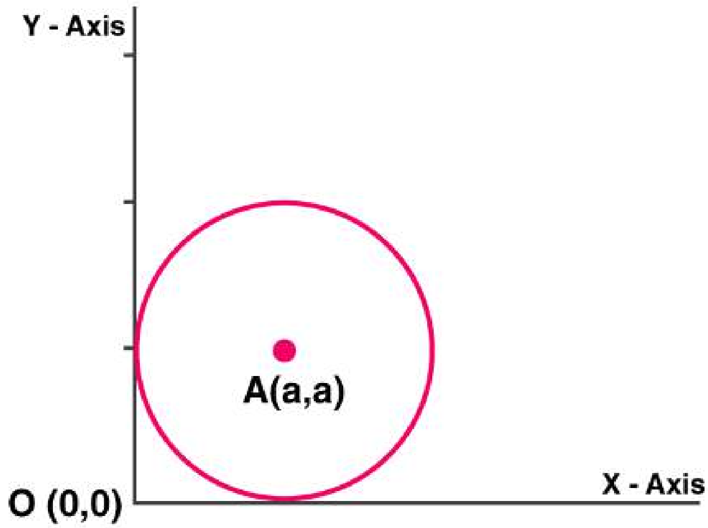
EXERCISE PAGE NO: 202

SHORT ANSWER TYPE:

1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

Solution:

The circle touches both the x and y axes in the first quadrant and the radius is a.



For a circle of radius a, the centre is (a, a).

The equation of a circle having centre (h, k), having radius as r units, is

$(x – h)^2 + (y – k)^2 = r^2$

Therefore, the equation of the circle becomes $(x – a)^2 + (y – a)^2 = a^2$

$x^2 - 2ax + a^2 + y^2 - 2ay + a^2 - a^2 = 0$

$x^2 - 2ax + y^2 - 2ay + a^2 = 0$

2. Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that –1 ≤ t ≤1 where a is any given real numbers.

Solution:

Given

$$x = \frac{2at}{1+t^2}, y = \frac{a(1-t^2)}{1+t^2}$$

Squaring both the equations,

$$x^2 = \left[\frac{2at}{1+t^2}\right]^2 \; \& y^2 = \left[\frac{a(1-t^2)}{1+t^2}\right]^2$$

Adding both the equations,

$$x^2 + y^2 = \frac{4a^2t^2}{(1+t^2)^2} + \frac{a^2(1-t^2)^2}{(1+t^2)^2}$$

Taking LCM and simplifying we get

$$= \frac{a^2(4t^2 + (1-t^2)^2)}{(1+t^2)^2}$$

$$= \frac{a^2(4t^2 + 1 + t^4 - 2t^2))}{(1+t^2)^2}$$

$$= \frac{a^2(2t^2 + 1 + t^4)}{(1+t^2)^2}$$

$$= \frac{a^2(1+t^2)^2}{(1+t^2)^2}$$

$$= a^2$$

Therefore $x^2+y^2= a^2$

The equation of a circle having centre (h, k), having radius as r units, is

$(x - h)^2 + (y - k)^2 = r^2$

Centre = (0, 0) Radius = a units

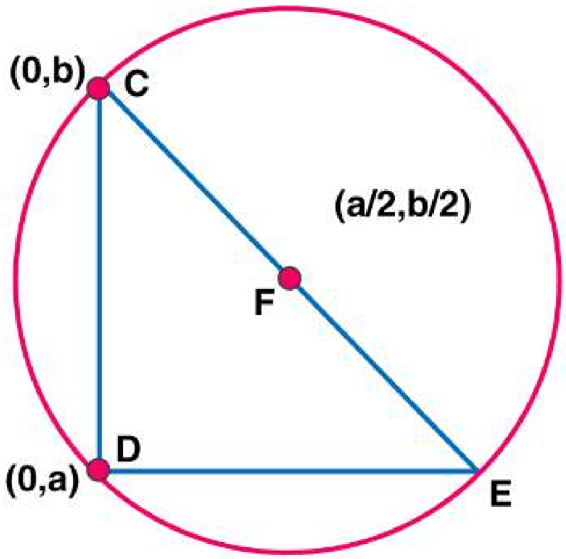
Hence proved.

3. If a circle passes through the point (0, 0) (a, 0), (0, b) then find the coordinates of its centre.

Solution:

The equation of a circle having centre (h, k), having radius as r units, is

$(x - h)^2 + (y - k)^2 = r^2$



Putting the values of given co-ordinates in the above expression,

(0, 0)

$(0 - h)^2 + (0 - k)^2 = r^2$

$h^2 + k^2 = r^2$

(a, 0)

$(a - h)^2 + (0 - k)^2 = r^2$

$a^2 + h^2 - 2ah + k^2 = r^2$ ------ (1)

(0, b)

$(0 - h)^2 + (b - k)^2 = r^2$

$h^2 + b^2 + k^2 - 2bk = r^2$ ------- (2)

On solving equations (1) & (2), respectively,

$a (a - 2h) = 0$

$b (b - 2k) = 0$

So, a = 0 or 2h

b = 0 or 2k respectively.

Since the circle passes through the centre (0, 0), so the co-ordinates are

a = 2h, h = a/2

b = 2k, k = b/2

The co-ordinates of the centre are (a/2, b/2)

4. Find the equation of the circle which touches x-axis and whose centre is (1, 2).

Solution:

Since the circle has a centre (1, 2) and also touches x-axis.

Radius of the circle is, r = 2

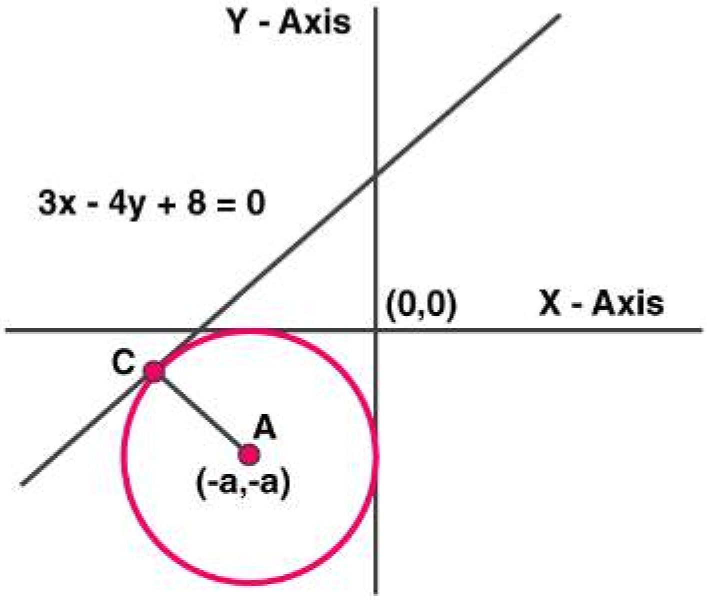
The equation of a circle having centre (h, k), having radius as r units, is
$(x - h)^2 + (y - k)^2 = r^2$
So, the equation of the required circle is:
$(x - 1)^2 + (y - 2)^2 = 2^2$
$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = 4$
$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$
The equation of the circle is $x^2 + y^2 - 2x - 4y + 1 = 0$.

5. If the lines 3x – 4y + 4 = 0 and 6x – 8y – 7 = 0 are tangents to a circle, then find the radius of the circle.

Solution:
Given both lines are parallel & tangent to the circle.
Then distance between these two lines must be the diameter of the circle.
Lines:
$3x - 4y + 4 = 0$
$3x - 4y - 3.5 = 0$ (Equating the co-efficient of the equation)
Distance (between 2 || lines) = $\left(\frac{|c-p|}{\sqrt{a^2+b^2}}\right)$ for the two given equations,
$ax + by + c = 0$ & $ax + by + p = 0$
Distance = $\left(\frac{|4-(-3.5)|}{\sqrt{3^2+(-4)^2}}\right) = \left(\frac{|7.5|}{\sqrt{9+16}}\right) = \frac{7.5}{5} = 1.5$ units
Radius = $\frac{\text{Diameter}}{2} = \frac{1.5}{2}$
=0.75 units.
Hence, the radius of the circle is 0.75 = ¾ units.

6. Find the equation of a circle which touches both the axes and the line 3x – 4y + 8 = 0 and lies in the third quadrant.

Solution:
As the circle lies in third quadrant, then the centre is (-a, -a).
Perpendicular Distance (Between a point and line) = $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$
Whereas the point is (x_1, y_1) and the line is expressed as $ax + by + c = 0$
The line which touches the circle is 3x−4y+8=0, which is a tangent to the circle.
The perpendicular distance = a units (radius of the circle)



$$a = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$a = \frac{|3(-a) - 4(-a) + 8|}{\sqrt{3^2 + 4^2}}$$

$$a = \frac{|-3a + 4a + 8|}{\sqrt{9 + 16}}$$

$$a = \frac{|a + 8|}{\sqrt{25}}$$

$$a = \frac{a + 8}{5}$$

$5a = a + 8$

$a = 2$

Co-ordinates of the centre of the circle = (-2,-2)

Since, the equation of a circle having centre (h, k), having radius as "r" units, is

$(x - h)^2 + (y - k)^2 = r^2$

$(x - (-2))^2 + (y - (-2))^2 = 2^2$

$(x + 2)^2 + (y + 2)^2 = 4$

$x^2 + 4x + 4 + y^2 + 4y + 4 - 4 = 0$

$x^2 + y^2 + 4x + 4y + 4 = 0$

The equation of the given circle is $x^2 + y^2 + 4x + 4y + 4 = 0$.

7. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the

coordinate of the other end of the diameter.

Solution:

Given equation of the circle,

$x^2 - 4x + y^2 - 6y + 11 = 0$

$x^2 - 4x + 4 + y^2 - 6y + 9 + 11 – 13 = 0$

the above equation can be written as

$x^2 – 2 (2) x + 2^2 + y^2 – 2 (3) y + 3^2 + 11 – 13 = 0$

on simplifying we get

$(x – 2)^2 + (y – 3)^2 = 2$

$(x – 2)^2 + (y – 3)^2 = (\surd 2)^2$

Since, the equation of a circle having centre (h, k), having radius as r units, is

$(x – h)^2 + (y – k)^2 = r^2$

We have centre = (2, 3)

The centre point is the mid-point of the two ends of the diameter of a circle.

Let the points be (p, q)

$(p + 3)/2 = 2$ and $(q + 4)/2 = 3$

$p + 3 = 4$ & $q + 4 = 6$

$p = 1$ & $q = 2$

Hence, the other ends of the diameter are (1, 2).

8. Find the equation of the circle having (1, –2) as its centre and passing through 3x + y = 14, 2x + 5y = 18

Solution:

Solving the given equations,

$3x + y = 14$ ……….1

$2x + 5y = 18$ …………..2

Multiplying the first equation by 5, we get

$15x + 5y = 70$…….3

$2x + 5y = 18$……..4

Subtract equation 4 from 3 we get

$13 x = 52$,

Therefore $x = 4$

Substituting $x = 4$, in equation 1, we get

$3 (4) + y = 14$

$y = 14 – 12 = 2$

So, the point of intersection is (4, 2)
Since, the equation of a circle having centre (h, k), having radius as r units, is
$(x - h)^2 + (y - k)^2 = r^2$
Putting the values of (4, 2) and centre co-ordinates (1,-2) in the above expression, we get
$(4 - 1)^2 + (2 - (-2))^2 = r^2$
$3^2 + 4^2 = r^2$
$r^2 = 9 + 16 = 25$
r = 5 units
So, the expression is
$(x - 1)^2 + (y - (-2))^2 = 5^2$
Expanding the above equation we get
$x^2 - 2x + 1 + (y + 2)^2 = 25$
$x^2 - 2x + 1 + y^2 + 4y + 4 = 25$
$x^2 - 2x + y^2 + 4y - 20 = 0$
Hence the required expression is $x^2 - 2x + y^2 + 4y - 20 = 0$.

9. If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.

Solution:
Since, the equation of a circle having centre (h, k), having radius as r units, is
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 0)^2 + (y - 0)^2 = 4^2$
Perpendicular Distance between a point (0, 0) & the line
$y = \sqrt{3}x + k$ or $\sqrt{3}x - y + k = 0$
Perpendicular Distance (Between a point and line) =
Whereas the point is (x_1, y_1) and the line is expressed as $ax + by + c = 0$

$$D = \frac{\{|\sqrt{3}(0) + (0)(-1) + k|\}}{\sqrt{(\sqrt{3})^2 + 1^2}} = \frac{\{|k|\}}{\sqrt{3+1}} = \frac{\{k\}}{\sqrt{4}} = \frac{k}{2} \quad \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$\frac{k}{2} = 4$ (Radius = 4, Given)
$k = 8$
Hence, the required value of k is 8.

10. Find the equation of a circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$

and has double of its area.

Solution:

Given equation of the circle is

$x^2 - 6x + y^2 + 12y + 15 = 0$

The above equation can be written as

$x^2 - 2(3)x + 3^2 + y^2 + 2(6)y + 6^2 + 15 - 9 + 36 = 0$

$(x - 3)^2 + (y-(-6))^2 - 30 = 0$

$(x - 3)^2 + (y-(-6))^2 = (\sqrt{30})^2$

Since, the equation of a circle having centre (h, k), having radius as r units, is

$(x - h)^2 + (y - k)^2 = r^2$

Centre = (3,-6)

Area of inner circle = $\pi r^2 = 22/7 \times 30 = 30\pi$ units square

Area of outer circle = $2 \times 30\pi = 60\pi$ units square

So, $\pi r^2 = 60\pi$

$r^2 = 60$

Equation of outer circle is,

$(x - 3)^2 + (y - (-6))^2 = (\sqrt{60})^2$

$x^2 - 6x + 9 + y^2 - 12y + 36 = 60$

$x^2 - 6x + y^2 + 12y + 45 - 60 = 0$

$x^2 - 6x + y^2 + 12y - 15 = 0$

Hence, the required equation of the circle is $x^2 - 6x + y^2 + 12y - 15 = 0$.

11. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

Solution:

Equation of an ellipse = $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Whereas

Length of latus rectum = $\frac{2b^2}{a}$

Length of minor axis = 2b

So,

$\frac{2b}{2} = \frac{2b^2}{a}$ (Given)

Ab = $2b^2$

$2b^2 - ab = 0$

b (2b - a) = 0

So, b = 0 or a = 2b

$b^2 = a^2 (1 - e^2)$

$b^2 = (2b)^2 (1 - e^2)$

$b^2 = 4b^2 (1 - e^2)$

On rearranging we get

$$1 - e^2 = \frac{1}{4}$$

$$e^2 = \frac{3}{4}$$

$$e = \frac{\sqrt{3}}{2}$$

Hence, the eccentricity is $\frac{\sqrt{3}}{2}$

12. Given the ellipse with equation $9x^2 + 25y^2 = 225$, find the eccentricity and foci.

Solution:

We know that equation of an ellipse is

$= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Also we have,

Length of latus rectum = $\frac{2b^2}{a}$

Length of minor axis = 2b

$9x^2 + 25y^2 = 225$

Dividing the above equation by 225,

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The above equation can be written as

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$$

From the above equation we have,

a = 5, b = 3

$b^2 = a^2 (1 - e^2)$

$3^2 = 5^2 (1 - e^2)$

On rearranging we get

$$\frac{9}{25} = 1 - e^2$$

$$e^2 = 1 - \frac{9}{25}$$

Taking LCM and simplifying we get

$$e^2 = \frac{16}{25}$$

$$e = \frac{4}{5}$$

We know foci = (±a e, 0)

$$= \left(\pm 5 \times \frac{4}{5}, 0\right) = (\pm 4, 0)$$

Hence, the eccentricity is 4/5 and foci is (±4, 0).

13. If the eccentricity of an ellipse is 5/8 and the distance between its foci is 10, then find latus rectum of the ellipse.

Solution:

We know that equation of an ellipse

$$= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

Also we have

Length of latus rectum = $\frac{2b^2}{a}$

Length of minor axis = 2b

$$e = \frac{5}{8}$$

We know that foci = (±a e, 0)

Given that distance between foci = 10

2ae = 10

$$2 \times a \times \frac{5}{8} = 10$$

a = 8

$b^2 = a^2 (1 - e^2)$

$b^2 = 8^2\left(1 - \left(\frac{5}{8}\right)^2\right)$

On simplifying we get

$b^2 = 64\left(1 - \frac{25}{64}\right)$

$b^2 = 64 \times \frac{39}{64} = 39$

Length of Latus Rectum = $\frac{2b^2}{a} = 2 \times \frac{39}{8} = 9.75$

Hence, the length of latus rectum is 9.75 units.

14. Find the equation of ellipse whose eccentricity is 2/3 , latus rectum is 5 and the centre is (0, 0).

Solution:

We know that equation of an ellipse

$= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Also we have,

Length of latus rectum = $\frac{2b^2}{a}$

Length of minor axis = 2b

Given

$e = \frac{2}{3}$

$b^2 = a^2 (1 - e^2)$ 1

Length of Latus Rectum = $\frac{2b^2}{a} = 5$

$b^2 = 2.5\ a$ 2

Now by substituting the values in equation 1 we get

$\therefore 2.5\ a\ =\ a^2\left(1 - \left(\frac{2}{3}\right)^2\right)$

On simplifying and rearranging we get

$2.5\ a\ =\ a^2\left(1 - \frac{4}{9}\right)$

$2.5\ a\ =\ a^2\left(\frac{5}{9}\right)$

$22.5\ a = 5a^2$

$5a^2 - 22.5a = 0$

$5a\ (a - 4.5) = 0$

a = 0 {Not Possible, as the length of latus rectum is 5 units} or 4.5 = 9/2

Now by substituting the values in equation 2 we get

$b^2 = 2.5 \times 4.5 = 11.25 = \frac{45}{4}$

Again we have equation of an ellipse

$= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

By substituting this values we get

$$\frac{x^2}{\left(\frac{9}{2}\right)^2} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

∴Equation of an ellipse is $\frac{4x^2}{81} + \frac{4y^2}{45} = 1$

15. Find the distance between the directrices of the ellipse $x^2/36 + y^2/20 = 1$

Solution:

We know that equation of an ellipse

$= \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Also we have Length of latus rectum = $\frac{2b^2}{a}$

Length of minor axis = 2b

$b^2 = a^2\ (1 - e^2)$ 1

Now by substituting we get

$$\frac{x^2}{(\sqrt{36})^2} + \frac{y^2}{(\sqrt{20})^2} = 1$$

$$\frac{x^2}{(6)^2} + \frac{y^2}{(2\sqrt{5}\,)^2} = 1$$

From equation 1, we have

$20 = 36\ (1 - e^2)$

On rearranging we get

$\frac{20}{36} = 1 - e^2$

$e^2 = 1 - \frac{20}{36}$

$e^2 = \frac{16}{36} = \frac{4}{9}$

$e = \frac{2}{3}$

Directrices = $\pm\frac{a}{e}$

Distance between directrices = $\frac{2a}{e} = \frac{2\times6}{\frac{2}{3}} = 18$

Hence the distance between directrices is 18 units.

16. Find the coordinates of a point on the parabola y2 = 8x whose focal distance is 4.

Solution:

We know that equation of an ellipse is $y^2 = 4ax$,

Also we have length of latus rectum = 4a

Now by comparing the above two equations,

4a = 8

Therefore

a = 2

$y^2 = 8 \times 2 = 16$

y = ± 4 and x = 2

Hence the co - ordinates are (2, 4) and (2, -4).

17. Find the length of the line-segment joining the vertex of the parabola y2 = 4axand a point on the parabola where the line-segment makes an angle q to the x-axis.

Solution:

We know that equation of an ellipse is $y^2 = 4ax$,

Also we have

Length of latus rectum = 4a

Let the point on parabola be (i, j)

From the figure, slope of OP = $\tan\theta = \frac{j}{i}$

$j = i\tan\theta$

Squaring the above equation on both sides we get

Squaring the above equation on both sides we get

$j^2 = i^2\tan^2\theta$ 1

$j^2 = 4ai$ 2

$OP = \sqrt{i^2 + j^2}$

By substituting the value of j then we get

$OP = \sqrt{i^2 + (i\tan\theta)^2}$

$= \sqrt{i^2 + i^2\tan^2\theta}$

Taking i^2 common we get

$= \sqrt{i^2(1 + \tan^2\theta)}$

We know that

$1 + \tan^2\theta = \mathrm{Sec}^2\theta$

$= \sqrt{i^2\mathrm{Sec}^2\theta}$

The above equation can be written as

$= i\mathrm{Sec}\theta$

Now by equating the equations 1 and 2 we get

$4ai = i^2\tan^2\theta$

On rearranging we get

$i^2\tan^2\theta - 4ai = 0$

Taking I common

$i(i\tan^2\theta - 4a) = 0$

i = 0 which is not Possible

Or

$i\tan^2\theta - 4a = 0$

$i = \frac{4a}{\tan^2\theta}$

Now OP = $i\mathrm{Sec}\theta = \frac{4a}{\tan^2\theta} \times \mathrm{Sec}\theta = \frac{4a \times \mathrm{Cos}^2\theta \times 1}{\mathrm{Sin}^2\theta \times \mathrm{Cos}\theta} = 4a\mathrm{Cot}\theta\mathrm{Cosec}\theta$

Hence, the length of line segment is $4a\mathrm{Cot}\theta\mathrm{Cosec}\theta$ units.

18. If the points (0, 4) and (0, 2) are respectively the vertex and focus of a parabola, then find the equation of the parabola.

Solution:

Given

Vertex = (0, 4) Focus = (0, 2)
So, the directrix of the parabola is y = 6,
Since, distance of (x, y) from (0, 2) and perpendicular distance from (x, y) to directrix are always equal.
Using Distance Formula & Perpendicular Distance Formula,
Perpendicular Distance (Between a point and line) is

$$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}},$$

Where the point is (x_1, y_1) and the line is expressed as ax + by + c = 0 i.e..,
x (0) + y – 6 = 0 & (x, y)
Distance between the point of intersection & centre
We know distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Now by substituting the values we get

$$\sqrt{((x-0)^2 + (y-2)^2)} = \frac{|x(0) + y(1) - 6|}{\sqrt{0^2 + (-1^2)}} = \frac{y-6}{1} = y - 6$$

Squaring both the sides,

$$\left[\sqrt{((x-0)^2 + (y-2)^2)}\right]^2 = (y-6)^2$$

$x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$
$x^2 + 8y - 32 = 0$
Hence, the required equation is $x^2 + 8y - 32 = 0$.

19. If the line y = mx + 1 is tangent to the parabola y2 = 4x then find the value of m.

Solution:
Given equations are,
y = mx + 1 & $y^2 = 4x$
By solving given equations we get
$(mx + 1)^2 = 4x$
Expanding the above equation we get
$m^2x^2 + 2mx + 1 = 4x$
On rearranging we get
$m^2x^2 + 2mx - 4x + 1 = 0$
$m+x^2 + x(2m - 4) + 1 = 0$
As the line touches the parabola, above equation must have equal roots,

Discriminant (D) = 0

$(2m - 4)^2 - 4(m^2)(1) = 0$

$4m^2 - 16m + 16 - 4m^2 = 0$

$-16m + 16 = 0$

$-m + 1 = 0$

$m = 1$

Hence, the required value of m is 1.

20. If the distance between the foci of a hyperbola is 16 and its eccentricity is √2, then obtain the equation of the hyperbola.

Solution:

We know that equation of Hyperbola

$= \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Also we have foci = (±a e, 0)

Given distance between foci is 2ae = 16

$e = \sqrt{2}$

$2 \times a \times \sqrt{2} = 16$

$$a = \frac{16}{2 \times \sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

$\because b^2 = a^2(e^2 - 1)$

$$b^2 = (4\sqrt{2})^2((\sqrt{2})^2 - 1)$$

$$= 32(2-1) = 32$$

$\therefore$ Equation is $\frac{x^2}{32} - \frac{y^2}{32} = 1$

21. Find the eccentricity of the hyperbola $9y^2 - 4x^2 = 36$.

Solution:

Given

$9y^2 - 4x^2 = 36$

Dividing the above equation by 36, we get

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

The above equation can be written as

$\frac{y^2}{2^2} - \frac{x^2}{3^2} = 1$

We know that equation of Hyperbola = $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Now by comparing the equations, we get

a = 2 and b = 3

$b^2 = a^2 (e^2 - 1)$

$3^2 = (2)^2[(e)^2 - 1)]$

$9 = 4 (e^2 - 1)$

$e^2 - 1 = \frac{9}{4}$

$e^2 = 1 + \frac{9}{4} = \frac{13}{4}$

$e = \frac{\sqrt{13}}{2}$

Hence, the eccentricity of given hyperbola is $\frac{\sqrt{13}}{2}$

22. Find the equation of the hyperbola with eccentricity 3/2 and foci at (± 2, 0).

Solution:

Given

$e = \frac{3}{2}$

We have foci = (± a e, 0) = (± 2, 0)

Therefore the hyperbola lies on x – axis,

Equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Given a e = 2

$a \times \frac{3}{2} = 2$

$a = \frac{4}{3}$

$\because b^2 = a^2 (e^2 - 1)$

$b^2 = \left(\frac{4}{3}\right)^2 \left(\left(\frac{3}{2}\right)^2 - 1\right)$

$= \frac{16}{9}\left(\frac{9}{4} - 1\right) = \frac{16}{9} \times \frac{5}{4} = \frac{20}{9}$

Equation is $\frac{x^2}{\left(\frac{4}{3}\right)^2} - \frac{y^2}{\frac{20}{9}} = 1$

$\frac{9x^2}{16} - \frac{9y^2}{20} = 1$

Hence, the required equation is $\frac{9x^2}{16} - \frac{9y^2}{20} = 1$

LONG ANSWER TYPE:

23. If the lines 2x – 3y = 5 and 3x – 4y = 7 are the diameters of a circle of area 154square units, then obtain the equation of the circle.

Solution:

Since, diameters of a circle intersect at the centre of a circle,

2x – 3y = 5 ………1

3x – 4y = 7 ………..2

Solving the above equations,

Multiplying equation 1 by 3 we get

6x – 9y = 15

Multiplying equation 2 by 2 we get

6x – 8y = 14

y = 1

y = -1

Putting y = -1, in equation 1, we get

2x – 3(-1) = 5

2x + 3 = 5

2x = 2

x = 1

Coordinates of centre = (1,-1)

Given area = 154

Area = πr^2 = 154

22/7 × r^2 = 154

r^2 = 154 × 7/22

r = 7 units

Since, the equation of a circle having centre (h, k), having radius as r units, is

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 1)^2 + (y - (-1))^2 = 7^2$

$x^2 - 2x + 1 + (y + 1)^2 = 49$
$x^2 - 2x + 1 + y^2 + 2y + 1 – 49 = 0$
$x^2 - 2x + y^2 + 2y – 47 = 0$
Hence the required equation of the given circle is $x^2 - 2x + y^2 + 2y – 47 = 0$.

24. Find the equation of the circle which passes through the points (2, 3) and (4, 5) and the centre lies on the straight line y – 4x + 3 = 0.

Solution:
Since, the equation of a circle having centre (h, k), having radius as r units, is
$(x – h)^2 + (y – k)^2 = r^2$………..1
Substituting (2, 3) & (4, 5) in the above equation, we get
$(2 – h)^2 + (3 – k)^2 = r^2$
$4 – 4h + h^2 + 9 + k^2 - 6k = r^2$
$h^2 - 4h + k^2 - 6k + 13 = r^2$ …………..2
$(4 – h)^2 + (5 – k)^2 = r^2$
$16 – 8h + h^2 + 25 + k^2 - 10k = r^2$
$h^2 - 8h + k^2 - 10k + 41 = r^2$ ………..3
Equating both the equations 2 & 3, as their RHS are equal, we get
$h^2 - 4h + k^2 - 6k + 13 = h^2 - 8h + k^2 - 10k + 41$
On simplifying we get
8h - 4h + 10k – 6k = 41 – 13
4h + 4k = 28
h + k = 7 ………..4
As centre lies on the given line, so it satisfies the values too,
k – 4h + 3 = 0…………5
Solving equations 3 and 4 simultaneously,
h + k = 7
-4h + k = -3
Subtracting both the equations, we get
5h = 10
h = 2
2 + k = 7
k = 5
Putting h = 2 & k = 5 in equation 2,
$h^2 - 4h + k^2 - 6k + 13 = r^2$
$2^2 - 4(2) + 5^2 - 6(5) + 13 = r^2$

$4 - 8 + 25 - 30 + 13 = r^2$

$r^2 = 4$

r = 2 units

Putting the values of h = 2, k = 5 & r = 2, respectively in equation 1,

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 2)^2 + (y - 5)^2 = 2^2$

$x^2 - 4x + 4 + y^2 - 10y + 25 = 4$

$x^2 - 4x + y^2 - 10y + 25 = 0$

Hence, the required equation is $x^2 - 4x + y^2 - 10y + 25 = 0$.

25. Find the equation of a circle whose centre is (3, –1) and which cuts off a chord of length 6 units on the line 2x – 5y + 18 = 0.

Solution:

Given equation of the chord is,

2x – 5y + 18 = 0 1

5y = 2x + 18

$$y = \frac{2}{5}x + \frac{18}{5}$$

As we have, y = mx + C

Where, m is the slope of the line,

$$m = \frac{2}{5}$$

Slope of the line perpendicular to the chord,

$$m' = -\frac{5}{2}$$

As the product of slope of perpendicular lines = -1,

$y - y_1 = m'(x - x_1)$

$$y - (-1) = -\frac{5}{2}(x - 3)$$

2y + 2 = -5x + 15

5x + 2y = 13 ....2[Equation of line passing from centre and cutting the chord]

Solving both the equations,

2x – 5y = -18 & 5x + 2y = 13

Multiplying the equation 1 & equation 2 by 2 & 5 respectively, we get

$4x - 10y = -36$
$25x + 10y = 65$
$29x = 29$
$x = 1$
$2(1) - 5y = -18$
$2 - 5y + 18 = 0$
$5y = 20$
$y = 4$
Point of intersection at chord and radius = (1, 4)
Distance between the point of intersection & centre
Distance formula = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(1-3)^2 + \left(4-(-1)\right)^2}$
$= \sqrt{(-2)^2 + (4+1)^2}$
$= \sqrt{4 + (5)^2}$
$= \sqrt{4 + 25}$
$=\sqrt{29}$ units
Using Pythagoras Theorem,
$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$
$= (3)^2 + (\sqrt{29})^2 = 29 + 9$
$= \sqrt{38}$
Hypotenuse = $\sqrt{38}$ units (radius)
Since, the radius bisects the chord into two equal halves,
Since, the equation of a circle having centre (h, k), having radius as r units, is
$(x - h)^2 + (y - k)^2 = r^2$
$(x - 3)^2 + (y - (-1))^2 = (\sqrt{38})^2$
$x^2 - 6x + 9 + (y + 1)^2 = 38$
$x^2 - 6x + y^2 + 2y + 1 + 9 - 38 = 0$
$x^2 - 6x + y^2 + 2y - 28 = 0$
Hence, the required equation of the circle is $x^2 - 6x + y^2 + 2y - 28 = 0$.

26. Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5, 5).

Solution:

Given $x^2 - 2x + y^2 - 4y - 20 = 0$

$x^2 - 2x + 1 + y^2 - 4y + 4 - 20 - 5 = 0$

$(x - 1)^2 + (y - 2)^2 = 25$

$(x - 1)^2 + (y - 2)^2 = 5^2$

Since, the equation of a circle having centre (h, k), having radius as r units, is

$(x - h)^2 + (y - k)^2 = r^2$

Centre = (1, 2)

Point of Intersection = (5, 5)

It intersects the line into 1: 1, as the radius of both the circles is 5 units.

Using Ratio Formula,

$$\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Ratio = m_1: m_2

Assuming the co-ordinates of the centre of the circle be (p, q)

$$5 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$5 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Now by substituting the values we get

$$\frac{(1)p + (1)1}{1 + 1} = \frac{p + 1}{2} = 5$$

$$\frac{1(q) + 1(2)}{1 + 1} = \frac{q + 2}{2} = 5$$

p + 1 = 10, q + 2 = 10

p = 9 & q = 8

Co-ordinates = (9, 8)

Therefore the equation is,

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 9)^2 + (y - 8)^2 = 5^2$

$x^2 - 18x + 81 + y^2 - 16y + 64 = 25$

$x^2 - 18x + y^2 - 16y + 145 - 25 = 0$

$x^2 - 18x + y^2 - 16y + 120 = 0$

Hence, the required equation is $x^2 - 18x + y^2 - 16y + 120 = 0$.

27. Find the equation of a circle passing through the point (7, 3) having radius 3 units

and whose centre lies on the line y = x – 1.

Solution:

Since, the equation of a circle having centre (h, k), having radius as r units, is

$(x - h)^2 + (y - k)^2 = r^2$Centre lies on the line i.e., y = x – 1,

Co – Ordinates are (h, k) = (h, h – 1)

$(x - h)^2 + (y - k)^2 = r^2$

$(7 - h)^2 + (3 - (h - 1))^2 = 3^2$

$49 + h^2 - 14h + (3 - h + 1)^2 = 9$

On rearranging we get

$h^2 - 14h + 49 + 16 + h^2 - 8h - 9 = 0$

$2h^2 - 22h + 56 = 0$

$h^2 - 11h + 28 = 0$

$h^2 - 4h - 7h + 28 = 0$

$h (h - 4) - 7 (h - 4) = 0$

$(h - 7) (h - 4) = 0$

h = 7 or 4

Centre = (7, 6) or (4, 3)

$(x - h)^2 + (y - k)^2 = r^2$

Equation, having centre (7, 6)

$(x - 7)^2 + (y - 6)^2 = 3^2$

$x^2 - 14x + 49 + y^2 - 12y + 36 - 9 = 0$

$x^2 - 14x + y^2 - 12y + 76 = 0$

Equation, having centre (4, 3)

$(x - 4)^2 + (y - 3)^2 = 3^2$

$x^2 - 8x + 16 + y^2 - 6y + 9 - 9 = 0$

$x^2 - 8x + y^2 - 6y + 16 = 0$

Hence, the required equation is $x^2 - 14x + y^2 - 12y + 76 = 0$ or $x^2 - 8x + y^2 - 6y + 16 = 0$.

28. Find the equation of each of the following parabolas

(a) Directrix x = 0, focus at (6, 0)

(b) Vertex at (0, 4), focus at (0, 2)

(c) Focus at (–1, –2), directrix x – 2y + 3 = 0

Solution:

(a) The distance of any point on the parabola from its focus and its directrix is same.

Given that, directrix, x = 0 and focus = (6, 0)

If a parabola has a vertical axis, the standard form of the equation of the parabola is $(x - h)^2 = 4p (y - k)$, where $p \neq 0$.
The vertex of this parabola is at (h, k).
The focus is at (h, k + p) & the directrix is the line y = k - p.
As the focus lies on x – axis,
Therefore the equation is $y^2 = 4ax$ or $y^2 = -4ax$
So, for any point P(x, y) on the parabola
Distance of point from directrix = Distance of point from focus
$x^2 = (x – 6)^2 + y^2$
$x^2 = x^2 - 12x + 36 + y^2$
$y^2 - 12x + 36 = 0$
Hence the required equation is $y^2 - 12x + 36 = 0$.

(b) Given Vertex = (0, 4) & Focus = (0, 2)
We know the distance between the vertex and directrix is same as the distance between the vertex and focus.
Directrix is y – 6 = 0
For any point of P(x, y) on the parabola
Distance of P from directrix = Distance of P from focus
Perpendicular Distance (Between a point and line) = $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$
Where the point is (x_1, y_1) and the line is expressed as ax + by + c = 0 i.e..,
x (0) + y – 6 = 0 & point = (x, y)
Distance between the point of intersection & centre
Distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ (between (x, y) & (0, 2))

$$\frac{|0(x)+(1)y-6|}{\sqrt{0^2+1^2}} = \sqrt{((x-0)^2+(y-2)^2)}$$

$$\frac{y-6}{1} = \sqrt{x^2+y^2-4y+4}$$

Squaring both the sides,
$x^2 + y^2 - 4y + 4 = (y – 6)^2$
$x^2 + y^2 - 4y + 4 = y^2 - 12y + 36$
$x^2 + 8y – 32 = 0$
Hence, the required equation is $x^2 + 8y – 32 = 0$.

(c) Focus = (–1, –2), directrix is x – 2y + 3 = 0

For any point (x, y) on parabola, the distance from focus to that point is always equal to the perpendicular distance from that point to the directrix,

Perpendicular Distance (Between a point and line) = $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Where the point is (x_1, y_1) and the line is expressed as ax + by + c = 0 i.e..,

x - 2y + 3 = 0 and point = (x, y)

Distance between the point of intersection & centre

Distance formula = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$\frac{|1(x)+(-2)y+3|}{\sqrt{(-2)^2+1^2}} = \sqrt{((x-(-1))^2+(y-(-2)^2)}$$

$$\frac{|x-2y+3|}{\sqrt{4+1}} = \sqrt{((x+1)^2+(y+2)^2)}$$

$$\frac{x-2y+3}{\sqrt{5}} = \sqrt{((x+1)^2+(y+2)^2)}$$

Squaring both the sides,

$$\left[\frac{x-2y+3}{\sqrt{5}}\right]^2 = \left[\sqrt{((x+1)^2+(y+2)^2)}\right]^2$$

$$\frac{(x-2y+3)^2}{5} = (x+1)^2+(y+2)^2$$

$x^2 + 4y^2 + 9 - 4xy + 6x - 12y = 5 [x^2 + 2x + 1 + y^2 + 4y + 4]$

$x^2 + 4y^2 + 9 - 4xy + 6x - 12y = 5x^2 + 10x + 5y^2 + 20y + 25$

$4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$

Hence the required equation is $4x^2 + y^2 + 4xy + 4x + 32y + 16 = 0$