## EXERCISE 18.4



**1. Find the absolute maximum and the absolute minimum values of the following functions in the given intervals:**
**(i) f (x) = 4x – $x^2$/2 in [–2, 45]**

**Solution:**

Given function is f(x) = $4x - \frac{x^2}{2}$

On differentiation we get
∴f'(x) = 4 – x

Now, for local minima and local maxima we have f'(x) = 0

4 – x = 0

x = 4

Then, we evaluate of f at critical points x = 4 and at the interval [– 2,$\frac{9}{2}$]

f (4) = $4(4) - \frac{(4)^2}{2}$ = 8

f (– 2) = $4(-2) - \frac{(-2)^2}{2} = -10$

f ($\frac{9}{2}$) = $4\left(\frac{9}{2}\right) - \frac{\left(\frac{9}{2}\right)^2}{2} = 18 - \frac{81}{8} = 7.875$

Hence, we can conclude that the absolute maximum value of f on [– 2, 9/2] is 8 occurring at x = 4 and the absolute minimum value of f on [– 2, 9/2] is – 10 occurring at x = – 2

**(ii) f (x) = (x – 1)$^2$ + 3 in [–3, 1]**

**Solution:**
Given function is f(x) = (x – 1)$^2$ + 3
On differentiation we get
∴f'(x) = 2(x – 1)

Now, for local minima and local maxima we have $f'(x) = 0$

$2(x - 1) = 0$

$x = 1$

Then, we evaluate of f at critical points x = 1 and at the interval [– 3, 1]

$f(1) = (1 - 1)^2 + 3 = 3$

$f(-3) = (-3 - 1)^2 + 3 = 19$

Hence, we can conclude that the absolute maximum value of f on [– 3, 1] is 19 occurring at x = – 3 and the minimum value of f on [– 3, 1] is 3 occurring at x = 1

**(iii) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ in [0, 3]**

**Solution:**

Given function is $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ in [0, 3]

On differentiating we get

$f'(x) = 12x^3 - 24x^2 + 24x - 48$

$f'(x) = 12(x^3 - 2x^2 + 2x - 4)$

$f'(x) = 12(x - 2)(x^2 + 2)$

Now, for local minima and local maxima we have $f'(x) = 0$

$x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Then, we evaluate of f at critical points x = 2 and at the interval [0, 3]

$f(2) = 3(2)^4 - 8(2)^3 + 12(2)^2 - 48(2) + 25$

$f(2) = 48 - 64 + 48 - 96 + 25 = -39$

$f(0) = 3(0)^4 - 8(0)^3 + 12(0)^2 - 48(0) + 25 = 25$

$f(3) = 3(3)^4 - 8(3)^3 + 12(3)^2 - 48(3) + 25 = 16$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at x = 0 and the minimum value of f on [0, 3] is – 39 occurring at x = 2

**(iv) $f(x) = (x - 2)\sqrt{x - 1}$ in [1, 9]**

**Solution:**

Given $f(x) = (x - 2)\sqrt{x - 1}$

$f'(x) = \sqrt{x - 1} + \frac{(x-2)}{2\sqrt{x-1}}$

Put $f'(x) = 0$

$\Rightarrow \sqrt{x-1} + \frac{(x-2)}{2\sqrt{x-1}} = 0$

$\Rightarrow \frac{2(x-1)+(x-2)}{2\sqrt{x-1}} = 0$

$\Rightarrow \frac{3x-4}{2\sqrt{x-1}} = 0$

$\Rightarrow x = \frac{4}{3}$

Now, f (1) = 0

f (4/3) = $\left(\frac{4}{3} - 2\right)\sqrt{\frac{4}{3} - 1} = -\frac{2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$

Hence, we can conclude that the absolute maximum value of f is 14 √2

occurring at x = 9 and the minimum value of is $-\frac{2\sqrt{3}}{9}$ occurring at x = 4/3

**2. Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [–3, –1].**

**Solution:**

Let $f(x) = 2x^3 - 24x + 107$

∴ $f'(x) = 6x^2 - 24 = 6(x^2 - 4)$

Now, for local maxima and local minima we have f'(x) = 0

$\Rightarrow 6(x^2 - 4) = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point x = 2 ∈ [1, 3] and at the end points of the interval [1, 3].

$f(2) = 2(2^3) - 24(2) + 107 = 75$

$f(1) = 2(1)^3 - 24(1) + 107 = 85$

$f(3) = 2(3)^3 - 24(3) + 107 = 89$

Hence, the absolute maximum value of f(x) in the interval [1, 3] is 89 occurring at x = 3,

Next, we consider the interval [– 3, – 1].

Evaluate the value of f at the critical point x = – 2 ∈ [1, 3]

$f(-3) = 2(-3)^3 - 24(-3) + 107 = 125$

$f(-2) = 2(-2)^3 – 24(-3) + 107 = 139$
$f(-1) = 2(-1)^3 – 24(-2) + 107 = 129$
Hence, the absolute maximum value when x = -2 is 139.