Exercise Solutions

Question 1: Find the dimensions of
(i) Linear Momentum
(ii) Frequency and
(iii) Pressure

Solution:
(i)
Linear momentum can be written as “mv”

Dimensions of Linear momentum: $mv = [MLT^{-1}]$

(ii) Frequency can be written as “1/T”, where T is time.

Dimensions of Frequency: $1/T = [M^0L^0T^{-1}]$

(iii) All units of pressure represent some ratio of force to area

So, dimensions of pressure: $\text{Force/Area} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

Question 2: Find the dimensions of
(a) Angular speed, $\omega$
(b) Angular acceleration, $\alpha$
(c) Torque, $\tau$ and
(d) Moment of Inertia, $I$

Some of the equations involving these quantities are.

Solution:

(a) Dimensions of Angular speed, $\omega$

\[
\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}, \quad \alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}, \quad \Gamma = F.r \quad \text{and} \quad I = mr^2.
\]
We know, $\omega = \theta/t$
Dimensions are $[M^0L^0T^{-1}]$

(b) Dimensions of Angular acceleration, $\alpha$
We know, $\alpha = \omega/t$
So required dimensions are $\frac{[M^0L^0T^{-1}]}{[T]} = [M^0L^0T^{-2}]$
[using (a) result])

(c) Dimensions of Torque, $\tau$ and
We know, $\tau = Fr$
So, $\tau = [MLT^{-2}][L] = [ML^2T^{-2}]$

(d) Dimensions of Moment of Inertia, $I$
Here $I = Mr^2 = [M][L^2] = [ML^2T^0]$

Question 3: Find the dimensions of
(a) Electric Field $E$
(b) Magnetic field $B$ and
(c) Magnetic permeability $\mu_0$
The relevant equations are

\[ F = qE, \quad F = qvB, \quad \text{and} \quad B = \frac{\mu_0 I}{2 \pi a}; \]

where $F$ is force, $q$ is charge, $v$ is speed, $I$ is current, and $a$ is distance.

Solution:
(a) Dimensions of Electric Field $E = F/q = \frac{[MLT^{-2}]}{[IT]} = [MLT^{-3}I^{-1}]$

(b) Dimensions of Magnetic field $B = F/qv = \frac{[MLT^{-2}]}{[IT][LT^{-1}]} = [MT^{-2}I^{-1}]$
(c) Dimensions of Magnetic permeability \( \mu_0 = \frac{(Bx \pi a)/I}{[I]} = \frac{[MT^{-2}I^{-1}][L]}{[I]} = [ML^{-2}I^{-2}] 

**Question 4:** Find the dimensions of
(a) Electric dipole moment \( p \) and
(b) Magnetic dipole moment \( M \).
The defining equations are \( p = qd \) and \( M = IA \)
Where \( d \) is distance, \( A \) is area, \( q \) is charge and \( I \) is current.

**Solution:**

a) Dimensions of Electric dipole moment \( p = qI = [IT][L] = [LTI] 
(b) Dimensions of Magnetic dipole moment \( M = IA = [I][L^2][L^2] 

**Question 5:** Find the dimensions of Planck's constant \( h \) from the equation \( E = hv \) where \( E \) is the energy and \( v \) is the frequency.

**Solution:**

Planck's constant can be written as, \( h = E/v \)
Where \( E \) = energy and \( v \) = frequency

\( => h = \frac{E}{v} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}] \)

**Question 6:** Find the dimensions of
(a) the specific heat capacity \( c \),
(b) the coefficient of linear expansion \( \alpha \) and
(c) the gas constant \( R \).
Some of the equations involving these quantities are

\( Q = mc(T_2 - T_1), \quad l_t = l_0[1 + \alpha(T_2 - T_1)] \) and \( PV = nRT \).

**Solution:**

(a) Dimensions of specific heat capacity,

\( c = \frac{Q}{m\Delta T} = \frac{[ML^2T^{-2}]}{[M][K]} = [L^2T^{-2}K^{-1}] \)
(b) Dimensions of coefficient of linear expansion,

\[ \alpha = \frac{L_1 - L_2}{L_0 \Delta T} = \frac{[L]}{[L][R]} = K^{-1} \]

(c) Dimensions of gas constant,

\[ R = \frac{PV}{nT} = \frac{[ML^{-1}T^{-2}][L^3]}{[mol][K]} = [ML^2 T^{-2}K^{-1} (mol)^{-1}] \]

**Question 7:** Taking force, length and time to be the fundamental quantities find the dimensions of

(a) Density
(b) Pressure
(c) Momentum and
(d) Energy

**Solution:**

As per given instruction, considering force, length and time to be the fundamental quantities

(a) Density = mass/volume ...(1)

and, mass = Force/acceleration = (Force \times time^2)/displacement.

(1)=> Density = \{((Force \times time^2)/displacement)/Volume

Dimensions of Density = [FL^{-4}T^2]

(b) Pressure = F/A

Dimensions of A = [L^2]

Dimensions of Pressure = [FL^{-2}]

(c) Momentum = mv = (force/acceleration) x velocity

= [F/LT^{-2}] \times [LT^{-1}] = [FT]
Dimensions of Momentum [FT]

(d) Energy = \( \frac{1}{2} mv^2 = \text{Force/acceleration} \times (\text{velocity})^2 \)

\[
= \left[ \frac{F}{LT^{-2}} \right] \left[LT^{-1}\right]^2 = [FL]
\]

Dimensions of Energy = [FL]

**Question 8:** Suppose the acceleration due to gravity at a place is 10 m/s\(^2\). Find its value in cm/(minute)\(^2\).

**Solution:**
Given: acceleration due to gravity at a place is 10 m/s\(^2\)
Convert units into cm/min\(^2\)

Here, \( g = 10 \text{ m/sec}^2 = 36 \times 10^5 \text{ cm/min}^2 \)

**Question 9:** The average speed of a snail is 0.020 miles/hour and that of a leopard is 70 miles/hour. Convert these speeds in SI units.

**Solution:**
Average speed of a snail = 0.020 miles/hour (Given)
Average speed of a leopard = 70 miles/hour (Given)

In SI Units:
0.020 miles/hour = \( \frac{(0.02 \times 1.6 \times 1000)}{3600} = 0.0089 \text{ m/s} \)
[Using 1 mile = 1.6 km = 1600m]
And,
70 miles/hr = \( \frac{(70 \times 1.6 \times 1000)}{3600} = 31 \text{ m/s} \)

**Question 10:** The height of mercury column in a barometer in a Calcutta laboratory was recorded to be 75 cm. Calculate this pressure in SI and CGS units using the following data.
Specific gravity of mercury = 13.6
Density of water = \( 10^3 \text{ kg/m}^3 \),
gravity, \( g = 9.8 \text{ m/s}^2 \) at Calcutta.
Pressure = \( h \rho g \) in usual symbols.

**Solution:**
The height of mercury column in a barometer in a Calcutta laboratory was recorded to be 75 cm. (Given)
Say, \( h = 75 \text{ cm} \)
Calculate pressure in SI and CGS units.
Pressure = \( h \rho g = 10 \times 10^4 \text{ N/m}^2 \) approx

In C.G.S. units, Pressure = \( 10 \times 10^5 \text{ dyne/cm}^2 \)

**Question 11:** Express the power of a 100 watt bulb in CGS unit.

**Solution:**
Write power 100 watt in CGS units.
In S.I. units: 100 watt = 100 Joule/sec
In C.G.S. unit = \( 10^9 \text{ erg/sec} \)

**Question 12:** The normal duration of I.Sc. Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury = \( 10^{-6} \times 100 \text{ years} \).
How many microcenturies did you sleep yesterday?

**Solution:**
Given: 1 microcentury = \( 10^{-6} \times 100 \text{ years} \).
1 year = 365 x 24 x 60 min
Or 1 microcentury = \( 10^{-4} \times 365 \times 24 \times 60 \text{ min} \)
So, 100 min = \( 10^5/52560 = 1.9 \text{ microcentury} \)

**Question 13:** The surface tension of water is 72 dyne/cm. Convert it in SI units.

**Solution:**
Given: surface tension of water is 72 dyne/cm
In S.I units: 72 dyne/cm = \( 0.072 \text{ N/m} \)
Question 14: The kinetic energy $K$ of a rotating body depends on its moment of inertia $I$ and its angular speed $\omega$. Assuming the relation to be $K = kI^a \omega^b$ where $k$ is a dimensionless constant, find $a$ and $b$. Moment of inertia of a sphere about its diameter is $\frac{2}{5} Mr^2$.

Solution:

$K = kI^a \omega^b$; where $k$ is a dimensionless constant, $K = \text{kinetic energy}$ and $\omega = \text{angular speed}$

To find: $a$ and $b$

Now, $K = [ML^2T^{-2}]$

$I^a = [ML^2]^a$ and $\omega^b = [T^{-1}]^b$

$\Rightarrow [ML^2T^{-2}] = [ML^2]^a [T^{-1}]^b$
[using principle of homogeneity of dimension]

Equating the dimensions, we get

$2a = 2 \Rightarrow a = 1$

$-b = -2 \Rightarrow b = 2$

Question 15: Theory of relativity reveals that mass can be converted into energy. The energy $E$ so obtained is proportional to certain powers of mass $m$ and the speed $c$ of light. Guess a relation among the quantities using the method of dimensions.

Solution:

The relationship between energy, mass and speed of light is,

$E \propto M^aC^b$

Where $M = \text{mass}$ and $C = \text{speed of light}$

or $E = K M^aC^b$ .... (1)

where $K$ = constant of proportionality

Find the dimensions of (1)

$[ML^2T^{-2}] = [M]^a [LT^{-1}]^b$

By comparing values, we have
Question 16: Let I = current through a conductor, R = its resistance and V = potential difference across its ends. According to Ohm's law, product of two of these quantities equals the third. Obtain Ohm's law from dimensional analysis. Dimensional formulae for R and V are $[ML^2T^{-2}]$ and $[ML^2T^{-3}I^{-1}]$ respectively.

Solution:
Given:
Dimensional formulae for $R = [ML^2T^{-2}]$ and $V = [ML^2T^{-3}I^{-1}]$
Therefore,

$$[ML^2T^{-3}I^{-1}] = [ML^2I^{-2}T^{-3}]$$

$=>$ $V = IR$

Question 17: The frequency of vibration of a string depends on the length L between the nodes, the tension F in the string and its mass per unit length m. Guess the expression for its frequency from dimensional analysis.

Solution:
$L$ = length, $M$ = mass and $F$ = Force
Here, $f = KL^aF^bM^c$ ...(1)

Dimension of frequency, $f = [T^{-1}]$ or $[M^0L^0T^{-1}]$
Dimension of length, $L = [L]$ 
Dimension of mass, $M = [ML^{-1}]$
Dimension of force, $F = [MLT^{-2}]$

(1)$=>$ $[M^0L^0T^{-1}] = K [L]^a [MLT^{-2}]^b [ML^{-1}]^c$
Equating both sides, we get

$b + c = 0$

$-c + a + b = 0$

$-2b = -1$

Solving above three equations, we have
a = -1, b = \frac{1}{2} and c = -\frac{1}{2}

Therefore, frequency is

\[ f = KL^{-1}F^{1/2}M^{-1/2} \]

\[ f = KL^{-1}F^{1/2}M^{-1/2} = \frac{K}{L} \sqrt{\frac{F}{M}} \]

**Question 18:** Test if the following equations are dimensionally correct:

(a) \[ h = \frac{2S \cos \theta}{\rho g} \]

(b) \[ v = \sqrt{\frac{P}{\rho}} \]

(c) \[ V = \frac{\pi Pr^4t}{8 \eta l} \]

(d) \[ v = \frac{1}{2} \pi \sqrt{\frac{mg}{l}} \]

where \( h = \) height, \( S = \) surface tension, \( \rho = \) density, \( P = \) pressure, \( V = \) volume, \( \eta = \) coefficient of viscosity, \( v = \) frequency and \( I = \) moment of inertia

**Solution:**

(a) Dimension of \( h = [L] \)

Dimension of \( S = \frac{F}{I} = ML^{-2}/L = [MT^{-2}] \)

Dimension of density = \( M/V = [ML^{-3}T^0] \)

Dimension of radius = \( [L] \)

Dimension of gravity = \( [LT^{-2}] \)

Now,

\[ \frac{2S \cos \theta}{\rho g} = \frac{[MT^{-2}]}{[ML^{-3}T^0][L][LT^{-2}]} = [M^0L^1T^0] = [L] \]

Relation is correct.

(b) Let velocity = \( v = \sqrt{\frac{P}{\rho}} \) \( \ldots \) (1)

Dimension of \( v = [LT^{-1}] \)

Dimension of \( p = \frac{F}{A} = [ML^{-1}T^{-2}] \)
Dimension of \( \rho = m/v = [ML^{-3}] \)

Substituting dimensions in (1), we have

\[
\sqrt{\frac{\rho}{\rho}} = \sqrt{\frac{[ML^{-3}]}{[ML^{-3}]}} = [L^{2}T^{-2}]^{1/2} = [LT^{-1}]
\]

Therefore, relation is correct.

(c)
Dimension of \( V = [L^3] \)
Dimension of \( p = [ML^{-1}T^{-2}] \)
Dimension of \( r^4 = [L^4] \)
Dimension of \( t = [T] \)
Dimension of \( \eta = [ML^{-1}T^{-1}] \)

\[
\frac{\pi r^4 t}{8\eta l} = \frac{[ML^{-1}T^{-2}][L^4][T]}{[ML^{-1}T^{-1}][L]} = [L^{-1}]
\]

Therefore, relation is correct.

(d)
Dimension of \( v = [T^{-1}] \)
Dimension of \( m = [M] \)
Dimension of \( g = [LT^{-2}] \)
Dimension of \( l = [L] \)
Dimension of inertia = \([ML^2]\)

\[
\sqrt{\frac{mg}{l}} = \sqrt{\frac{[M][LT^{-2}][L]}{[ML^2]}} = [T^{-1}]
\]

Therefore, relation is correct.

**Question 19:** Let \( x \) and \( a \) stand for distance. Is the below equation dimensionally correct?

\[
\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a} \sin^{-1} \frac{a}{x}
\]
Solution:

Dimensions of $a = [L]$  
Dimensions of $x = [L]$  

LHS  
\[
\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{L}{\sqrt{L^2 - L^2}} = [L^0]
\]

RHS  
\[
\frac{1}{a} \sin^{-1}\left(\frac{a}{x}\right) = [L^{-1}]
\]

\[
\int \frac{dx}{\sqrt{a^2 - x^2}} \neq \frac{1}{a} \sin^{-1}\left(\frac{a}{x}\right)
\]