BYJU'S Home Learning Program

Topic covered:

- **Mathematical Tools (session - 2) - NEET**

Worksheet

1. Change the given exponential expressions into logarithmic expressions
   a. $3^x = m$
   b. $x^2 = a$

2. Evaluate the following logarithmic expression without calculator
   a. $\log_2 16$
   b. $\log_3 27$

3. Simplify: $\log_2 5 + \log_2 1.6$

4. Find the derivative of x^5.

5. If $y = x^3 + 2x^2 + 7x + 8$, then $\frac{dy}{dx}$ will be
   a. $3x^2 + 2x + 15$
   b. $3x^2 + 4x + 7$
   c. $x^3 + 2x^2 + 15$
   d. $x^3 + 4x + 7$

6. If $y = 2\sin x$, then $\frac{dy}{dx}$ will be
   a. $2\cos x$
   b. $-2\cos x$
   c. $\cos x$
   d. None

7. If $y = \frac{1}{x^4}$, then $\frac{dy}{dx}$ will be
   a. $\frac{4}{x^3}$
   b. $4x^3$
   c. $-\frac{4}{x^5}$
   d. $\frac{4}{x^5}$

8. If $y = \ln x$, then $\frac{dy}{dx}$ will be
   a. x
   b. $\ln x$
   c. $\frac{1}{x}$
   d. $-\frac{1}{x^2}$

9. If $y = e^x \cot x$, then $\frac{dy}{dx}$ will be
   a. $e^x \cot x - \operatorname{cosec}^2 x$
   b. $e^x \operatorname{cosec}^2 x$
   c. $e^x[\cot x - \operatorname{cosec}^2 x]$
   d. $e^x \cot x$

10. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx}$ will be
    a. $\frac{1-\ln x}{x}$
    b. $\frac{1+\ln x}{x^2}$
    c. $\frac{1-\ln x}{x^2}$
    d. $\frac{\ln x - 1}{x^2}$

11. Differentiation of $\sin x^2$ w.r.t. x is
    a. $\cos x^2$
    b. $2x \cos x^2$
    c. $x^2 \cos x^2$
    d. $-\cos 2x$

12. If $y = x^2 \sin x$, then $\frac{dy}{dx}$ will be
    a. $x^2 \cos x + 2x \sin x$
    b. $2x \sin x$
    c. $x^2 \cos x$
    d. $2x \cos x$

13. Find $f'(x)$ if $f(x)$ is
    (i) xe^x
    (ii) $x \sin x$

14. If $f(x) = \sqrt{x}\, g(x)$, where $g(4) = 2$ and $g'(4) = 3$, find $f'(4)$.

15. Find the derivative of $y = \frac{t^2-1}{t^2+1}$.

16. Find the derivative of $y(x) = \frac{x^3}{(x+1)^2}$ with respect to x.

17. Find $\frac{dy}{dx}$ of $y = \tan x$.

18. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so, where?

19. Find the slope of the tangent to the curve $y = 2\sqrt{x}$ at the point (1,2).

20. If surface area of a cube is changing at a rate of $5\ m^2/s$, find the rate of change of body diagonal at the moment when side length is $1\ m$.
    a. $5\ m/s$
    b. $5\sqrt{3}\ m/s$
    c. $\frac{5}{2}\sqrt{3}\ m/s$
    d. $\frac{5}{4\sqrt{3}}\ m/s$

Answer Key

Question Number	1	2	3	4	5
Answer Key	a. $x = \log_3 m$ b. $2 = \log_x a$	a. 4 b. 3	3	$5x^4$	(b)

Question Number	6	7	8	9	10
Answer Key	(a)	(c)	(c)	(c)	(c)

Question Number	11	12	13	14
Answer Key	(b)	(a)	(i) $(x+1)e^x$ (ii) $x\cos x + \sin x$	6.5

Question Number	15	16	17	18	19	20
Answer Key	$\frac{4t}{(t^2+1)^2}$	$\frac{3x^2}{(x+1)^2} - \frac{2x^3}{(x+1)^3}$	$\sec^2 x$	(0, 2), (1, 1), (-1, 1)	1	(d)

Solutions

1.

a. We know, $y = a^x$ can be written as $x = \log_a y$.

$3^x = m$

$x = \log_3 m$

b. $x^2 = a$

$2 = \log_x a$

2.

a. $\log_2 16 = \log_2 2^4 = 4\log_2 2 = 4$

b. $\log_3 27 = \log_3 3^3 = 3\log_3 3 = 3$

3. $\log_b m + \log_b n = \log_b (mn)$

$\log_2 5 + \log_2 1.6 = \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3$

4. $y = x^5$

$\frac{dy}{dx} = 5x^4$

5. (b)

$y = x^3 + 2x^2 + 7x + 8$

$\frac{dy}{dx} = 3x^2 + 4x + 7$

6. (a)

$y = 2\sin x$

$\frac{dy}{dx} = 2\cos x$

7. (c)

$y = x^{-4}$

$\frac{dy}{dx} = (-4)x^{-4-1} = -\frac{4}{x^5}$

8. (c)

$\text{y} = \ln x$

$$\frac{dy}{dx} = \frac{1}{x}$$

9. (c)

$y = e^x . \cot x$

$\frac{dy}{dx} = e^x \frac{d}{dx}(\cot x) + \cot x \frac{d}{dx}(e^x)$

$= e^x(-\text{cosec}^2 x) + \cot x (e^x)$

$= e^x[\cot x - \text{cosec}^2 x]$

10. (c)

$y = \frac{\ln x}{x}$

$\frac{dy}{dx} = \frac{x\frac{d}{dx}(\ln x) - \ln x \frac{d}{dx}(x)}{x^2}$

$= \frac{x\left(\frac{1}{x}\right) - \ln x}{x^2}$

$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$

11. (b)

$y = \sin x^2$

$\frac{dy}{dx} = \cos(x^2)\frac{d}{dx}\,(x^2)$

$= 2x\cos(x^2)$

12. (a)

$y = x^2 \sin x$

$\frac{dy}{dx} = x^2\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x^2)$

$= x^2\cos x + 2x\sin x$

13.

(i) $f'(x) = xe^x + e^x.1 = (x+1)e^x$

(ii) $f'(x) = x.\frac{d}{dx}(\sin x) + \sin x.\frac{dx}{dx}$

$f'(x) = x\cos x + \sin x$

14. $f'(4) = \sqrt{x}g'(x) + \frac{g(x)}{2\sqrt{x}}$

$f'(4) = \sqrt{4}g'(4) + \frac{g(4)}{2\sqrt{4}} = 2 \times 3 + \frac{2}{2\times 2} = 6.5$

15. We apply the Quotient Rule with $u = t^2 - 1$ and $v = t^2 + 1$

$$\frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dt}\right) - u\left(\frac{dv}{dt}\right)}{v^2}$$

$\frac{dy}{dt} = \frac{(t^2+1).2t - (t^2-1).2t}{(t^2+1)^2}$

$= \frac{2t^3+2t-2t^3+2t}{(t^2+1)^2} = \frac{4t}{(t^2+1)^2}$

16. We can rewrite this function as $y(x) = x^3(x+1)^{-2}$ and apply product rule,

$\frac{dy}{dx} = (x+1)^{-2}\frac{d}{dx}(x^3) + x^3\frac{d}{dx}((x+1)^{-2})$

$= (x+1)^{-2}3x^2 + x^3(-2)(x+1)^{-3}$

$\frac{dy}{dx} = \frac{3x^2}{(x+1)^2} - \frac{2x^3}{(x+1)^3}$

17. $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{\cos^2 x}$

$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

18. The horizontal tangents, if any, occur where the slope $\frac{dy}{dx}$ is zero.

To find these points,

1. Calculate $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2) = 4x^3 - 4x$
2. Solve the equation : $\frac{dy}{dx} = 0$

$$4x^3 - 4x = 0$$
$$4x(x^2 - 1) = 0$$
$$x = 0, 1, -1$$

The curve $y = x^4 - 2x^2 + 2$ have horizontal tangents at $x = 0, 1$ and -1. The corresponding points on the curve are $(0,2), (1,1)$ and $(-1,1)$.

19. $y = 2\sqrt{x}$

$$\frac{dy}{dx} = 2\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{\sqrt{x}}$$

$\frac{dy}{dx}$ is called slope of the tangent to the curve $y = 2\sqrt{x}$ at the point (x, y). For the point $x = 1$ and $y = 2$. Putting $x = 1$ and $y = 2$ in the equation $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$, we get $\frac{dy}{dx} = \frac{1}{\sqrt{1}} = 1$. Hence slope is 1.

20. (d)

Surface area of cube $S = 6a^2$ (where, a = side of cube)

Body diagonal $l = \sqrt{3}a$. Therefore $S = 2l^2$

Differentiating it w.r.t. time,

$\frac{dS}{dt} = 2(2l)\frac{dl}{dt}$

$\Rightarrow \frac{dl}{dt} = \frac{1}{4(\sqrt{3}a)}\frac{dS}{dt} = \frac{5}{4\sqrt{3}}\ m/s$