

## EXERCISE 22.1

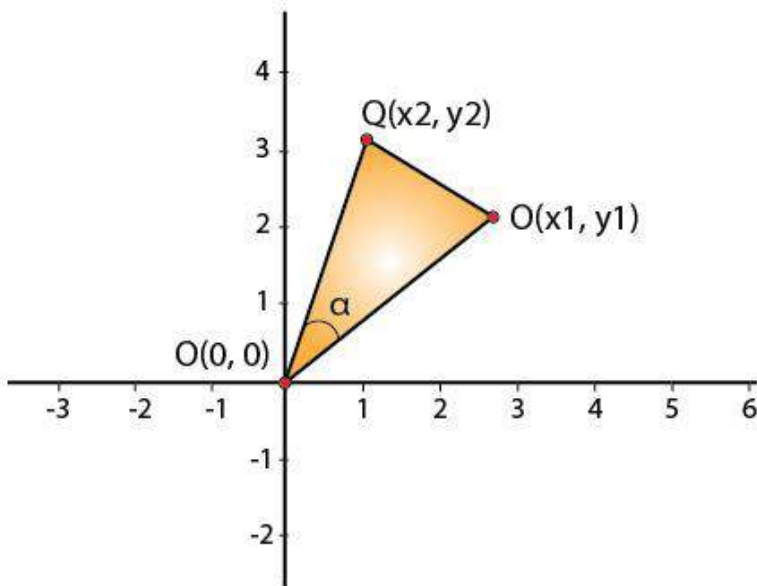
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1. If the line segment joining the points  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  subtends an angle  $\alpha$  at the origin  $O$ , prove that :  $OP \cdot OQ \cos \alpha = x_1 x_2 + y_1 y_2$ .

**Solution:**

Given,

Two points  $P$  and  $Q$  subtends an angle  $\alpha$  at the origin as shown in figure:



From figure we can see that points  $O$ ,  $P$  and  $Q$  forms a triangle. Clearly in  $\triangle OPQ$  we have:

$$\cos \alpha = \frac{OP^2 + OQ^2 - PQ^2}{2OP.OQ} \quad \{\text{from cosine formula}\}$$

$$2 OP.OQ \cos \alpha = OP^2 + OQ^2 - PQ^2 \dots \text{equation (1)}$$

We know that the, coordinates of O are (0, 0)  $\Rightarrow x_2 = 0$  and  $y_2 = 0$

Coordinates of P are  $(x_1, y_1) \Rightarrow x_1 = x_1$  and  $y_1 = y_1$

By using distance formula we have:

$$\begin{aligned} OP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} \\ &= \sqrt{x_1^2 + y_1^2} \end{aligned}$$

$$\begin{aligned} \text{Similarly, } OQ &= \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2} \\ &= \sqrt{x_2^2 + y_2^2} \end{aligned}$$

$$\text{And, } PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore OP^2 + OQ^2 - PQ^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2 - \{(x_2 - x_1)^2 + (y_2 - y_1)^2\}$$

By using  $(a-b)^2 = a^2 + b^2 - 2ab$

$$\therefore OP^2 + OQ^2 - PQ^2 = 2x_1 x_2 + 2y_1 y_2 \dots \text{Equation (2)}$$

So now from equation (1) and (2) we have:

$$2OP.OQ \cos \alpha = 2x_1 x_2 + 2y_1 y_2$$

$$OP.OQ \cos \alpha = x_1 x_2 + y_1 y_2$$

Hence Proved.

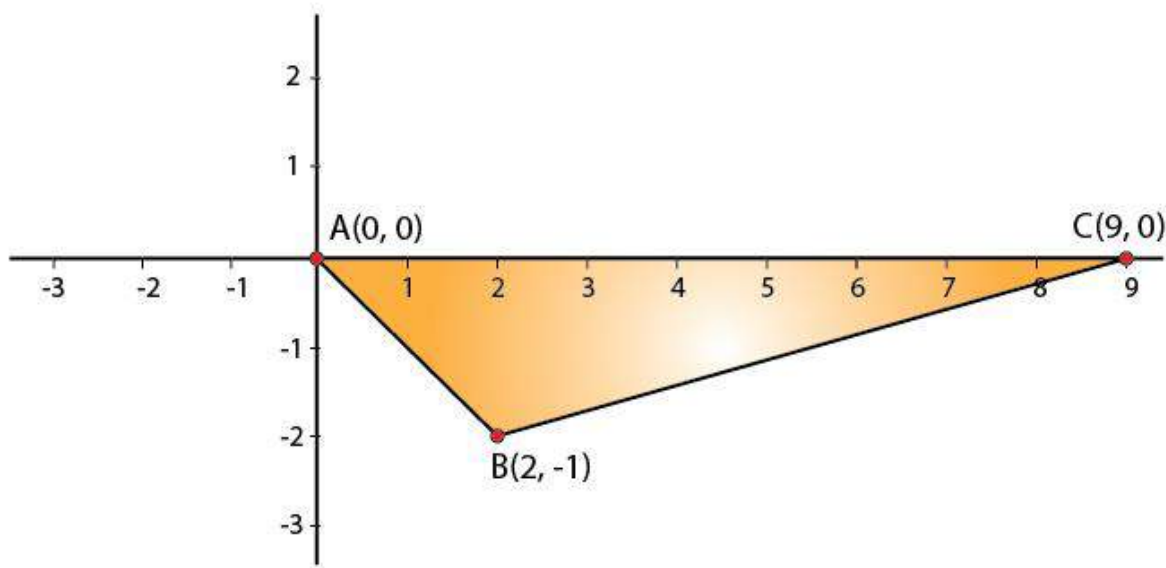
**2. The vertices of a triangle ABC are A(0, 0), B (2, -1) and C (9, 0). Find cos B.**

**Solution:**

Given:

The coordinates of triangle.

From the figure,



By using cosine formula,

In  $\triangle ABC$ , we have:

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2AB \cdot BC}$$

Now by using distance formula we have:

$$AB = \sqrt{(2 - 0)^2 + (-1 - 0)^2} = \sqrt{5}$$

$$BC = \sqrt{(9 - 2)^2 + (0 - (-1))^2} = \sqrt{7^2 + 1^2} = \sqrt{50}$$

$$\text{And, } AC = \sqrt{(9 - 0)^2 + (0 - 0)^2} = 9$$

Now substitute the obtained values in the cosine formula, we get

$$\therefore \cos B = \frac{(\sqrt{5})^2 + (\sqrt{50})^2 - 9^2}{2\sqrt{5}\sqrt{50}} = \frac{55 - 81}{2\sqrt{5}\sqrt{2 \times 25}} = \frac{-26}{10\sqrt{10}} = \frac{-13}{5\sqrt{10}}$$

3. Four points A (6, 3), B (-3, 5), C (4, -2) and D (x, 3x) are given in such a way

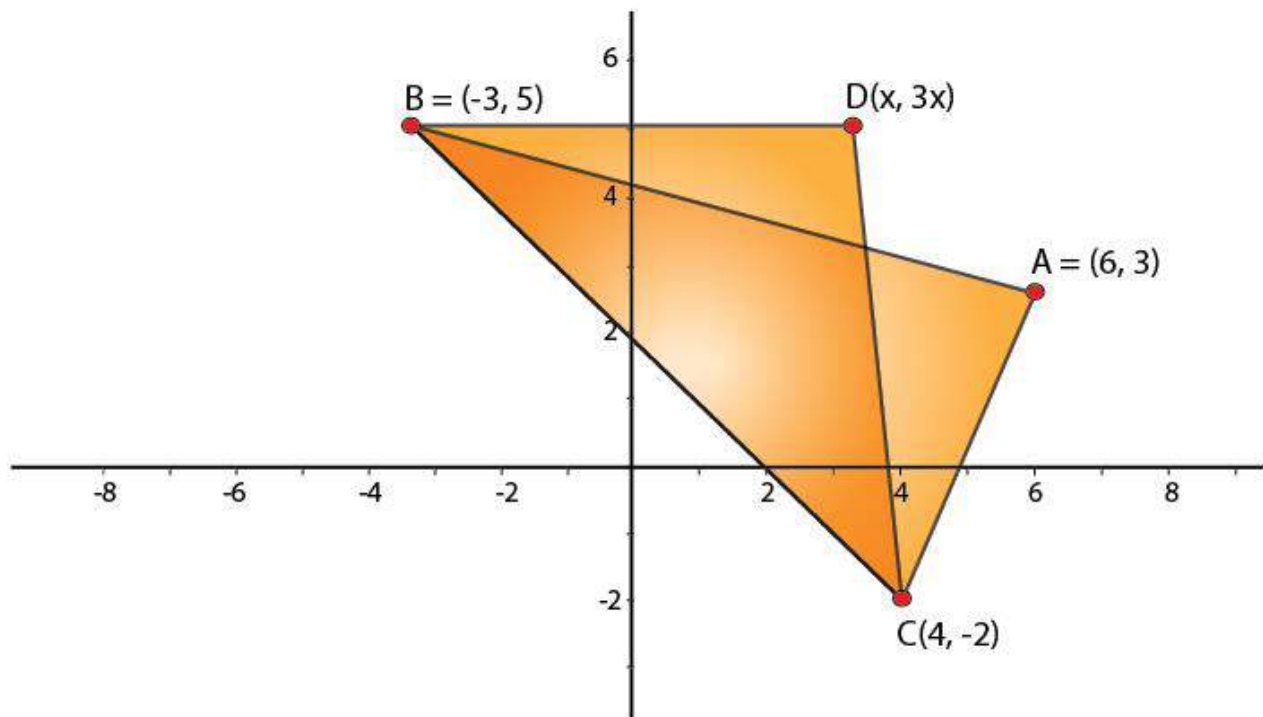
that  $\frac{\triangle DBC}{\triangle ABC} = \frac{1}{2}$ , find x.

**Solution:**

Given:

The coordinates of triangle are shown in the below figure.

$$\text{Also, } \frac{\triangle DBC}{\triangle ABC} = \frac{1}{2}$$



Now let us consider Area of a  $\Delta PQR$

Where,  $P(x_1, y_1)$ ,  $Q(x_2, y_2)$  and  $R(x_3, y_3)$  be the 3 vertices of  $\Delta PQR$ .

So, Area of  $(\Delta PQR) = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\begin{aligned} \text{Area of } (\Delta DBC) &= \frac{1}{2} [x(5 - (-2)) + (-3)(-2 - 3x) + 4(3x - 5)] \\ &= \frac{1}{2} [7x + 6 + 9x + 12x - 20] = 14x - 7 \end{aligned}$$

$$\begin{aligned} \text{Similarly, area of } (\Delta ABC) &= \frac{1}{2} [6(5 - (-2)) + (-3)(-2 - 3) + 4(3 - 5)] \\ &= \frac{1}{2} [42 + 15 - 8] = \frac{49}{2} = 24.5 \end{aligned}$$

$$\therefore \frac{\Delta DBC}{\Delta ABC} = \frac{1}{2} = \frac{14x-7}{24.5}$$

$$24.5 = 28x - 14$$

$$28x = 38.5$$

$$x = 38.5/28$$

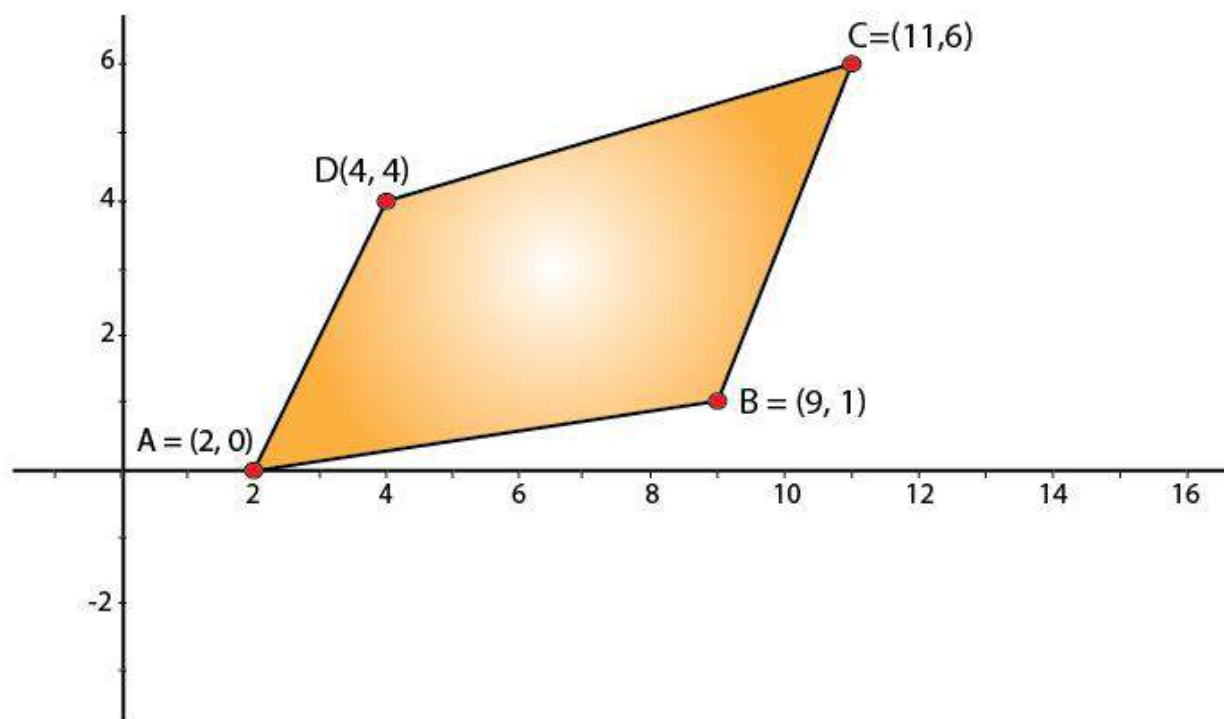
$$= 1.375$$

**4. The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.**

**Solution:**

Given:

The coordinates of 4 points that form a quadrilateral is shown in the below figure



Now by using distance formula, we have:

$$AB = \sqrt{(9 - 2)^2 + (1 - 0)^2} = \sqrt{7^2 + 1} = \sqrt{50}$$

$$BC = \sqrt{(11 - 9)^2 + (6 - 1)^2} = \sqrt{2^2 + 5^2} = \sqrt{29}$$

It is clear that,  $AB \neq BC$  [quad ABCD does not have all 4 sides equal.]

$\therefore$  ABCD is not a Rhombus