EXERCISE 20.2 PAGE NO: 20.16

1. Find three numbers in G.P. whose sum is 65 and whose product is 3375.
Solution:
Let the three numbers be a/r, a, ar
So, according to the question
a/r + a + ar = 65 … equation (1)
a/r × a × ar = 3375 … equation (2)
From equation (2) we get,
$a^3 = 3375$
a = 15.

From equation (1) we get,
$(a + ar + ar^2)/r = 65$
$a + ar + ar^2 = 65r$ … equation (3)
Substituting a = 15 in equation (3) we get
$15 + 15r + 15r^2 = 65r$
$15r^2 – 50r + 15 = 0$… equation (4)

Dividing equation (4) by 5 we get
$3r^2 – 10r + 3 = 0$
$3r^2 – 9r – r + 3 = 0$
3r(r – 3) – 1(r – 3) = 0
r = 3 or r = 1/3
Now, the equation will be
15/3, 15, 15×3 or
15/(1/3), 15, 15×1/3
So the terms are 5, 15, 45 or 45, 15, 5
∴ The three numbers are 5, 15, 45.

2. Find three number in G.P. whose sum is 38 and their product is 1728.
Solution:
Let the three numbers be a/r, a, ar
So, according to the question
a/r + a + ar = 38 … equation (1)
a/r × a × ar = 1728 … equation (2)
From equation (2) we get,
$a^3 = 1728$
a = 12.

From equation (1) we get,
$(a + ar + ar^2)/r = 38$
$a + ar + ar^2 = 38r$ … equation (3)

Substituting a = 12 in equation (3) we get
$12 + 12r + 12r^2 = 38r$
$12r^2 – 26r + 12 = 0$… equation (4)
Dividing equation (4) by 2 we get
$6r^2 – 13r + 6 = 0$
$6r^2 – 9r – 4r + 6 = 0$
$3r(3r – 3) – 2(3r – 3) = 0$
r = 3/2 or r = 2/3
Now the equation will be
12/(3/2) = 8 or
12/(2/3) = 18
So the terms are 8, 12, 18
∴ The three numbers are 8, 12, 18

3. The sum of first three terms of a G.P. is 13/12, and their product is – 1. Find the G.P.
Solution:
Let the three numbers be a/r, a, ar
So, according to the question
$a/r + a + ar = 13/12$ … equation (1)
$a/r \times a \times ar = -1$ … equation (2)
From equation (2) we get,
$a^3 = -1$
a = -1

From equation (1) we get,
$(a + ar + ar^2)/r = 13/12$
$12a + 12ar + 12ar^2 = 13r$ … equation (3)
Substituting a = – 1 in equation (3) we get
$12(– 1) + 12(– 1)r + 12(– 1)r^2 = 13r$
$12r^2 + 25r + 12 = 0$
$12r^2 + 16r + 9r + 12 = 0$… equation (4)
$4r (3r + 4) + 3(3r + 4) = 0$
r = -3/4 or r = -4/3

Now the equation will be
-1/(-3/4), -1, -1×-3/4 or -1/(-4/3), -1, -1×-4/3
4/3, -1, ¾ or ¾, -1, 4/3
∴ The three numbers are 4/3, -1, ¾ or ¾, -1, 4/3

4. The product of three numbers in G.P. is 125 and the sum of their products taken in pairs is 87 ½ . Find them.
Solution:
Let the three numbers be a/r, a, ar
So, according to the question
a/r × a × ar = 125 … equation (1)
From equation (1) we get,
$a^3 = 125$
a = 5
a/r × a + a × ar + ar × a/r = 87 ½
a/r × a + a × ar + ar × a/r = 195/2
$a^2/r + a^2r + a^2 = 195/2$
$a^2 (1/r + r + 1) = 195/2$
Substituting a = 5 in above equation we get,
$5^2 [(1+r^2+r)/r] = 195/2$
$1+r^2+r = (195r/2×25)$
$2(1+r^2+r) = 39r/5$
$10 + 10r^2 + 10r = 39r$
$10r^2 – 29r + 10 = 0$
$10r^2 – 25r – 4r + 10 = 0$
5r(2r-5) – 2(2r-5) = 0
r = 5/2, 2/5
So G.P is 10, 5, 5/2 or 5/2, 5, 10
∴ The three numbers are 10, 5, 5/2 or 5/2, 5, 10

5. The sum of the first three terms of a G.P. is 39/10, and their product is 1. Find the common ratio and the terms.
Solution:
Let the three numbers be a/r, a, ar
So, according to the question
a/r + a + ar = 39/10 … equation (1)
a/r × a × ar = 1 … equation (2)
From equation (2) we get,
$a^3 = 1$

a = 1

From equation (1) we get,
$(a + ar + ar^2)/r = 39/10$
$10a + 10ar + 10ar^2 = 39r$ … equation (3)
Substituting a = 1 in 3 we get
$10(1) + 10(1)r + 10(1)r^2 = 39r$
$10r^2 – 29r + 10 = 0$
$10r^2 – 25r – 4r + 10 = 0$… equation (4)
$5r(2r – 5) – 2(2r – 5) = 0$
r = 2/5 or 5/2
so now the equation will be,
1/(2/5), 1, 1×2/5 or 1/(5/2), 1, 1×5/2
5/2, 1, 2/5 or 2/5, 1, 5/2
∴ The three numbers are 2/5, 1, 5/2