


नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 15 हैं।	(I) Please check that this question paper contains 15 printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II) Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 36 प्रश्न हैं।	(III) Please check that this question paper contains 36 questions.
(IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।	(IV) Please write down the Serial Number of the question in the answer-book before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित   
**MATHEMATICS**

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

.65/2/2

1

P.T.O.

**General Instructions :**

Read the following instructions very carefully and strictly follow them :

- (i) This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.
- (ii) **Section A** - Questions no. **1 to 20** comprises of **20** questions of **1** mark each.
- (iii) **Section B** - Questions no. **21 to 26** comprises of **6** questions of **2** marks each.
- (iv) **Section C** - Questions no. **27 to 32** comprises of **6** questions of **4** marks each.
- (v) **Section D** - Questions no. **33 to 36** comprises of **4** questions of **6** marks each.
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is **not** permitted.

**SECTION A**

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. The area of a triangle formed by vertices O, A and B, where

$$\vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{OB} = -3\hat{i} - 2\hat{j} + \hat{k} \text{ is}$$

- (A)  $3\sqrt{5}$  sq. units
- (B)  $5\sqrt{5}$  sq. units
- (C)  $6\sqrt{5}$  sq. units
- (D) 4 sq. units

2. If  $\cos\left(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x\right) = 0$ , then x is equal to

- (A)  $\frac{1}{\sqrt{5}}$
- (B)  $-\frac{2}{\sqrt{5}}$
- (C)  $\frac{2}{\sqrt{5}}$
- (D) 1

3. The interval in which the function  $f$  given by  $f(x) = x^2e^{-x}$  is strictly increasing, is

- (A)  $(-\infty, \infty)$
- (B)  $(-\infty, 0)$  *True*
- (C)  $(2, \infty)$
- (D)  $(0, 2)$

4. The function  $f(x) = \frac{x-1}{x(x^2-1)}$  is discontinuous at

- (A) exactly one point
- (B) exactly two points
- (C) exactly three points
- (D) no point

5. The function  $f: \mathbb{R} \rightarrow [-1, 1]$  defined by  $f(x) = \cos x$  is

- (A) both one-one and onto
- (B) not one-one, but onto
- (C) one-one, but not onto
- (D) neither one-one, nor onto

6. The coordinates of the foot of the perpendicular drawn from the point  $(2, -3, 4)$  on the  $y$ -axis is

- (A)  $(2, 3, 4)$
- (B)  $(-2, -3, -4)$
- (C)  $(0, -3, 0)$
- (D)  $(2, 0, 4)$

7. The relation  $R$  in the set  $\{1, 2, 3\}$  given by  $R = \{(1, 2), (2, 1), (1, 1)\}$  is

- (A) symmetric and transitive, but not reflexive
- (B) reflexive and symmetric, but not transitive
- (C) symmetric, but neither reflexive nor transitive
- (D) an equivalence relation

8. The angle between the vectors  $\hat{i} - \hat{j}$  and  $\hat{j} - \hat{k}$  is

- (A)  $-\frac{\pi}{3}$
- (B)  $0$
- (C)  $\frac{\pi}{3}$
- (D)  $\frac{2\pi}{3}$

9. If  $A$  is a non-singular square matrix of order 3 such that  $A^2 = 3A$ , then value of  $|A|$  is

- (A)  $-3$
- (B)  $3$
- (C)  $9$
- (D)  $27$

10. If  $|\vec{a}| = 4$  and  $-3 \leq \lambda \leq 2$ , then  $|\lambda \vec{a}|$  lies in

- (A)  $[0, 12]$
- (B)  $[2, 3]$
- (C)  $[8, 12]$
- (D)  $[-12, 8]$

Fill in the blanks in question numbers 11 to 15.

11. If the radius of the circle is increasing at the rate of  $0.5$  cm/s, then the rate of increase of its circumference is \_\_\_\_\_.

12. If  $\begin{vmatrix} 2x & -9 \\ -2 & x \end{vmatrix} = \begin{vmatrix} -4 & 8 \\ 1 & -2 \end{vmatrix}$ , then value of  $x$  is \_\_\_\_\_.

13. The corner points of the feasible region of an LPP are  $(0, 0)$ ,  $(0, 8)$ ,  $(2, 7)$ ,  $(5, 4)$  and  $(6, 0)$ . The maximum profit  $P = 3x + 2y$  occurs at the point \_\_\_\_\_.

14. The range of the principal value branch of the function  $y = \sec^{-1} x$  is \_\_\_\_\_.

**OR**

The principal value of  $\cos^{-1} \left( -\frac{1}{2} \right)$  is \_\_\_\_\_.

65/2/2

15. The distance between parallel planes  $2x + y - 2z - 6 = 0$  and  $4x + 2y - 4z = 0$  is \_\_\_\_\_ units.

OR

If  $P(1, 0, -3)$  is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is \_\_\_\_\_.

Question numbers 16 to 20 are very short answer type questions.

16. Evaluate :

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x \, dx$$

17. Find the coordinates of the point where the line  $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$  cuts the  $xy$ -plane.

18. Find the value of  $k$ , so that the function  $f(x) = \begin{cases} kx^2 + 5 & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases}$  is continuous at  $x = 1$ .

19. Find the integrating factor of the differential equation


$$x \frac{dy}{dx} = 2x^2 + y$$

20. Differentiate  $\sec^2(x^2)$  with respect to  $x^2$ .

OR

If  $y = f(x^2)$  and  $f'(x) = e^{\sqrt{x}}$ , then find  $\frac{dy}{dx}$ .

85/2/2



**SECTION B**

Question numbers 21 to 26 carry 2 marks each.

21. Find a vector  $\vec{r}$  equally inclined to the three axes and whose magnitude is  $3\sqrt{3}$  units.

**OR**

Find the angle between unit vectors  $\vec{a}$  and  $\vec{b}$  so that  $\sqrt{3}\vec{a} - \vec{b}$  is also a unit vector.

22. If  $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , find scalar  $k$  so that  $A^2 + I = kA$ .

23. If  $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ , find  $f'\left(\frac{\pi}{3}\right)$ .

**OR**

Find  $f'(x)$  if  $f(x) = (\tan x)^{\tan x}$ .

24. Find :

$$\int \frac{\tan^3 x}{\cos^3 x} dx$$

25. Show that the plane  $x - 5y - 2z = 1$  contains the line  $\frac{x-5}{3} = y = 2 - z$ .
26. A fair dice is thrown two times. Find the probability distribution of the number of sixes. Also determine the mean of the number of sixes.

**SECTION C**

Question numbers 27 to 32 carry 4 marks each.

27. Solve the following differential equation :

$$(1 + e^{y/x}) dy + e^{y/x} \left(1 - \frac{y}{x}\right) dx = 0 \quad (x \neq 0).$$

28. A cottage industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table :

Item	Machine Time	Craftsman time	Profit (in ₹)
Pedestal lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an LPP and solve it graphically.

29. Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$$

30. Check whether the relation R in the set N of natural numbers given by

$$R = \{(a, b) : a \text{ is divisor of } b\}$$

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

**OR**

Prove that  $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \sin^{-1} \left( \frac{4}{5} \right)$ .

31. Find the equation of the plane passing through the points (1, 0, -2), (3, -1, 0) and perpendicular to the plane  $2x - y + z = 8$ . Also find the distance of the plane thus obtained from the origin.

32. If  $\tan^{-1} \left( \frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$ , prove that  $\frac{dy}{dx} = \frac{x+y}{x-y}$ .

**OR**

If  $y = e^{a \cos^{-1} x}$ ,  $-1 < x < 1$ , then show that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2y = 0$$

65/2/2



**SECTION D**

Question numbers 33 to 36 carry 6 marks each.

33. Amongst all open (from the top) right circular cylindrical boxes of volume  $125\pi \text{ cm}^3$ , find the dimensions of the box which has the least surface area.
34. Using integration, find the area lying above x-axis and included between the circle  $x^2 + y^2 = 8x$  and inside the parabola  $y^2 = 4x$ .

**OR**

Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).

35. If  $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$ , find  $A^{-1}$  and use it to solve the following system of equations :

$$5x - y + 4z = 5$$

$$2x + 3y + 5z = 2$$

$$5x - 2y + 6z = -1$$

**OR**

If x, y, z are different and  $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ , then using properties of

determinants show that  $1 + xyz = 0$ .

36. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly one-by-one without replacement and are found to be both kings. Find the probability of the lost card being a king.