BYJU'S Home Learning Program

Topic covered:

- **Relations and Functions (Session- 1)**

Daily Practice Problems

1. Let $A = \{2,3,4,5.....17,18\}$. Let "$\simeq$" be the equivalence relation on $A \times A$, cartesian product of A with itself, defined by $(a, b) \simeq (c, d)$ if $ad = bc$. Then the number of ordered pairs of the equivalence class of $(3, 2)$ is

   a. 5
   b. 7
   c. 9
   d. 6

2. Let $Y = \{1, 2, 3, 4, 5\}, A = \{1, 2\}, B = \{3, 4, 5\}$. If $A \times B$ denotes the cartesian product of sets A and B, then $(Y \times A) \cap (Y \times B)$ is

   a. Y
   b. A
   c. B
   d. Null set

3. The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b): |a^2 - b^2| < 16\}$ has ____ no. of elements.

   a. 19
   b. 25
   c. 20
   d. 16

4. Let R be a relation on the set N be defined by $\{(x, y)|x, y \in \mathbf{N}, 2x + y = 41\}$. Then R is

   a. Symmetric only
   b. Symmetric and transitive only
   c. Not symmetric, not reflexive, not transitive
   d. Equivalence

5. Let $X = \{1, 2, 3,4, 5\}, Y = \{1, 3, 5, 7, 9\}$. Which of the following can't be called relation from X to Y ?

   a. $R_1 = \{(x, y)|y = 2 + x, x \in X, y \in Y\}$
   b. $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
   c. $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
   d. $R_4 = \{(1, 3), (2, 5), (3, 7), (2, 4), (7, 9)\}$

6. If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}, B = \{1, 3, 5, 6, 7, 8, 9\}$ then $n\big((A \,\Delta\, B) \times (A \,\Delta\, B)\big)$ is equal to

   a. 16
   b. 18
   c. 9
   d. 12

7. Let $A = \{1, 2, 3, 4, 5\}, B = \{4, 5, 6, 7, 8\}, C = \{8, 9, 10, 11, 12\}$ then $n(A \times (B' \cup C')')$ is equal to

   a. 5
   b. 125
   c. 80
   d. 60

8. Let $A = \{1, 2, 3\}$ and R, S be two equivalence relations on A given by $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$, $S = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$ then $R \cup S$ is

   a. Equivalence
   b. It is reflexive and transitive only
   c. It is not transitive
   d. Symmetric and transitive only

9. Let **N** denote the set of all-natural numbers and R be a relation on $\mathbf{N} \times \mathbf{N}$ defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Then on $\mathbf{N} \times \mathbf{N}$, R is ______ relation.

   a. An equivalence
   b. Reflexive and symmetric only
   c. Transitive only
   d. Symmetric and transitive only

10. Let $S = \{1, 2, 3, 4, 5\}$. The total number of unordered pairs of disjoint subsets of S is equal to.

   a. 122
   b. 242
   c. 123
   d. 240

ANSWER KEY

Question No.	1	2	3	4	5	6	7	8	9	10
Correct Answer	(d)	(d)	(a)	(c)	(d)	(c)	(a)	(c)	(a)	(a)

SOLUTIONS

Answer 1 :

The number of ordered pairs in the equivalence class of $(3, 2)$ is the no. of ordered pairs (a, b) satisfying $(a, b) \simeq (3,2)$ i.e.

$2a = 3b \Rightarrow \frac{\mathrm{a}}{\mathrm{b}} = \frac{3}{2}$

$\therefore$ Ordered pairs are $(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)$

Answer 2 :

$(Y \times A) \cap (Y \times B) = Y \times \phi = \phi$

Answer 3 :

$(a, b) \in \mathbf{R} \Leftrightarrow |a^2 - b^2| < 16$

$a = 1 \Rightarrow |1 - b^2| < 16 \Rightarrow -16 < 1 - b^2 < 16 \Rightarrow -15 < b^2 < 17 \Rightarrow \mathrm{b} = 1, 2, 3, 4$

$a = 2 \Rightarrow |4 - b^2| < 16 \Rightarrow -16 < 4 - b^2 < 16 \Rightarrow -12 < b^2 < 20 \Rightarrow \mathrm{b} = 1, 2, 3, 4$

$a = 3 \Rightarrow |9 - b^2| < 16 \Rightarrow -16 < 9 - b^2 < 16 \Rightarrow -7 < b^2 < 25 \Rightarrow \mathrm{b} = 1, 2, 3, 4$

$a = 4 \Rightarrow |16 - b^2| < 16 \Rightarrow -16 < 16 - b^2 < 16 \Rightarrow 0 < b^2 < 32 \Rightarrow \mathrm{b} = 1, 2, 3, 4, 5$

$a = 5 \Rightarrow |25 - b^2| < 16 \Rightarrow -16 < 25 - b^2 < 16 \Rightarrow 9 < b^2 < 41 \Rightarrow \mathrm{b} = 4, 5$

$\therefore$ Total relations = 19

Answer 4 :

$\mathrm{R} = \{(1, 39), (2, 37), (3, 35), (4, 33), (5, 31), (6, 29), (7, 27), (8, 25), (9, 23), (10,21),$
$(11, 19), (12, 17), (13, 15), (14, 13), (15, 11), (16, 9), (17, 7), (18, 5), (19, 3), (20, 1)\}$

i. Not reflexive because $(1, 1)$ not present
ii. Not symmetric because $(1, 39) \in \mathrm{R}$ but $(39, 1) \notin \mathrm{R}$
iii. Not Transitive, because $(15, 11) \in \mathrm{R}$, $(11, 19) \in \mathrm{R}$ but $(15, 19) \notin R$

Answer 5 :

$(7, 9) \in R_4$ but $(7, 9) \notin A \times B$.

Answer 6 :

$A - B = \{2, 4\}, B - A = \{9\}, A \,\Delta\, B = (A - B) \cup (B - A) = \{2, 4, 9\}$

$\therefore \quad (A \,\Delta\, B) \times (A \,\Delta\, B) = 3^2 = 9$

Answer 7 :

$A \times (B' \cup C')' = A \times (B \cap C) = \{1, 2, 3, 4, 5\} \times \{8\}$

$n(A \times (B' \cup C')') = 5$

Answer 8 :

$R \cup S = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$

$\therefore$ It is reflexive, symmetric but not transitive because $(1,1),(2,3) \in R \cup S$ but $(1,3) \notin R \cup S$.

Answer 9 :

i. $(a,b)\ R\ (c,d) = ab(b+a) = ba(a+b)$ which is true

$\therefore$ reflexive

ii. $(a,b)\ R\ (c,d) = ad(b+c) = bc(a+d)$

$= cb(d+a) = da(c+b)$

$= (c,d)\ R\ (a,b)$

$\therefore$ symmetric

iii. $(a,b)\ R\ (c,d) = ad(b+c) = bc(a+d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad}$$

$$\Rightarrow \frac{1}{b}+\frac{1}{c} = \frac{1}{a}+\frac{1}{d} \qquad ....(1)$$

$(c,d)\ R\ (e,f) = cf(d+e) = de(c+f)$

$$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf}$$

$$\Rightarrow \frac{1}{e}+\frac{1}{d} = \frac{1}{f}+\frac{1}{c} \qquad ....(2)$$

$$(1)+(2) \Rightarrow \left(\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}\right) = \left(\frac{1}{a}+\frac{1}{d}+\frac{1}{f}+\frac{1}{c}\right)$$

$$\Rightarrow \frac{1}{b}+\frac{1}{e} = \frac{1}{a}+\frac{1}{f}$$

$$\Rightarrow \frac{b+e}{be} = \frac{a+f}{af} \Rightarrow af(b+e) = be(a+f)$$

$$\Rightarrow (a,b)\ R\ (e,f)$$

$\therefore$ Transitive

Hence equivalence relation

Answer 10 :

Set $S = \{1,2,3,4,5\}$. Let P & Q be disjoint subsets of S. Now for any element of a, we have 3 cases.

Case 1: $a \in P, a \notin Q$

Case 2: $a \notin P, a \in Q$

Case 3: $a \notin P, a \notin Q$

$\therefore$ Total cases $= 3^5 = 243$

Here $P \neq Q$ except the case $P = \phi, Q = \phi$

$\therefore$ $243 - 1 = 242$

$\therefore$ Number of ordered pairs $= \frac{242}{2} + 1 = 122$