BYJU'S Home Learning Program

Topic covered:

- **Relations and Functions (Session- 1)**

Worksheet

1. Let $X = \{1, 2, 3, 4, 5\}$. The number of different orders pairs (Y, Z) that can be formed such that $Y \subseteq X, Z \subseteq X,\ Y \cap Z$ empty is

   a. 5^2 b. 3^5

   c. 2^5 d. 5^3

2. Let R be the set of real numbers
   Statement 1: $A = \{(x, y)\ R \times R : y - z$ is an integer$\}$ is an equivalence relation on R.
   Statement 2: $B = \{(x, y)\ R \times R : x = y$ for some rational number $\}$ is an equivalence relation on R.

   a. Statement 1, 2 are true, but statement 2 is not a correct explanation for statement 1.

   b. Statement 1, 2 are true, statement 2 is a correct explanation for statement 1.

   c. Statement 1 is true, statement 2 is false.

   d. Statement 1 is false, statement 2 is true.

3. Consider the following relations:
   $R = \{(x, y) | x, y$ are real numbers and $x = wy$ for same rational number $\omega\}$
   $S = \left\{\left(\frac{m}{n}, \frac{p}{\dotplus}\right) \middle| m, n, p, \dotplus\right.$ are integers such that $n, \dotplus \neq 0$ and $\dotplus m = pn\}$. Then

   a. Neither R nor S is an equivalence relation

   b. S is an equivalence relation but R is not an equivalence relation.

   c. R and S both are equivalence relations.

   d. R is an equivalence relation and S is not an equivalence relation.

4. Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \mid$ the words x and y have atleast one letter in common.$\}$ Then R is

   a. Symmetric, transitive and not reflexive.

   b. Reflexive, symmetric and not transitive.

   c. Reflexive, symmetry and not symmetric

   d. Reflexive, transitive and not symmetric

5. Let $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. Then the relation is

   a. Reflexive and transitive only b. Reflexive only

   c. An equivalence relation d. Reflexive and symmetric only

6. If $n(A) = 5, n(B) = 7$ be two sets having 3 elements in common then $n((A \times B) \cap (B \times A)) =$

   a. 32 b. 15

   c. 21 d. 9

7. Let A be a non-empty set such that $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Then $A =$

   a. $\{-1, 0\}$ b. $\{-1,0,1\}$

   c. $\{0,1\}$ d. $\{-1,1\}$

8. If a relation R is defined on the set II of integers as follows: $(a, b) \in R \iff a^2 + b^2 = 25$. Then domain $(R) =$

   a. $\{3,4,5\}$ b. $\{0,3,4,5\}$

   c. $\{0 \pm 3, \pm 4, \pm 5\}$ d. $\{\pm 3, \pm 4, \pm 5\}$

9. Let $A = \{a, b, c\}$ and let $R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$, $S = \{(a, a), (b, b), (c, c), (b, c), (c, b)\}$ be equivalence relations, then

   a. $R \cup S$ is an equivalence relation

   b. $R \cup S$ is not transitive

   c. $R \cup S$ is not reflexive

   d. $R \cup S$ is transitive and reflexive but not symmetric

10. Let a relation R, on the set R of real numbers of defined as $(a, b) \in R, \iff 1 + ab > 0$ $\forall\, a, b \in R$. Then R, is

    a. Equivalence relation

    b. Reflexive and transitive only

    c. Not transitive

    d. Symmetric and transitive only.

11. Let N be a set of all-natural numbers and let P be a relation and $N \times N$, defined by $(a, b)\ R\ (c, d) \iff ad = bc\ \forall\, (a, b), (c, d) \in N \times N$. Then R

    a. An equivalence relation b. Reflexive and symmetric only

    c. Reflexive and transitive only d. Not symmetric

12. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to _____

    a. 41 b. 42

    c. 25 d. 34

13. The number of unordered pairs (A, B) of subsets of the set $S = \{1, 2, 3, 4, 5, 6\}$ such that $A \cap B = \phi$ and $A \cup B = S$ is

a. 32 b. 64

c. 128 d. 63

14. Let A & B be two sets containing from and two elements respectively. Then the number of subsets of set $A \times B$ each having at least five elements is

a. 90 b. 120

c. 93 d. 125

15. If $A = \{\alpha, \beta, \gamma\}, B = \{1, 2, 3, 4\}$, then the number of elements in the set $A \times B \times B$ is

a. 48 b. 36

c. 10 d. $2^{3\times4\times4}$

16. Let $A = \{x\,;\, x$ is a root of the eqn. $x^3 + 2x^2 - x - 2 = 0\}$
$B = \{x :\ x$ is a prime division of $720\}$ then $n(A \times B)$

a. 6 b. 9

c. 12 d. 10

17. $A = \{1, 2, 3, 4, 5, 6\}, B = \{3, 6, 9, 12\}, C = \{6, 12, 18, 20\}$ then $n\{(A \times B) \cap (A \times C)\} =$

a. 12 b. 24

c. 48 d. 36

18. For real numbers x and y, we write $x\,R\,y \Leftrightarrow x - y + \sqrt{12}$ is an irrational number. Then the relation R is

a. Reflexive b. Equivalence

c. Transitive d. Reflexive and transitive

19. Let R be a relation on the set N of natural number defined by $n\,R\,m \Leftrightarrow n$ is a factor of m $(i.e, n|m)$. The R is

a. Reflexive and Symmetric b. Transitive and symmetric

c. Equivalence d. Reflexive, transitive but not symmetric

20. The relation $R = \{(1, 1), (2, 2), (3, 3)\}$ on the set $\{1, 2, 3\}$ is

a. Reflexive only b. Symmetric and reflexive only

c. Equivalence relation d. Not transitive but reflexive.

ANSWER KEY

Question No.	1	2	3	4	5	6	7	8	9	10
Correct Answer	(b)	(c)	(b)	(b)	(a)	(d)	(b)	(c)	(b)	(c)

Question No.	11	12	13	14	15	16	17	18	19	20
Correct Answer	(a)	(a)	(a)	(c)	(a)	(b)	(a)	(a)	(d)	(c)

SOLUTIONS

Answer 1 :

Given $n(x) = 5$, Each element has 3 options. Either set y and z or none $(y \cap z \neq 0)$

$\therefore$ Number of ordered pairs = 3^5

Answer 2 :

Statement 1

(i) $x - x = 0$ is an integer $\therefore$ Reflexive

(ii) $x - y \in Z$ $y - z \in Z$ $\Rightarrow$ Symmetric

(iii) $x - y \in Z, y - z \in Z$

$\Rightarrow (x - y) + (y - z) \in Z \Rightarrow x - z \in Z \Rightarrow$ Transitive

$\therefore$ Statement 1 is equivalence

Statement 2.

(i) $x = \alpha x \Rightarrow \alpha = 1 \in Z$ rational number $\therefore$ Reflexive

(ii) $\frac{1}{b} = \alpha$ is true for $\frac{y}{x} = \alpha$ may not be true

Let $x = 0, y = 1$ then $\frac{x}{y} = 0$ But $\frac{y}{x}$ not feasible.

$\therefore$ Statement 2 is not equivalence.

Answer 3 :

For R: $x = \omega x \Rightarrow 1 \Rightarrow$ Reflexive relation.

$x = \omega x \Rightarrow y = \omega x$ is in correct as

$10 = 2.5 \nRightarrow 5 = 2.10 \Rightarrow$ Not Symmetric

$\therefore$ Not an equivalence relation.

For S: $\frac{m}{n} = \frac{p}{+}$

(i) $\frac{m}{n} = \frac{m}{n}$ $\therefore$ Reflexive

(ii) $\frac{m}{n} = \frac{p}{+} \Rightarrow \frac{p}{+} = \frac{m}{n}$ $\therefore$ Symmetric

(iii) $\frac{m}{n} = \frac{p}{+}$, $\frac{p}{+} = \frac{r}{s} \Rightarrow \frac{m}{n} = \frac{r}{s} \Rightarrow ms = nr$ $\therefore$ Transitive

Hence equivalence relation.

Answer 4 :

(i) $(x, x) \in R, \forall x \in W$, So R is reflexive
(ii) $(x, y) \in R$, then $(y, x) \in R$, So R is symmetric
(iii) Let $x = CAP, y = RAT, z = TOY$, then
$x\ R\ y$ (A common)
$y\ R\ z$ (T common)
But $x\ R\ z$ does not exist as no letter is common.

Answer 5 :

(i) It is reflexive as $\{(3,3), (6,6), (9,9), (12,12)\}$ present.
(ii) It is not symmetric as $(6,12) \in \mathrm{R}$ but $(12,6) \notin R$
(iii) It is transitive as $\{(3,3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6)\} \in R$.

Answer 6 :

$(\mathrm{A} \times \mathrm{B}) \cap (\mathrm{B} \times \mathrm{A}) = (\mathrm{A} \cap \mathrm{B}) \times (\mathrm{A} \cap \mathrm{B}) = 3^2 = 9.$

Answer 7 :

$(-1, 0) \in A \times A\ (0, 1) \in A \times A$
$\therefore \quad A = \{-1,0,1\}$

Answer 8 :

$(a, b) \in R \Leftrightarrow a^2 + b^2 = 25 \Leftrightarrow b = \pm\sqrt{25 - a^2}$
$\therefore \quad a = 0 \Rightarrow b = \pm 5$
$a = \pm 3 \Rightarrow b = \pm 5$ $\qquad \therefore$ Domain $= \{0, \pm 3, \pm 4, \pm 5\}$
$a = \pm 4 \Rightarrow b = \pm 3$
$a = \pm 5 \Rightarrow b = \pm 0$

Answer 9 :

$(a, b) \in R \cup S, (b, c) \in R \cup S$ but $(a, c) \notin R \cup S$
Hence not transitive.

Answer 10 :

(i) $(a, a) \in R, \Rightarrow \quad 1 + a^2 > 0 \quad \therefore$ Reflexive
(ii) $(a, b) \in R, \Rightarrow \quad 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow (b, a) \in R, \quad \therefore$ Symmetric
(iii) $\left(1, \frac{1}{2}\right) \in R, \quad \Rightarrow \quad 1 + 1\left(\frac{1}{2}\right) > 0$
$\left(\frac{1}{2}, -1\right) \in R, \Rightarrow \quad 1 + \frac{1}{2}(-1) > 0$
But $(1, -1) \notin R$, because $1 + 1(-1) = 0, \ngtr 0 \quad \therefore$ Not transitive.

Answer 11 :

(i) $(a, b) \in N \times N$

$(a, b)\ R\ (a, b) \Leftrightarrow ab = ba$. Which is true

Hence Reflexive

(ii) $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$

$\Leftrightarrow bc = ad$

$(c, d)\ R\ (a, b) \Leftrightarrow cb = da$ which is true

$$\left.\begin{aligned}&\text{(iii) } (a, b)\ R\ (c, d) \Rightarrow ad = bc\\ &(c, d)\ R\ (e, f) \Rightarrow cf = de\\ &(a, d)\,(c, f) = (b, c)\,(d, e)\end{aligned}\right\}$$

$\Rightarrow af = be$

$\Rightarrow (a, b)\ R\ (e, f)$

$\therefore (a, b)\ R\ (c, d), (c, d)\ R\ (e, f) \Rightarrow (a, b)\ R\ (e, f)$

$\Rightarrow$ Transitive

Hence equivalence relation.

Answer 12 :

Let P & Q be disjoint subsets of S. Now for any element a, we have 3 cases

Case 1: $a \in P, a \notin Q$,

Case 2: $a \notin P, a \in Q$

Case 3: $a \notin P, a \notin Q$

$\therefore$ Total options $= 3^4$

Here $P \neq Q$ except the case $P = \phi, Q = \phi$

$\therefore 81 - 1 = 80,$

No. of unordered pairs $= \dfrac{80}{2} + 1 = 41$

Answer 13 :

$A \cup B = S$ i.e. if set A contains x elements then set B must contain $6 - x$ elements. Now let us hold

the subsets starting from selecting 0 element for A, b for B, 1 element for A, 5 for B …..

$\therefore {}^6C_0 \cdot 1 + {}^6C_1 \cdot 1 + {}^6C_2 \cdot 1 + \ldots.. + {}^6C_6 = 2^6$

$\therefore$ Unordered pairs $= \dfrac{2^6}{2} = 2^5 = 32.$

Answer 14 :

$n(A \times B) = 8$

No. of subsets of $A \times B$ having at least 5 elements will be

${}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8 = {}^8C_3 + {}^8C_2 + {}^8C_1 + {}^8C_0 = 1 + 8 + 28 + 56$

$= 93$

Answer 15 :

$N(A \times B \times B) = n(A)\, n(B)\, n(B) = 3 \cdot 4 \cdot 4 = 48$

Answer 16 :

$$x^3 + 2x^2 - x - 2 = (x-1)\,(x+1)(x+2) = 0$$
$$\therefore \quad A = \{-1, 1, -2\}$$
$$B = \{2, 3, 5\}$$
$$n = (A \times B) = 9$$

Answer 17 :

$$(A \times B) \cap (A \times C) = (A \cap A) \times (B \cap C)$$
$$= \{1, 2, 3, 4, 5, 6\} \times \{12, 6\}$$
$n((A \times B) \cap (A \times C)) = 12$ elements

Answer 18 :

$x \, R \, y \iff x - y + \sqrt{2}, x, y \in R$ is irrational
(i) $x \, R \, y \Rightarrow x - x + \sqrt{2} = \sqrt{2}$ is irrational $\quad \therefore$ Reflexive
(ii) Not symmetric; $\sqrt{2} \, R \, 1$ but $1 \, R \, \sqrt{2}$
(iii) Not transitive $\sqrt{2} \; R \; 1, IR \; 2\sqrt{2}$ but $\sqrt{2} \, R \; - \; 2\sqrt{2}$

Answer 19 :

(i) n|n $\quad \therefore$ Reflexive
(ii) not symmetric because 2|6 but 6|2
(iii) $n \, R \, m, m \, R \, p \Rightarrow \; n \, R \, p \quad \therefore$ transitive

Answer 20 :

R is an identity relation on the set $\{1, 2, 3\}$ and the identity relation is always an equivalence relation.
Simple: 1, 6, 7, 9, 15
Av: 4, 5, 8, 14, 16, 17, 19, 20
Diff: 2, 3, 10, 11, 12, 13, 18