^{-1}\left(\frac{d}{1 + a_{n-1} a_n}\right)\right]$$

Solution:

As $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d.
$d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots = a_n - a_{n-1}$
So,

$$\tan^{-1}\frac{d}{1 + a_1 a_2} = \tan^{-1}\frac{a_2 - a_1}{1 + a_1 a_2} = \tan^{-1} a_2 - \tan^{-1} a_1$$

Similarly $\tan^{-1}\frac{d}{1+a_2a_3} = \tan^{-1}\frac{a_3-a_2}{1+a_2a_3} = \tan^{-1}a_3 - \tan^{-1}a_2$

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$$\tan^{-1}\frac{d}{1+a_{n-1}a_n} = \tan^{-1}\frac{a_n-a_{n-1}}{1+a_{n-1}a_n} = \tan^{-1}a_n - \tan^{-1}a_{n-1}$$

$$\therefore \quad \tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\cdots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

$$= \tan[(\tan^{-1}a_2 - \tan^{-1}a_1) + (\tan^{-1}a_3 - \tan^{-1}a_2) + \ldots + (\tan^{-1}a_n - \tan^{-1}a_{n-1})]$$

$$= \tan[\tan^{-1}a_n - \tan^{-1}a_1]$$

$$= \tan\left[\tan^{-1}\frac{a_n-a_1}{1+a_na_1}\right] \quad \left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \frac{a_n-a_1}{1+a_na_1} \qquad [\because \tan(\tan^{-1}x) = x]$$

20. Which of the following is the principal value branch of cos-1 x?
(a) [-π/2, π/2] (b) (0, π) (c) [0. π] (d) [0, π] – {π/2}
Solution:

(c) [0. π]
As we know that the principal value branch $\cos^{-1}$ x is [0, π].

21. Which of the following is the principal value branch of cosec^{-1} x?
(a) (-π/2, π/2) (b) [0, π] – {π/2} (c) [-π/2, π/2] (d) [-π/2, π/2] – {0}
Solution:

(d) [-π/2, π/2] – {0}
As the principal branch of cosec^{-1} x is [-π/2, π/2] – {0}.

22. If 3 $\tan^{-1}$ x + $\cot^{-1}$ x = π, then x equals
(a) 0 (b) 1 (c) -1 (d) ½
Solution:

(b) 1
Given, 3 $\tan^{-1}$ x + $\cot^{-1}$ x = π

$2\tan^{-1}x + \tan^{-1}x + \cot^{-1}x = \pi$
$2\tan^{-1}x + \pi/2 = \pi$ (As $\tan^{-1} + \cot^{-1} = \pi/2$)
$2\tan^{-1}x = \pi/2$
$\tan^{-1}x = \pi/4$
$x = 1$

23. The value of $\sin^{-1}\cos 33\pi/5$ is
(a) $3\pi/5$ (b) $-7\pi/5$ (c) $\pi/10$ (d) $-\pi/10$
Solution:

(d) $-\pi/10$

$$\sin^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\cos\left(6\pi+\frac{3\pi}{5}\right)\right) = \sin^{-1}\left(\cos\frac{3\pi}{5}\right)$$
$$= \sin^{-1}\left[\sin\left(\frac{\pi}{2}-\frac{3\pi}{5}\right)\right]$$
$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{10}\right)\right)$$
$$= -\frac{\pi}{10} \quad \left(\because \sin^{-1}(\sin x) = x, \text{ for } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)$$

24. The domain of the function cos-1 (2x - 1) is
(a) [0, 1] (b) [-1, 1] (c) [-1, 1] (d) [0, π]
Solution:

(a) [0, 1]
Since, cos-1 x is defined for $x \in [-1, 1]$
So, f(x) = cos-1 (2x - 1) is defined if
$-1 \le 2x - 1 \le 1$
$0 \le 2x \le 2$
Hence,
$0 \le x \le 1$

25. The domain of the function by $f(x) = \sin^{-1}\sqrt{(x - 1)}$ is
(a) [1, 2] (b) [-1, 1] (c) [0, 1] (d) none of these
Solution:

(a) [1, 2]
We know that, $\sin^{-1}x$ is defined for $x \in [-1, 1]$
So, $f(x) = \sin^{-1}\sqrt{(x - 1)}$ is defined if
$0 \le \sqrt{(x - 1)} \le 1$
$0 \le x - 1 \le 1$
$1 \le x \le 2$
Hence,

$x \in [1, 2]$

26. If cos ($\sin^{-1}$ 2/5 + $\cos^{-1}$ x) = 0, then x is equal to
(a) 1/5 (b) 2/5 (c) 0 (d) 1
Solution:

(b) 2/5
Given,
$\cos(\sin^{-1} 2/5 + \cos^{-1} x) = 0$
So, this can be rewritten as
$\sin^{-1} 2/5 + \cos^{-1} x = \cos^{-1} 0$
$\sin^{-1} 2/5 + \cos^{-1} x = \pi/2$
$\cos^{-1} x = \pi/2 - \sin^{-1} 2/5$
$\cos^{-1} x = \cos^{-1} 2/5$ [Since, $\cos^{-1} x + \sin^{-1} x = \pi/2$]
Hence,
$x = 2/5$

27. The value of sin (2 $\tan^{-1}$(0.75)) is equal to
(a) 0.75 (b) 1.5 (c) 0.96 (d) sin 1.5
Solution:

(c) 0.96
We have, $\sin(2\tan^{-1}(0.75))$

$$= \sin\left(2\tan^{-1}\frac{3}{4}\right) = \sin\left(\sin^{-1}\frac{2\cdot\frac{3}{4}}{1+\frac{9}{16}}\right) \quad \left(\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right)$$

$$= \sin\left(\sin^{-1}\frac{3/2}{25/16}\right) = \sin\left(\sin^{-1}\frac{24}{25}\right) = \frac{24}{25} = 0.96$$