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1. Find the equation of the parabola whose:
(i) focus is (3, 0) and the directrix is 3x + 4y = 1
(ii) focus is (1, 1) and the directrix is x + y + 1 = 0
(iii) focus is (0, 0) and the directrix is 2x - y – 1 = 0
(iv) focus is (2, 3) and the directrix is x - 4y + 1 = 0
Solution:
(i) focus is (3, 0) and the directrix is 3x + 4y = 1
Given:
The focus S(3, 0) and directrix(M) 3x + 4y - 1 = 0.
Let us assume P(x, y) be any point on the parabola.
The distance between two points (x_1, y_1) and (x_2, y_2) is given as:
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
And the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
So by equating both, we get
$$(x-3)^2 + (y-0)^2 = \left(\frac{|3x+4y-1|}{\sqrt{3^2+4^2}}\right)^2$$
$$x^2 - 6x + 9 + y^2 = \frac{(|3x+4y-1|)^2}{3^2+4^2}$$
$$x^2 + y^2 - 6x + 9 = \frac{(9x^2 + 16y^2 + 1 - 6x - 8y + 24xy)}{9+16}$$
Upon cross multiplication, we get
$25x^2 + 25y^2 - 150x + 225 = 9x^2 + 16y^2 - 6x - 8y + 24xy + 1$
$16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
∴The equation of the parabola is $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$

(ii) focus is (1, 1) and the directrix is x + y + 1 = 0
Given:
The focus S(1, 1) and directrix(M) x + y + 1 = 0.
Let us assume P(x, y) be any point on the parabola.
The distance between two points (x_1, y_1) and (x_2, y_2) is given as:
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
And the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
So by equating both, we get

$$(x-1)^2 + (y-1)^2 = \left(\frac{|x+y+1|}{\sqrt{1^2+1^2}}\right)^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = \frac{(|x+y+1|)^2}{1+1}$$

$$x^2 + y^2 - 2x - 2y + 2 = \frac{(x^2+y^2+1+2x+2y+2xy)}{2}$$

Upon cross multiplication, we get

$2x^2 + 2y^2 - 4x - 4y + 4 = x^2 + y^2 + 2x + 2y + 2xy + 1$

$x^2 + y^2 + 2xy - 6x - 6y + 3 = 0$

∴ The equation of the parabola is $x^2 + y^2 + 2xy - 6x - 6y + 3 = 0$

(iii) focus is (0, 0) and the directrix is $2x - y – 1 = 0$

Given:

The focus S(0, 0) and directrix(M) $2x - y - 1 = 0$.

Let us assume P(x, y) be any point on the parabola.

The distance between two points (x_1, y_1) and (x_2, y_2) is given as:

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

And the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

So by equating both, we get

$$(x-0)^2 + (y-0)^2 = \left(\frac{|2x-y-1|}{\sqrt{2^2+(-1)^2}}\right)^2$$

$$x^2 + y^2 = \frac{(|2x-y-1|)^2}{4+1}$$

$$x^2 + y^2 = \frac{(4x^2+y^2+1-4x+2y-4xy)}{5}$$

Upon cross multiplication, we get

$5x^2 + 5y^2 = 4x^2 + y^2 - 4x + 2y - 4xy + 1$

$x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$

∴ The equation of the parabola is $x^2 + 4y^2 + 4xy + 4x - 2y - 1 = 0$

(iv) focus is (2, 3) and the directrix is $x - 4y + 1 = 0$

Given:

The focus S(2, 3) and directrix(M) $x - 4y + 3 = 0$.

Let us assume P(x, y) be any point on the parabola.

The distance between two points (x_1, y_1) and (x_2, y_2) is given as:

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

And the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$

is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So by equating both, we get

$$(x-2)^2 + (y-3)^2 = \left(\frac{|x-4y+3|}{\sqrt{1^2+(-4)^2}}\right)^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{(|x-4y+3|)^2}{1+16}$$

$$x^2 + y^2 - 4x - 6y + 13 = \frac{(x^2 + 16y^2 + 9 + 6x - 24y - 8xy)}{17}$$

Upon cross multiplication, we get

$17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 6x - 24y - 8xy + 9$

$16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

∴ The equation of the parabola is $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

2. Find the equation of the parabola whose focus is the point (2, 3) and directrix is the line x – 4y + 3 = 0. Also, find the length of its latus - rectum.

Solution:

Given:

The focus S(2, 3) and directrix(M) $x - 4y + 3 = 0$.

Let us assume P(x, y) be any point on the parabola.

The distance between two points (x_1, y_1) and (x_2, y_2) is given as:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

And the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$

is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So by equating both, we get

$$(x-2)^2 + (y-3)^2 = \left(\frac{|x-4y+3|}{\sqrt{1^2+(-4)^2}}\right)^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{(|x-4y+3|)^2}{1+16}$$

$$x^2 + y^2 - 4x - 6y + 13 = \frac{(x^2 + 16y^2 + 9 + 6x - 24y - 8xy)}{17}$$

Upon cross multiplication, we get

$17x^2 + 17y^2 - 68x - 102y + 221 = x^2 + 16y^2 + 6x - 24y - 8xy + 9$

$16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$

∴ The equation of the parabola is $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$.

Now, let us find the length of the latus rectum,

We know that the length of the latus rectum is twice the perpendicular distance from the

focus to the directrix.
So by using the formula,

$$L = 2\frac{|2-4(3)+3|}{\sqrt{1^2+(-4)^2}}$$
$$= 2\frac{|-7|}{\sqrt{1+16}}$$
$$= \frac{14}{\sqrt{17}}$$

∴ The length of the latus rectum is $14/\sqrt{17}$

3. Find the equation of the parabola, if
(i) the focus is at (-6, 6) and the vertex is at (-2, 2)
(ii) the focus is at (0, -3) and the vertex is at (0, 0)
(iii) the focus is at (0, -3) and the vertex is at (-1, -3)
(iv) the focus is at (a, 0) and the vertex is at (a′, 0)
(v) the focus is at (0, 0) and the vertex is at the intersection of the lines x + y = 1 and x – y = 3.

Solution:

(i) the focus is at (-6, 6) and the vertex is at (-2, 2)
Given:
Focus = (-6, 6)
Vertex = (-2, 2)
Let us find the slope of the axis $(m_1) = (6-2)/(-6-(-2))$
$= 4/-4$
$= -1$
Let us assume m_2 be the slope of the directrix.
$m_1m_2 = -1$
$-1m_2 = -1$
$m_2 = 1$
Now, let us find the point on directrix.
(-2, 2) = [(x-6/2), (y+6)/2]
By equating we get,
(x-6/2) = -2 and (y+6)/2 = 2
x-6 = -4 and y+6 = 4
x = -4+6 and y = 4-6
x = 2 and y = -2
So the point of directrix is (2, -2).
We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$
y - (-2) = 1(x - 2)

y + 2 = x – 2

x - y - 4 = 0

Let us assume P(x, y) be any point on the parabola.

The distance between two points (x_1, y_1) and (x_2, y_2) is given as:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

And the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So by equating both, we get

$$(x - (-6))^2 + (y - 6)^2 = \left(\frac{|x-y-4|}{\sqrt{1^2 + (-1)^2}}\right)^2$$

$$x^2 + 12x + 36 + y^2 - 12y + 36 = \frac{(|x-y-4|)^2}{1+1}$$

$$x^2 + y^2 + 12x - 12y + 72 = \frac{(x^2 + y^2 + 16 - 8x + 8y - 2xy)}{2}$$

Now by cross multiplying, we get

$2x^2 + 2y^2 + 24x - 24y + 144 = x^2 + y^2 - 8x + 8y - 2xy + 16$

$x^2 + y^2 + 2xy + 32x - 32y + 128 = 0$

∴ The equation of the parabola is $x^2 + y^2 + 2xy + 32x - 32y + 128 = 0$

(ii) the focus is at (0, -3) and the vertex is at (0, 0)

Given:

Focus = (0, -3)

Vertex = (0, 0)

Let us find the slope of the axis (m_1) = (-3-0)/(0-0)

= -3/0

= ∞

Since the axis is parallel to the x-axis, the slope of the directrix is equal to the slope of x-axis = 0

So, $m_2 = 0$

Now, let us find the point on directrix.

(0, 0) = [(x-0/2), (y-3)/2]

By equating we get,

(x/2) = 0 and (y-3)/2 = 0

x = 0 and y - 3 = 0

x = 0 and y = 3

So the point on directrix is (0, 3).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$

y - 3 = 0(x - 0)
y - 3 = 0
Now, let us assume P(x, y) be any point on the parabola.
The distance between two points (x_1, y_1) and (x_2, y_2) is given as:
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
And the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
So by equating both, we get
$$(x - 0)^2 + (y - (-3))^2 = \left(\frac{|y-3|}{\sqrt{1^2}}\right)^2$$
$$x^2 + y^2 + 6y + 9 = \frac{(|y-3|)^2}{1}$$
Now by cross multiplying, we get
$x^2 + y^2 + 6y + 9 = y^2 – 6y + 9$
$x^2 + 12y = 0$
∴ The equation of the parabola is $x^2 + 12y = 0$

(iii) the focus is at (0, -3) and the vertex is at (-1, -3)
Given:
Focus = (0, -3)
Vertex = (-1, -3)
Let us find the slope of the axis (m_1) = (-3-(-3))/(0-(-1))
= 0/1
= 0
We know, the products of the slopes of the perpendicular lines is - 1 for non - vertical lines.
Here the slope of the axis is equal to the slope of the x - axis. So, the slope of directrix is equal to the slope of y - axis i.e., ∞.
So, $m_2 = \infty$
Now let us find the point on directrix.
(-1, -3) = [(x+0/2), (y-3)/2]
By equating we get,
(x/2) = -1 and (y-3)/2 = -3
x = -2 and y – 3 = -6
x = -2 and y = -6+3
x = -2 and y = -3
So, the point on directrix is (-2, -3)
We know that the equation of the lines passing through (x_1, y_1) and having slope m is y -

$y_1 = m(x - x_1)$
$y - (-3) = \infty(x - (-2))$
$(y+3)/\infty = x + 2$
$x + 2 = 0$
Now, let us assume P(x, y) be any point on the parabola.
The distance between two points (x_1, y_1) and (x_2, y_2) is given as:
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
And the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0
is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
So by equating both, we get
$$(x - 0)^2 + (y - (-3))^2 = \left(\frac{|x+2|}{\sqrt{1^2}}\right)^2$$
$$x^2 + y^2 + 6y + 9 = \frac{(|x+2|)^2}{1}$$
By cross multiplying, we get
$x^2 + y^2 + 6y + 9 = x^2 + 4x + 4$
$y^2 - 4x + 6y + 5 = 0$
∴ The equation of the parabola is $y^2 - 4x + 6y + 5 = 0$

(iv) the focus is at (a, 0) and the vertex is at (a′, 0)
Given:
Focus = (a, 0)
Vertex = (a′, 0)
Let us find the slope of the axis (m_1) = (0-0)/(a′, a)
= 0/(a′, a)
= 0
We know, the products of the slopes of the perpendicular lines is - 1 for non - vertical lines.
Here the slope of the axis is equal to the slope of the x - axis. So, the slope of directrix is equal to the slope of y - axis i.e., ∞.
So, $m_2 = \infty$
Now let us find the point on directrix.
(a′, 0)= [(x+a/2), (y+0)/2]
By equating we get,
(x+a/2) = a′ and (y)/2 = 0
x + a = 2a′ and y = 0
x = (2a′ - a) and y = 0
So the point on directrix is (2a′ - a, 0).

We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$

$y - (0) = \infty(x - (2a' - a))$

$y/\infty = x + a – 2a'$

$x + a – 2a' = 0$

Now, let us assume P(x, y) be any point on the parabola.

The distance between two points (x_1, y_1) and (x_2, y_2) is given as:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

And the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

So by equating both, we get

$$(x - a)^2 + (y - 0)^2 = \left(\frac{|x + a - 2a'|}{\sqrt{1^2}}\right)^2$$

$$x^2 - 2ax + a^2 + y^2 = \frac{(|x + a - 2a'|)^2}{1}$$

By cross multiplying we get,

$x^2 + y^2 - 2ax + a^2 = x^2 + a^2 + 4(a')^2 + 2ax - 4aa' - 4a'x$

$y^2 - (4a - 4a')x + a^2 - 4(a')^2 + 4aa' = 0$

∴ The equation of the parabola is $y^2 - (4a - 4a')x + a^2 - 4(a')^2 + 4aa' = 0$

(v) the focus is at (0, 0) and the vertex is at the intersection of the lines $x + y = 1$ and $x – y = 3$.

Given:

Focus = (0, 0)

Vertex = intersection of the lines $x + y = 1$ and $x – y = 3$.

So the intersecting point of above lines is (2, -1)

Vertex = (2, -1)

Slope of axis $(m_1) = (-1-0)/(2-0)$

$= -1/2$

We know that the products of the slopes of the perpendicular lines is - 1.

Let us assume m_2 be the slope of the directrix.

$m_1.m_2 = -1$

$-1/2 \,.\, m_2 = -1$

So $m_2 = 2$

Now let us find the point on directrix.

$(2, -1) = [(x+0)/2, (y+0)/2]$

$(x)/2 = 2$ and $y/2 = -1$

$x = 4$ and $y = -2$

The point on directrix is (4, - 2).
We know that the equation of the lines passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$
$y - (-2) = 2(x - 4)$
$y + 2 = 2x - 8$
$2x - y - 10 = 0$
Now, let us assume P(x, y) be any point on the parabola.
The distance between two points (x_1, y_1) and (x_2, y_2) is given as:
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
And the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$
So by equating both, we get
$$(x - 0)^2 + (y - 0)^2 = \left(\frac{|2x - y - 10|}{\sqrt{2^2 + (-1)^2}}\right)^2$$
$$x^2 + y^2 = \frac{(|2x - y - 10|)^2}{4 + 1}$$
$$x^2 + y^2 = \frac{(4x^2 + y^2 + 100 - 40x + 20y - 4xy)}{5}$$
By cross multiplying, we get
$5x^2 + 5y^2 = 4x^2 + y^2 - 40x + 20y - 4xy + 100$
$x^2 + 4y^2 + 4xy + 40x - 20y - 100 = 0$
∴ The equation of the parabola is $x^2 + 4y^2 + 4xy + 40x - 20y - 100 = 0$

4. Find the vertex, focus, axis, directrix and lotus - rectum of the following parabolas
(i) $y^2 = 8x$
(ii) $4x^2 + y = 0$
(iii) $y^2 – 4y – 3x + 1 = 0$
(iv) $y^2 – 4y + 4x = 0$
(v) $y^2 + 4x + 4y – 3 = 0$
Solution:
(i) $y^2 = 8x$
Given:
Parabola = $y^2 = 8x$
Now by comparing with the actual parabola $y^2 = 4ax$
Then,
$4a = 8$
$a = 8/4 = 2$
So, the vertex is (0, 0)
The focus is (a, 0) = (2, 0)

The equation of the axis is y = 0.
The equation of the directrix is x = - a i.e., x = - 2
The length of the latus rectum is 4a = 8.

(ii) $4x^2 + y = 0$
Given:
Parabola => $4x^2 + y = 0$
Now by comparing with the actual parabola $y^2 = 4ax$
Then,
4a = ¼
a = 1/(4 × 4)
= 1/16
So, the vertex is (0, 0)
The focus is = (0, -1/16)
The equation of the axis is x = 0.
The equation of the directrix is y = 1/16
The length of the latus rectum is 4a = ¼

(iii) $y^2 – 4y – 3x + 1 = 0$
Given:
Parabola $y^2 – 4y – 3x + 1 = 0$
$y^2 - 4y = 3x - 1$
$y^2 - 4y + 4 = 3x + 3$
$(y - 2)^2 = 3(x + 1)$
Now by comparing with the actual parabola $y^2 = 4ax$
Then,
4b = 3
b = ¾
So, the vertex is (-1, 2)
The focus is = (3/4 – 1, 2) = (-1/4, 2)
The equation of the axis is y - 2 = 0.
The equation of the directrix is (x - c) = -b
(x – (-1)) = -3/4
x = -1 – ¾
= -7/4
The length of the latus rectum is 4b = 3

(iv) $y^2 – 4y + 4x = 0$
Given:

Parabola $y^2 – 4y + 4x = 0$
$y^2 - 4y = - 4x$
$y^2 - 4y + 4 = - 4x + 4$
$(y - 2)^2 = - 4(x - 1)$
Now by comparing with the actual parabola $y^2 = 4ax \Rightarrow (y - a)^2 = - 4b(x - c)$
Then,
$4b = 4$
$b = 1$
So, the vertex is $(c, a) = (1, 2)$
The focus is $(b + c, a) = (1-1, 2) = (0, 2)$
The equation of the axis is $y - a = 0$ i.e., $y - 2 = 0$
The equation of the directrix is $x - c = b$
$x - 1 = 1$
$x = 1 + 1$
$= 2$
Length of latus rectum is $4b = 4$

(v) $y^2 + 4x + 4y – 3 = 0$
Given:
The parabola $y^2 + 4x + 4y – 3 = 0$
$y^2 + 4y = - 4x + 3$
$y^2 + 4y + 4 = - 4x + 7$
$(y + 2)^2 = - 4(x – 7/4)$
Now by comparing with the actual parabola $y^2 = 4ax \Rightarrow (y - a)^2 = - 4b(x - c)$
Then,
$4b = 4$
$b = 4/4 = 1$
So, The vertex is $(c, a) = (7/4, -2)$
The focus is $(- b + c, a) = (-1 + 7/4, -2) = (3/4, -2)$
The equation of the axis is $y - a = 0$ i.e., $y + 2 = 0$
The equation of the directrix is $x - c = b$
$x – 7/4 = 1$
$x = 1 + 7/4$
$= 11/4$
Length of latus rectum is $4b = 4$.

5. For the parabola, $y^2 = 4px$ find the extremities of a double ordinate of length 8p. Prove that the lines from the vertex to its extremities are at right angles.
Solution:

Given:
The parabola, $y^2 = 4px$ and a double ordinate of length 8p.
Let AB be the double ordinate of length 8p for the parabola $y^2 = 4px$.
Now, let us compare to the actual parabola, $y^2 = 4ax$
Then,
axis is $y = 0$
vertex is O(0, 0).

We know that double ordinate is perpendicular to the axis.
So, let us assume that the point at which the double ordinate meets the axis is $(x_1, 0)$.
Then the equation of the double ordinate is $y = x_1$. It meets the parabola at the points $(x_1, 4p)$ and $(x_1, - 4p)$ as its length is 8p.
Now, let us find the value of x_1 by substituting in the parabola.
$(4p)^2 = 4p(x_1)$
$x_1 = 4p$.

The extremities of the double ordinate are A(4p, 4p) and B(4p, - 4p).
Assume the slopes of OA and OB be m_1 and m_2. Let us find their values.
$m_1 = (4p - 0)/(4p - 0)$
$= 4p/4p$
$= 1$

$m_2 = (4p - 0)/(-4p - 0)$
$= 4p/-4p$
$= -1$
So, $m_1.m_2 = 1. - 1$
$= - 1$
The product of slopes is -1. So, the lines OA and OB are perpendicular to each other.
Hence the extremities of double ordinate make right angle with the vertex.

6. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus - rectum.
Solution:
Given:
The parabola, $x^2 = 12y$
Now, let us compare to the actual parabola, $y^2 = 4ax$
Then,
Vertex is O(0, 0)
Ends of latus rectum is (2b, b), (-2b, b)

4b = 12
b = 12/4
= 3

Ends of latus rectum = (2(3), 3), (- 2(3), 3)
Ends of latus rectum is A(6, 3), B(- 6, 3)
We know that area of the triangle with the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

$$\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 0-6 & 0-(-6) \\ 0-3 & 0-3 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -6 & 6 \\ -3 & -3 \end{vmatrix}$$

$$= \frac{1}{2}|(-3 \times -6) - (-3 \times 6)|$$

$$= \frac{1}{2}|+18+18|$$

$$= \frac{1}{2}|36|$$

$$= 18$$

∴The area of the triangle is 18 sq.units.

7. Find the coordinates of the point of intersection of the axis and the directrix of the parabola whose focus is (3, 3) and directrix is 3x – 4y = 2. Find also the length of the latus - rectum.

Solution:

Given:
Focus = (3, 3)
Directrix = 3x – 4y = 2
Firstly let us find the slope of the directrix.
The slope of the line ax + by + c = 0 is –a/b
So, m_1 = -3/-4
= ¾
Let us assume the slope of axis is m_2.
$m_1.m_2 = -1$
¾ . m_2 = -1
m_2 = -4/3
We know that the equation of the line passing through the point (x_1, y_1) and having slope m is $(y - y_1) = m(x - x_1)$
y – 3 = -4/3 (x - 3)
3(y - 3) = - 4(x - 3)

3y - 9 = - 4x + 12

4x + 3y = 21

On solving the lines, the intersection point is (18/5, 11/5)

By using the formula to find the length is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$L = 2\frac{|3(3)-4(3)-2|}{\sqrt{3^2+(-4)^2}}$$

$$= 2\frac{|9-12-2|}{\sqrt{9+16}}$$

$$= \frac{2|-5|}{\sqrt{25}}$$

$$= \frac{2\times5}{5}$$

$$= 2$$

∴The length of the latus rectum is 2.

8. At what point of the parabola $x^2 = 9y$ is the abscissa three times that of ordinate?

Solution:

Given:

The parabola, $x^2 = 9y$

Let us assume the point be $(3y_1, y_1)$.

Now by substituting the point in the parabola we get,

$(3y_1)^2 = 9(y_1)$

$9y_1^2 = 9y_1$

$y_1^2 - y_1 = 0$

$y_1(y_1 - 1) = 0$

$y_1 = 0$ or $y_1 - 1 = 0$

$y_1 = 0$ or $y_1 = 1$

The points is B (3(1), 1) => (3, 1)

∴The point is (3, 1).