EXERCISE 19.9

Evaluate the following integrals:

1. $\int \frac{\log x}{x}$ **dx**

Solution:

Assume log x = t

$\Rightarrow$ d (log x) = dt

$\Rightarrow \frac{1}{x}dx = dt$

Substituting t and dt in above equation we get

$\Rightarrow \int t.dt$

$\Rightarrow \frac{t^2}{2} + c$

But t = log(x)

$\Rightarrow \frac{\log^2 x}{2} + c$.

2. $\int \frac{\log\left(1+\frac{1}{x}\right)}{x(1+x)}dx$

Solution:

Assume $\log\left(1 + \frac{1}{x}\right) = t$

$\Rightarrow d(\log\left(1 + \frac{1}{x}\right)) = dt$

$\Rightarrow \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2}dx = dt$

$\Rightarrow \frac{x}{x+1} \times \frac{-1}{x^2}dx = dt$

$\Rightarrow \frac{-1.dx}{x(x+1)} = dt$

$\Rightarrow \frac{dx}{x(x+1)} = -dt$

∴ Substituting t and dt in the given equation we get

$\Rightarrow \int -t.\,dt$

$\Rightarrow -\int t.\,dt$

$\Rightarrow \frac{-t^2}{2} + c$

But $\log\left(1 + \frac{1}{x}\right) = t$

$\Rightarrow -\frac{1}{2}\left\{\log\left(1+\frac{1}{x}\right)^2 + c\right\}$

3. $\int \frac{(1+\sqrt{x})^2}{\sqrt{x}}\,dx$

Solution:

Assume $1 + \sqrt{x} = t$

$\Rightarrow d(1 + \sqrt{x}) = dt$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}}dx = 2dt$

∴ Substituting t and dt in the given equation we get

$\Rightarrow \int 2t^2.\,dt$

$\Rightarrow 2\int t^2.\,dt$

$\Rightarrow \frac{2t^3}{3} + c$

But $1 + \sqrt{x} = t$

$\Rightarrow \frac{2(1+\sqrt{x})^3}{3} + c.$

4. $\int \sqrt{1+e^x}\, e^x \, dx$

Solution:

Assume $1 + e^x = t$

$\Rightarrow d(1 + e^x) = dt$

$\Rightarrow e^x \, dx = dt$

∴ Substituting t and dt in given equation we get

$\Rightarrow \int \sqrt{t}.dt$

$\Rightarrow \int t^{1/2}.dt$

$\Rightarrow \frac{2t^{\frac{3}{2}}}{3} + c$

But $1 + e^x = t$

$\Rightarrow \frac{2(1+e^x)^{3/2}}{3} + c.$

5. $\int \sqrt[3]{\cos^2 x} \sin x \, dx$

Solution:

Assume $\cos x = t$

$\Rightarrow d(\cos x) = dt$

$\Rightarrow -\sin x \, dx = dt$

$\Rightarrow dx = \frac{-dt}{\sin x}$

∴ Substituting t and dt in the given equation we get

$\Rightarrow \int \sqrt[3]{t^2} \sin x . \frac{dt}{\sin x}$

$\Rightarrow \int t^{3/2} . dt$

$\Rightarrow -\frac{3}{5} t^{\frac{5}{3}} x + c$

But cos x = t

$\Rightarrow -\frac{3}{5} \cos^{\frac{5}{3}} x + c$.

6. $\int \frac{e^x}{(1+e^x)^2} dx$

Solution:

Assume $1 + e^x = t$

$\Rightarrow d(1 + e^x) = dt$

$\Rightarrow e^x dx = dt$

∴ Substituting t and dt in given equation we get

$\Rightarrow \int \frac{1}{t^2} dt$

$\Rightarrow \int t^{-2} . dt$

$\Rightarrow \frac{-1}{t} + c$

But $1 + e^x = t$

$\Rightarrow \frac{-1}{1+e^x} + c$.

7. $\int \cot^3 x \; cosec^2 \; x \, dx$

Solution:

Assume cot x = t

$\Rightarrow$ d (cot x) = dt

$\Rightarrow$ - cosec^2x.dx = dt

$\Rightarrow \text{dt} = \frac{-\text{dt}}{\csc^2 x}$

∴ Substituting t and dt in the given equation we get

$\Rightarrow \int t^3 \csc^2 x . \frac{-dt}{\csc^2 x}$

$\Rightarrow \int -t^3 . dt$

$\Rightarrow -\int t^3 . dt$

$\Rightarrow \frac{-t^4}{4} + c$

But t = cot x

$\Rightarrow \frac{-\cot^4 x}{4} + c.$

8. $\int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1-x^2}} dx$

Solution:

Assume $\sin^{-1}$x = t

$\Rightarrow$ d ($\sin^{-1}$x) = dt

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

∴ Substituting t and dt in the given equation we get

$\Rightarrow \int e^{t^2} dt$

$\Rightarrow \int e^{2t} . dt$

$\Rightarrow \frac{e^{2t}}{2} + c$

But t = $\sin^{-1}x$

$\Rightarrow \frac{1}{2}\left\{e^{\sin^{-1}x}\right\}^2 + c$

9. $\int \frac{1+\sin x}{\sqrt{x-\cos x}}\,dx$

Solution:

Assume x – cos x = t

⇒ d (x – cos x) = dt

⇒ (1 + sin x) dx = dt

∴ Substituting t and dt in given equation we get

$\Rightarrow \int \frac{1}{\sqrt{t}}\,dt$

$\Rightarrow \int t^{-1\backslash 2}.\,dt$

$\Rightarrow 2t^{1\backslash 2} + c$

But t = x – cos x.

⇒ $2(x - \cos x)^{1/2} + c$.

10. $\int \frac{1}{\sqrt{1-x^2}(\sin^{-1}x)^2}\,dx$

Solution:

Assume $\sin^{-1}x$ = t

⇒ d ($\sin^{-1}x$) = dt

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

∴ Substituting t and dt in the given equation we get

$\Rightarrow \int \frac{1}{t^2} dt$

$\Rightarrow \int t^{-2} . dt$

Om integrating the above equation we get

$\Rightarrow \frac{t^{-1}}{-1} + c$

But t = sin $^{-1}$x

$\Rightarrow \frac{-1}{\sin^{-1} x} + c$