Practice set 1.1 Page 5

1. Base of a triangle is 9 and height is 5. Base of another triangle is 10 and height is 6. Find the ratio of areas of these triangles.

Solution:
Let base of the first triangle is b_1 and height is h_1. Let base of second triangle is b_2 and height is h_2.
Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
Here $b_1 = 9$
$h_1 = 5$
$b_2 = 10$
$h_1 = 6$
Then ratio of their areas = $b_1 \times h_1 / b_2 \times h_2$
= $9 \times 5/10 \times 6$
= $3/4$
Hence the ratio of the areas of these triangles is 3:4

2. In figure 1.13 BC⊥ AB, AD⊥ AB, BC = 4, AD = 8, then find A(△ABC) /A(△ ADB) .

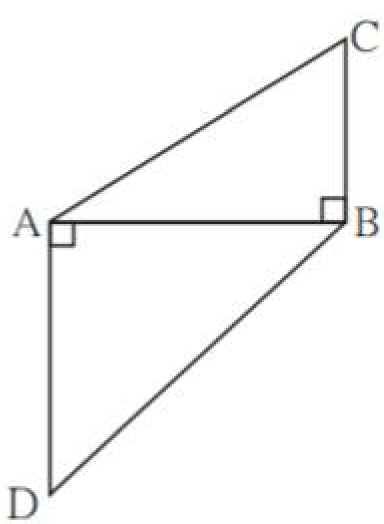


Fig. 1.13

Solution:
Here △ABC and △ADB have the same base AB.
Areas of triangles with equal bases are proportional to their corresponding heights.
Since bases are equal, areas are proportional to heights.

Given BC = 4 and AD = 8

So, A(△ABC) /A(△ ADB) = BC/AD

= 4/8

= 1/2

Hence ratio of areas of △ABC and △ADB is 1:2.

3. In adjoining figure 1.14 seg PS ⊥ seg RQ, seg QT ⊥ seg PR. If RQ = 6, PS = 6 and PR = 12, then find QT.

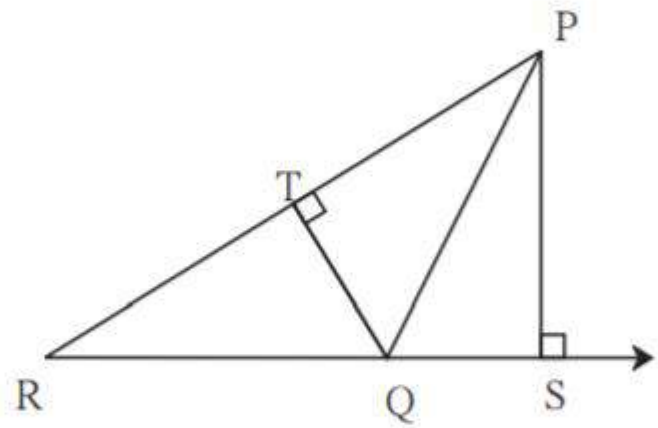


Fig. 1.14

Solution:

Given PS ⊥ RQ and QT ⊥ PR.

RQ = 6

PS = 6

PR = 12

Area of △PQR with base PR and height QT = (1/2)×PR×QT

Area of △PQR with base QR and height PS = (1/2)×QR×PS

A(△PQR)/ A(△PQR) = (1/2)×PR×QT /(1/2)×QR×PS

1 = PR×QT/ QR×PS

⇒1 = 12×QT/6×6

⇒6×6 = QT×12

⇒QT = 36/12

⇒QT = 3

Hence measure of side QT is 3 units.

4. In adjoining figure, AP ⊥ BC, AD || BC, then find A(△ ABC):A(△ BCD).

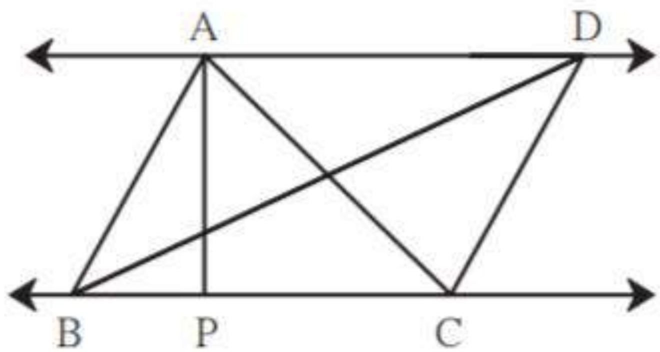


Fig. 1.15

Solution:
Given , AP ⊥ BC, and AD || BC. △ ABC and △ BCD has same base BC.
Areas of triangles with equal bases are proportional to their corresponding heights.
Since AP is the perpendicular distance between parallel lines AD and BC, height of △ABC and height of △BCD are same.
∴ A(△ ABC) /A(△ BCD) = AP/AP = 1
Hence A(△ ABC): A(△ BCD) = 1:1

5. In adjoining figure PQ ⊥ BC, AD ⊥ BC then find following ratios.
(i) A(△ PQB) /A(△ PBC)
(ii) A(△PBC) / A(△ ABC)
(iii) A(△ ABC) /A(△ ADC)
(iv) A(△ ADC) /A(△ PQC)

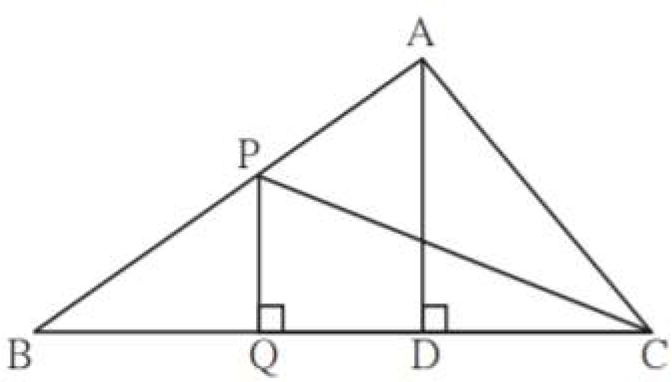


Fig. 1.16

Solution:
(i) △PQB and △PBC have same height PQ.
Ratio of areas of triangles with equal heights are proportional to their corresponding bases.
∴A(△PQB)/A(△PBC) = BQ/BC

(ii) $\triangle$PBC and $\triangle$ABC have same base BC.
Ratio of areas of triangles with equal bases are proportional to their corresponding heights.
$\therefore$ A($\triangle$PBC) / A($\triangle$ ABC) = PQ/AD

(iii) $\triangle$ ABC) and $\triangle$ ADC have equal heights AD.
Ratio of areas of triangles with equal heights are proportional to their corresponding bases.
$\therefore$A($\triangle$ ABC) /A($\triangle$ ADC) = BC/DC

(iv) Ratio of areas of two triangles is equal to the ratio of the products of their bases and corresponding heights.
$\therefore$ A($\triangle$ ADC) /A($\triangle$ PQC) = DC×AD/QC×PQ

Practice set 1.2 Page 13

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of ∠ QPR.

(1)

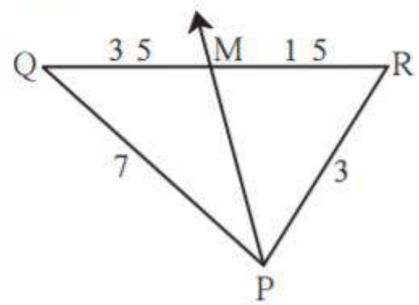


Fig. 1.33

(2)

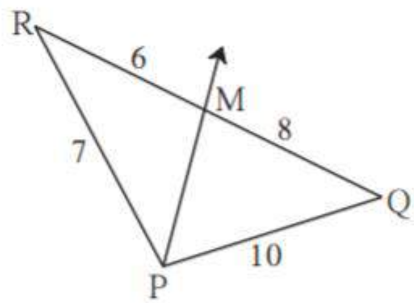


Fig. 1.34

(3)

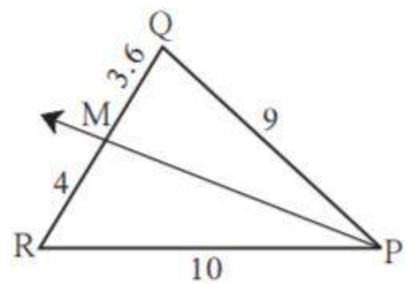


Fig. 1.35

Solution:

(i) In △PQR
QM/RM = 35/15 = 7/3 …..(i)
PQ/PR = 7/3……………….(ii)
From (i) and (ii) QM/RM = PQ/PR
∴ By Converse of angle bisector theorem , Ray PM is the bisector of ∠QPR.

(ii) In△PQR
PR/PQ = 7/10……..(i)
RM/QM = 6/8……..(ii)
From (i) and (ii) PR/PQ ≠ RM/QM
∴Ray PM is not the bisector of ∠QPR

(iii) In △PQR
PR/PQ = 10/9………(i)
RM/QM = 4/3.6 = 40/36 = 10/9 ……..(ii)
From (i) and (ii) PR/PQ = RM/QM
∴ By Converse of angle bisector theorem , Ray PM is the bisector of ∠QPR.

2. In △ PQR, PM = 15, PQ = 25 PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.

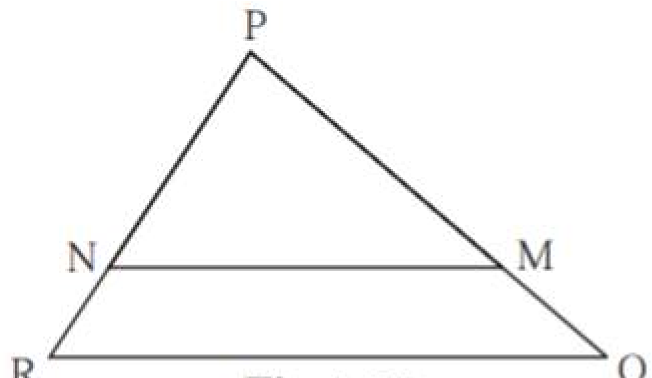


Fig. 1.36

Solution:
Given PM = 15, PQ = 25, PR = 20, NR = 8
PQ = PM+MQ
25 = 15+MQ
$\Rightarrow$MQ = 25-15
$\Rightarrow$MQ = 10
PR = PN+NR
20 = PN+8
$\Rightarrow$PN = 20-8
$\Rightarrow$PN = 12
PM/MQ = 15/10 = 3/2
PN/NR = 12/8 = 3/2
In $\triangle$PQR , PM/MQ = PN/NR.
$\therefore$ By Converse of basic proportionality theorem , line NM $\parallel$ side RQ.

3. In $\triangle$ MNP, NQ is a bisector of $\angle$ N. If MN = 5, PN = 7 MQ = 2.5 then find QP .

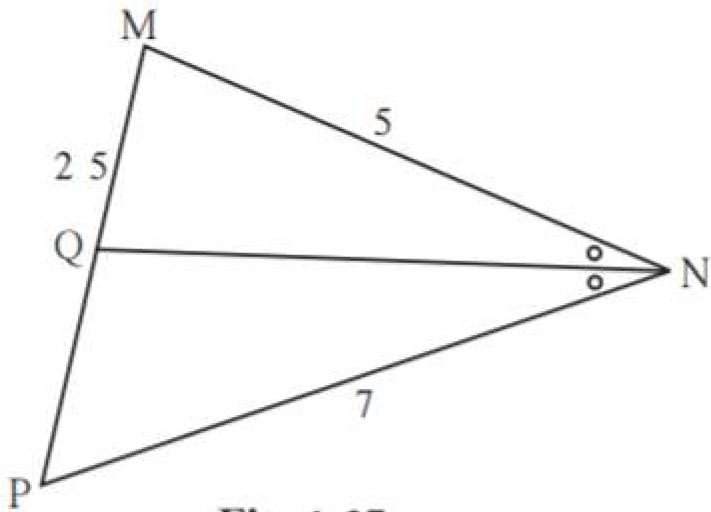


Fig. 1.37

Solution:
Given MN = 5, PN = 7, MQ = 2.5
Since NQ is a bisector of ∠N , PN/MN = QP/MQ [Angle bisector theorem]
7/5 = QP/2.5
⇒ QP = 7×2.5/5
⇒ QP = 3.5
Hence measure of QP is 3.5 units.

4. Measures of some angles in the figure are given. Prove that AP/PB = AQ/QC

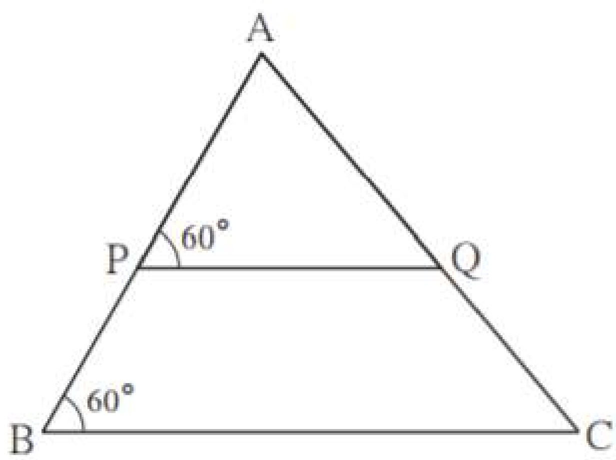


Fig. 1.38

Solution:
∠ABC = 60°   [Given]
∠APQ = 60°   [Given]

Since the corresponding angles are equal, line PQ || BC.
In △ABC , PQ || BC.
∴ By basic proportionality theorem , AP/PB = AQ/QC
Hence proved.

5. In trapezium ABCD, side AB ||side PQ ||side DC, AP = 15, PD = 12, QC = 14, find BQ.

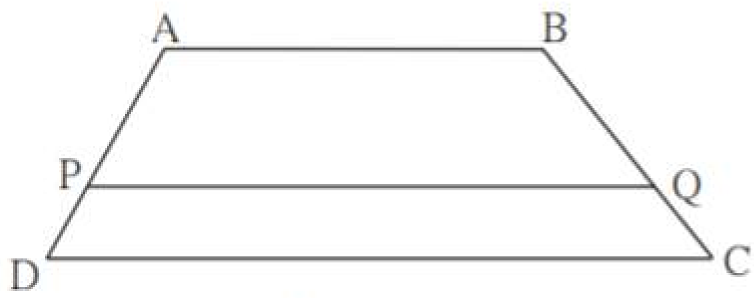


Fig. 1.39

Solution:
Given AB || PQ || DC.
AP = 15
PD = 12
QC = 14
AP/PD = BQ/QC [Property of three parallel lines and their transversals]
15/12 = BQ /14
⇒ BQ = 15×14/12
⇒ BQ = 17.5 units.
Hence measure of BQ is 17.5 units.

6. Find QP using given information in the figure.

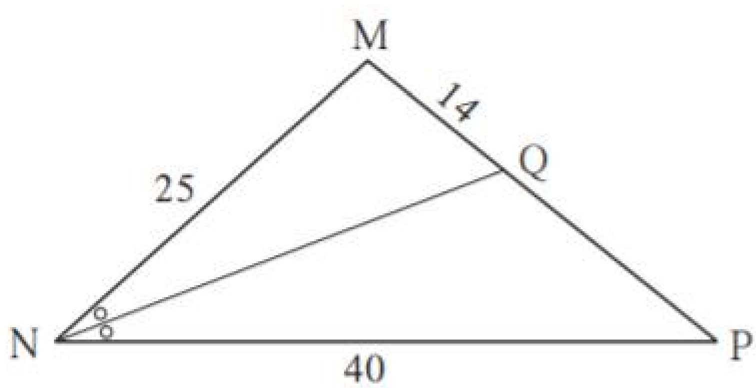


Fig. 1.40

Solution:
From figure MN = 25, NP = 40, MQ = 14
Given NQ bisects ∠MNP.

∴ MN/NP = MQ/QP [Angle bisector theorem]
25/40 = 14/QP
⇒ QP = 40×14/25
⇒ QP = 22.5
Hence measure of QP is 22.5 units.

7. In figure 1.41, if AB || CD || FE then find x and AE.

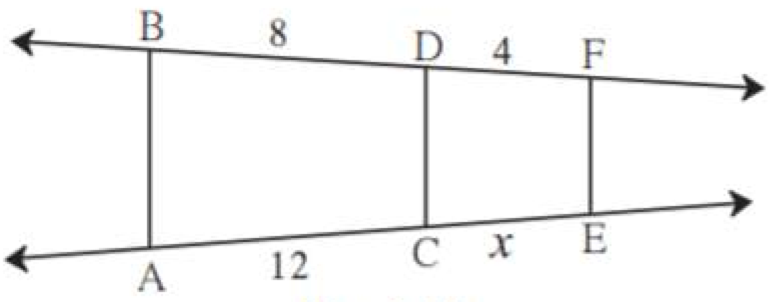


Fig. 1.41

Solution:
From figure BD = 8, DF = 4, AC = 12 and CE = x
Given AB ll CD ll FE
∴ BD/DF = AC/CE [Property of three parallel lines and their transversals]
8/4 = 12/x
x = 12×4/8
⇒ x = 6
∴CE = 6
AE = AC+CE
∴ AE = 12+6
∴ AE = 18
Hence measure of x is 6 units and AE is 18 units.

8. In D LMN, ray MT bisects ∠ LMN. If LM = 6, MN = 10, TN = 8, then find LT.

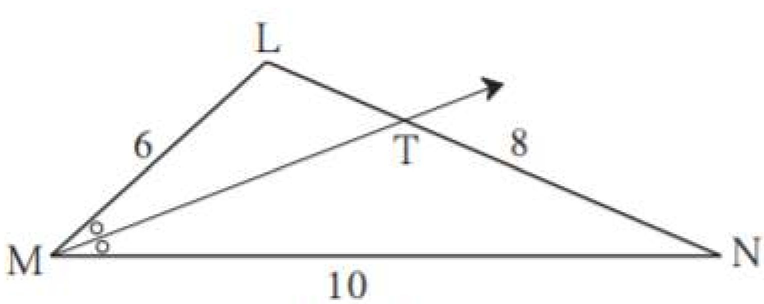


Fig. 1.42

Solution:
Given ray MT bisects ∠LMN.

LM = 6
MN = 10
TN = 8
Since ray MT bisects ∠LMN , LM/MN = LT/TN [Angle bisector theorem]
6/10 = LT/8
⇒ LT = 6×8/10
⇒ LT = 4.8
Hence measure of LT is 4.8 units.

9. In △ABC, seg BD bisects ∠ ABC. If AB = x, BC = x+5, AD = x-2, DC = x+2, then find the value of x.

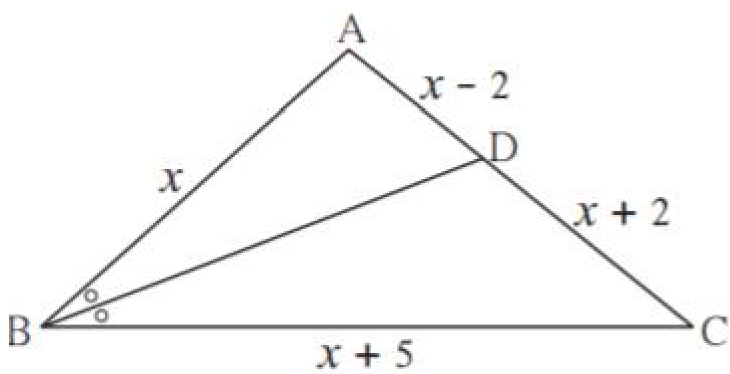


Fig. 1.43

Solution:
Given BD bisects ∠ABC.
Also AB = x, BC = x+5
AD = x-2, DC = x+2
Since BD bisects ∠ABC, AB/BC = AD/DC [Angle bisector theorem]
x/(x+5) = (x-2)/(x+2)
Cross multiplying, we get
x(x+2) = (x+5)(x-2)
$x^2+2x = x^2+5x-2x-10$
$x^2+2x = x^2+3x-10$
x = 10
∴ the value of x is 10 .

10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ || seg DE, seg QR || seg EF. Fill in the blanks to prove that, seg PR || seg DF.

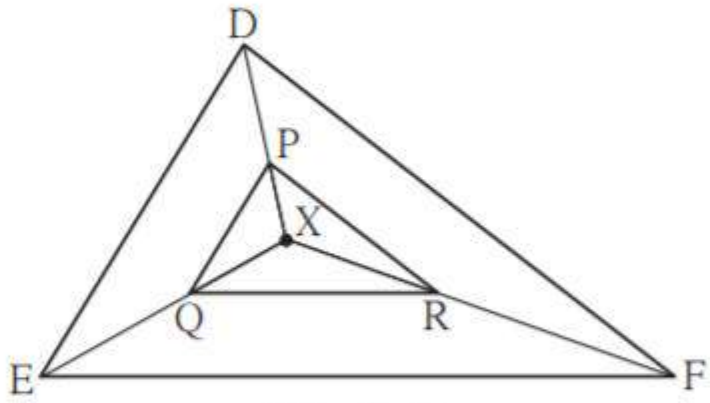


Fig. 1.44

Proof : In △ XDE, PQ || DE ...._______

∴ XP/________ = _______/ QE .... (I) (Basic proportionality theorem)

In △ XEF, QR || EF .... ________

∴ _______ /_______= _______ /_______ ....(II) _______

∴ _______ /_______= _______ /_______ .... from (I) and (II)

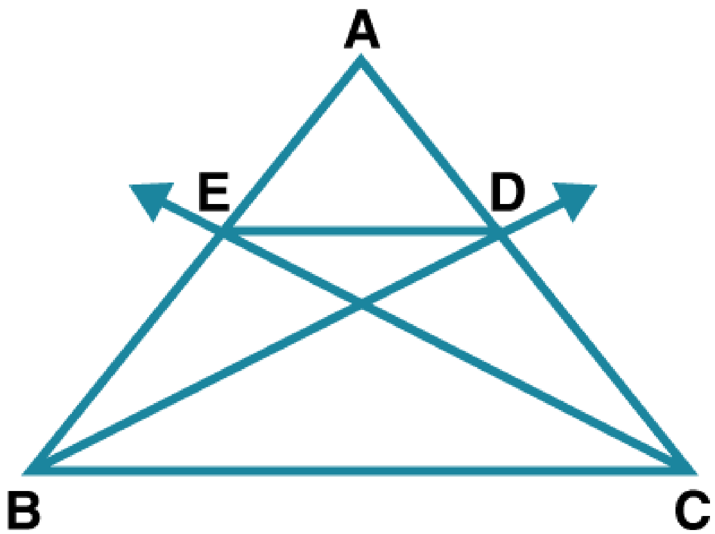
seg PR || seg DE .... (converse of basic proportionality theorem)

Solution:

In △ XDE, PQ || DE……. Given
∴XP/PD = XQ/QE…….. (I) (Basic proportionality theorem)
In △ XEF, QR || EF……. Given
∴XR/RF = XQ/QE……. .(II) (Basic proportionality theorem)
∴ XP/PD = XR/RF .... from (I) and (II)
∴ seg PR || seg DE .... (converse of basic proportionality theorem)

11*. In △ ABC, ray BD bisects ∠ABC and ray CE bisects ∠ACB. If seg AB ≅ seg AC then prove that ED || BC.

Solution:



Given , In $\triangle ABC$ ray BD bisects $\angle ABC$.
$\therefore$ AB/BC = AD/CD ………..(i) [Angle bisector theorem]
Since ray CE bisects $\angle ACB$
AC/BC = AE/BE …………….(ii) [Angle bisector theorem]
Given seg AB = seg AC.
Substitute AB in (ii)
AB/BC = AE/BE…..(iii)
From (i) AD/CD = AE/BE [in (i) AB/BC = AD/CD]
$\therefore$ ED $\parallel$ BC [converse of basic proportionality theorem]
Hence proved.

Practice set 1.3 Page 21

1. In figure1.55, ∠ ABC = 75°, ∠ EDC = 75° state which two triangles are similar and by which test? Also write the similarity of these two triangles by a proper one to one correspondence.

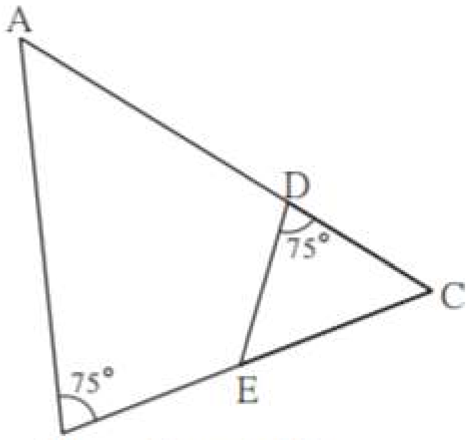


Fig. 1.55

Solution:

Given ∠ABC = 75˚ , ∠EDC = 75˚
Consider △ABC and △EDC
∠ABC = ∠EDC [Given ∠ABC = 75˚ , ∠EDC = 75˚]
∠ACB = ∠DCE [Common angle]
∴ △ABC ~ △EDC [AA test of similarity]
One to one correspondence is ABC↔ EDC

2. Are the triangles in figure 1.56 similar? If yes, by which test ?

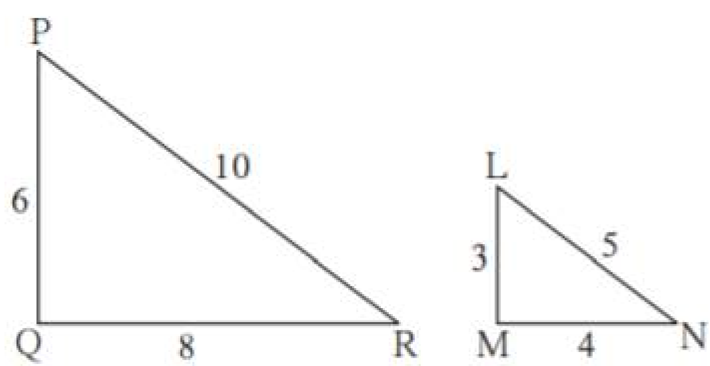


Fig. 1.56

Solution:

Consider △PQR and △LMN ,
PQ/LM = 6/3 = 2/1 …………(i)

QR/MN = 8/4 = 2/1 ……….(ii)
PR/LN = 10/5 = 2/1 ……….(iii)
∴ PQ/LM = QR/MN = PR/LN
∴ △PQR ~ △LMN [SSS test of similarity]

3. As shown in figure 1.57, two poles of height 8 m and 4 m are perpendicular to the ground. If the length of shadow of smaller pole due to sunlight is 6 m then how long will be the shadow of the bigger pole at the same time ?

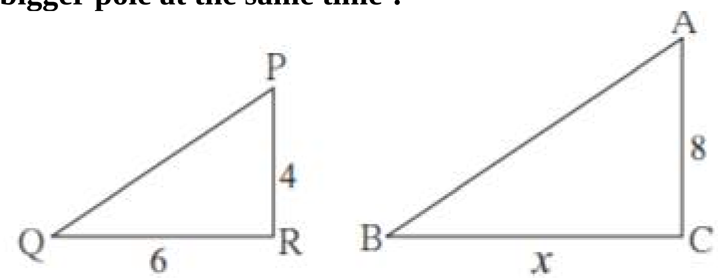


Fig. 1.57

Solution:
Here PR and AC represents the smaller and bigger poles, and QR and BC represents their shadows respectively.
Given PR = 4m, QR = 6m , AC = 8m, BC = x
△PRQ ~ △ACB [∵ Vertical poles and their shadows form similar figures]
∴PR/AC = QR/BC [Corresponding sides of similar triangles]
4/8 = 6/x
⇒ x = 6×8/4
⇒ x = 12
Hence the length of shadow of the bigger pole is 12 m.

4. In △ ABC, AP ⊥ BC, BQ ⊥ AC B- P-C, A-Q - C then prove that, △ CPA ~ △ CQB. If AP = 7, BQ = 8, BC = 12 then find AC.

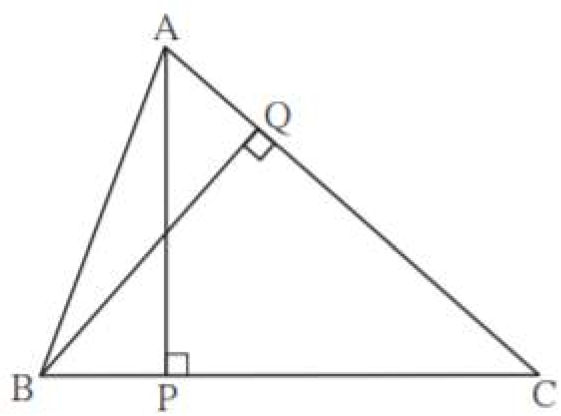


Fig. 1.58

Solution:
Consider $\triangle CPA$ and $\triangle CQB$,
$\angle CPA \cong \angle CQB$ [From figure, angle is equal to 90˚]
$\angle PCA \cong \angle QCB$ [Common angle]
$\therefore \triangle CPA \sim \triangle CQB$, [AA test of similarity]
Hence proved.
AC/BC = AP/BQ [corresponding sides of similar triangles]
Given AP = 7, BQ = 8, BC = 12
AC/12 = 7/8
$\Rightarrow$ AC = 12×7/8
$\Rightarrow$ AC = 10.5
Hence measure of AC is 10.5 units.

5. Given : In trapezium PQRS, side PQ || side SR, AR = 5AP, AS = 5AQ then prove that, SR = 5PQ

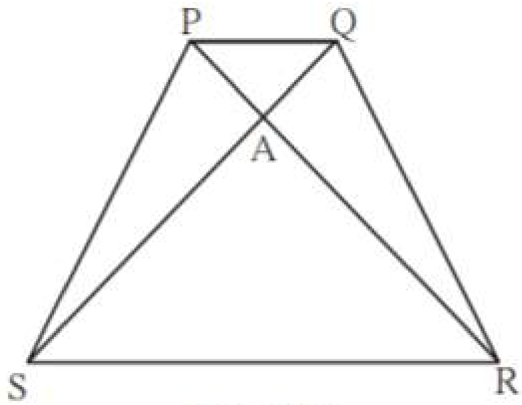


Fig. 1.59

Solution:
Given side PQ ∥ side SR.
Also AR = 5AP, AS = 5AQ
SQ is the transversal of parallel sides PQ and SR.
$\angle QSR = \angle PQS$ [Alternate interior angles]
$\angle ASR = \angle AQP$….(i) [Alternate interior angles]
Consider $\triangle ASR$ and $\triangle AQP$
$\angle ASR = \angle AQP$ From (i)
$\angle SAR = \angle QAP$ [vertical opposite angles]
$\triangle ASR \sim \triangle AQP$ [AA test of similarity]
AS/AQ = SR/PQ [Corresponding sides of similar triangles]
AS = 5AQ [Given]
AS/AQ = 5/1
SR/PQ = 5/1
$\therefore$ SR = 5PQ
Hence proved.

Practice set 1.4 Page 25

1. The ratio of corresponding sides of similar triangles is 3:5; then find the ratio of their areas

Solution:
When two triangles are similar, the ratio of areas of those triangles is equal to the ratio of the squares of their corresponding sides.
Given , the ratio of corresponding sides of the triangle is 3:5.
$\therefore$ Ratio of their areas $= 3^2/5^2$ [Theorem of areas of similar triangles]
$= 9/25$
Hence ratio of their areas = 9:25

2. If △ABC ~ △PQR and AB: PQ = 2:3, then fill in the blanks.
A(△ABC)/ A(△PQR) = AB^2/____ = $2^2/3^2$ = ____/____

Solution:
A(△ABC)/ A(△PQR) = AB^2/PQ^2
$= 2^2/3^2 = 4/9$ [Theorem of areas of similar triangles]

3. If △ ABC ~ △ PQR, A(△ ABC) = 80, A(△ PQR) = 125, then fill in the blanks.
A(△ ABC) /A(△…) = 80/125 ∴AB/PQ = ____/____

Solution:
Given A(△ ABC) = 80, A(△ PQR) = 125
(△ ABC) / A(△ PQR) = 80/125 = 16/25
(△ ABC) / A(△ PQR) = AB^2/PQ^2 [Theorem of areas of similar triangles]
$\therefore$ $AB^2/PQ^2 = 16/25$
Taking square root on both sides
$\therefore$ AB/PQ = 4/5
Hence AB/PQ = 4/5

4. △ LMN ~ △ PQR, 9 ×A(△PQR) = 16 ×A(△LMN). If QR = 20 then find MN.

Solution:
Given 9 ×A(△PQR) = 16 ×A(△LMN)
$\therefore$ A(△PQR)/ A(△LMN) = 16/9…………(i)
△ LMN ~ △ PQR
$\therefore$ A(△PQR)/ A(△LMN) = QR^2/MN^2 …..(ii)
From (i) and (ii)
$QR^2/MN^2 = 16/9$
Given QR = 20
$\therefore 20^2/MN^2 = 16/9$
Taking square root on both sides

20/MN = 4/3
MN = 20×3/4
MN = 15
Hence the measure of MN is 15 units.

Problem Set 1

1. Select the appropriate alternative.
(1) In △ ABC and △ PQR, in a one to one correspondence AB/QR = BC/ PR = CA/ PQ then
(A) △ PQR ~ △ ABC
(B) △ PQR ~ △ CAB
(C) △ CBA ~ △ PQR
(D) △ BCA ~ △ PQR

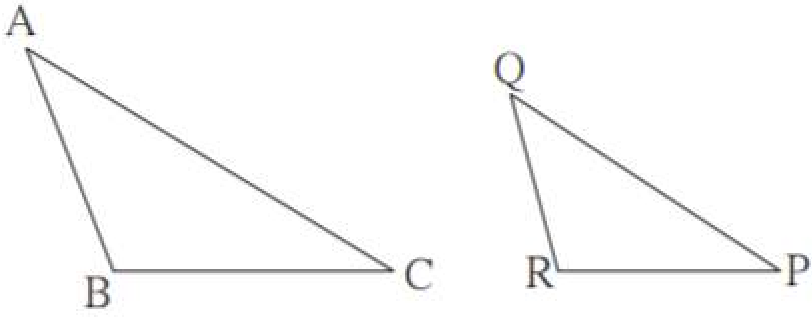


Fig. 1.67

Solution:
Given AB/QR = BC/ PR = CA/ PQ
By SSS test of similarity , △ PQR ~ △ CAB **.**
∴Correct option is (B).

(2) If in △ DEF and △ PQR, ∠ D ≅ ∠ Q, ∠ R ≅ ∠ E then which of the following statements is false?
(A) EF/PR = DF/ PQ
(B) DE/ PQ = EF/ RP
(C) DE/ QR = DF /PQ
(D) EF/ RP = DE/ QR

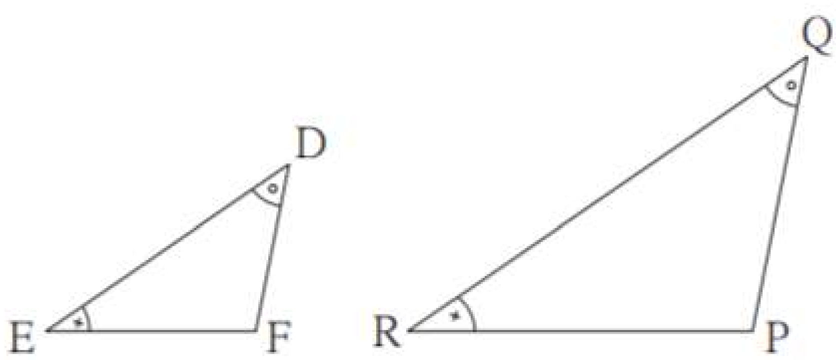


Fig. 1.68

Solution:

Given $\angle D \cong \angle Q$, $\angle R \cong \angle E$
$\therefore \triangle DEF \sim \triangle QRP$…. [AA test of similarity]
$\therefore DE/QR = EF/RP = DF/QP$ [Corresponding sides of similar triangles]
$\therefore$ $DE/PQ \neq EF/RP$
Hence option (B) is false.

(3) In $\triangle$ ABC and $\triangle$ DEF $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3DE then which of the statements regarding the two triangles is true?
(A)The triangles are not congruent and not similar
(B)The triangles are similar but not congruent.
(C)The triangles are congruent and similar.
(D) None of the statements above is true.

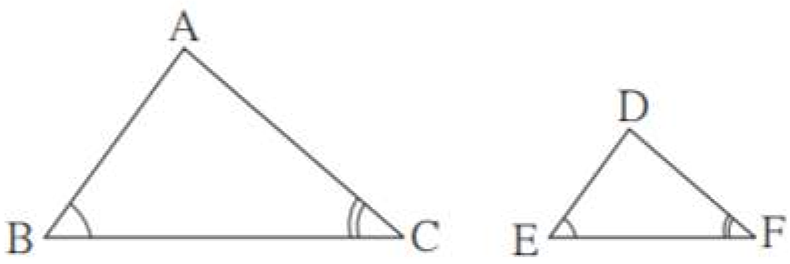


Fig. 1.69

Solution:
Given $\angle B = \angle E$
$\angle F = \angle C$
$\therefore \triangle ABC \sim \triangle DEF$ [AA test of similarity]
Hence option B is the true statement.

(4) $\triangle$ ABC and $\triangle$ DEF are equilateral triangles, A ($\triangle$ABC):A($\triangle$DEF)=1:2
If AB = 4 then what is length of DE?
(A)$2\sqrt{2}$
(B)4
(C)8
(D)$4\sqrt{2}$

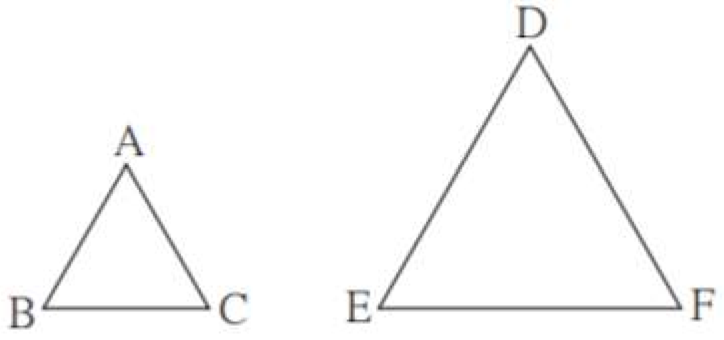


Fig. 1.70

Solution:
Given A (△ABC):A(△DEF) = 1:2
△ABC and △DEF are equilateral triangles.
∠A = ∠D [Angle equals 60˚]
∠B = ∠E [Angle equals 60˚]
∴△ABC ~ △DEF [AA test of similarity]
∴ A (△ABC):A(△DEF) = AB^2/DE^2 [Theorem of areas of similar triangles]
$1/2 = 4^2/DE^2$
Taking square root on both sides
$1/\sqrt{2} = 4/DE$
∴DE = $4\sqrt{2}$
Hence option (D) is the correct answer.

(5) In figure 1.71, seg XY || seg BC, then which of the following statements is true?
(A) AB / AC = AX / AY
(B) AX / XB = AY / AC
(C) AX / YC = AY / XB
(D) AB / YC = AC / XB

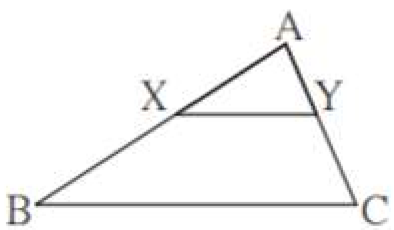


Fig. 1.71

Solution:
Given seg XY || seg BC
AX/BX = AY/YC [Basic proportionality theorem]

⇒ (BX/AX) +1 = (YC/AY) +1
⇒ (BX+AX)/AX = (YC+AY)/AY
⇒ AB/AX = AC/AY
⇒AB/AC = AX/AY
Hence correct option is (A).

2. In △ ABC, B - D – C and BD = 7, BC = 20 then find following ratios.
(1) A(△ ABD) /A(△ ADC)
(2) A(△ ABD) /A(△ ABC)
(3) A(△ ADC) /A(△ ABC)

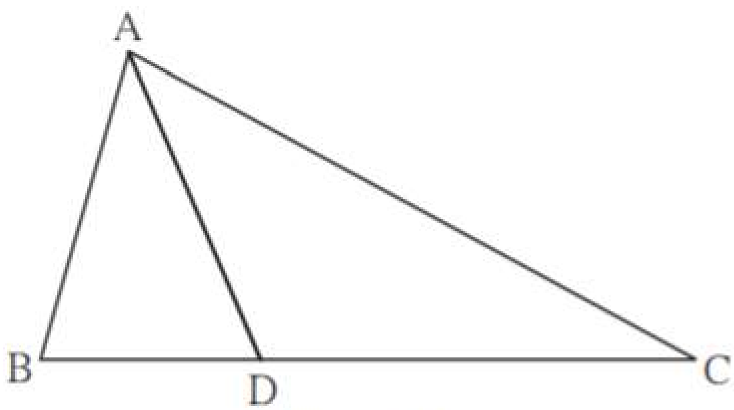
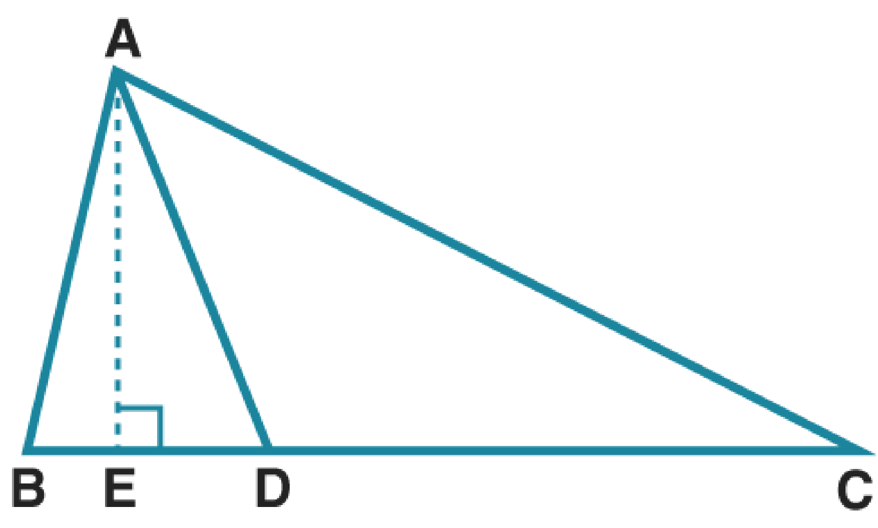


Fig. 1.72

Solution:



Given BD = 7, BC = 20
Construction:
Draw a perpendicular from A to BC meeting at E.

BC = BD+DC
20 = 7+DC
$\therefore$DC = 13

(1)A($\triangle$ ABD) /A($\triangle$ ADC) = BD/DC [Triangles having same height]
$\therefore$ A($\triangle$ ABD) /A($\triangle$ ADC) = 7/13

(2) A($\triangle$ ABD) /A($\triangle$ ABC) = BD/BC [Triangles having same height]
$\therefore$ A($\triangle$ ABD) /A($\triangle$ ABC) = 7/20

(3) A($\triangle$ ADC) /A($\triangle$ ABC) = DC/BC [Triangles having same height]
$\therefore$ A($\triangle$ ADC) /A($\triangle$ ABC) = 13/20

3. Ratio of areas of two triangles with equal heights is 2:3. If the base of the smaller triangle is 6cm then what is the corresponding base of the bigger triangle ?

Solution:
Given ratio of two triangles with equal height is 2:3
Let b_1 be base of smaller triangle and b_2 be base of bigger triangle.
$b_1 = 6$ cm
Let a_1 and a_2 be areas of the triangles.
Since triangles have equal height , $a_1/a_2 = b_1/b_2$
$\therefore 2/3 = 6/b_2$
$\therefore b_2 = 3\times6/2$
$\therefore b_2 = 9$
Hence base of bigger triangle is 9 cm.

4. In figure 1.73, ∠ABC = ∠DCB = 90° AB = 6, DC = 8 then A(△ ABC) /A(△ DCB) = ?

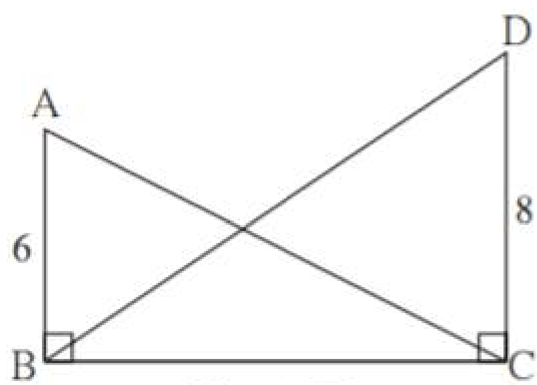


Fig. 1.73

Solution:
Given ∠ABC = ∠DCB = 90° AB = 6, DC = 8
BC is the common base of $\triangle$ ABC and $\triangle$ DCB

$\therefore A(\triangle ABC)/A(\triangle DCB) = AB/DC$
$= 6/8$
$= 3/4$

5. In figure 1.74, PM = 10 cm A(△PQS) = 100 sq.cm A(△QRS) = 110 sq.cm then find NR.

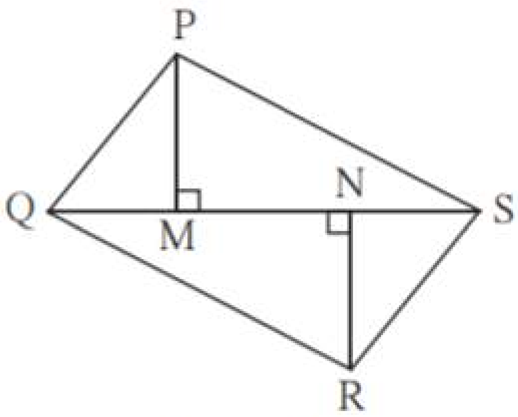
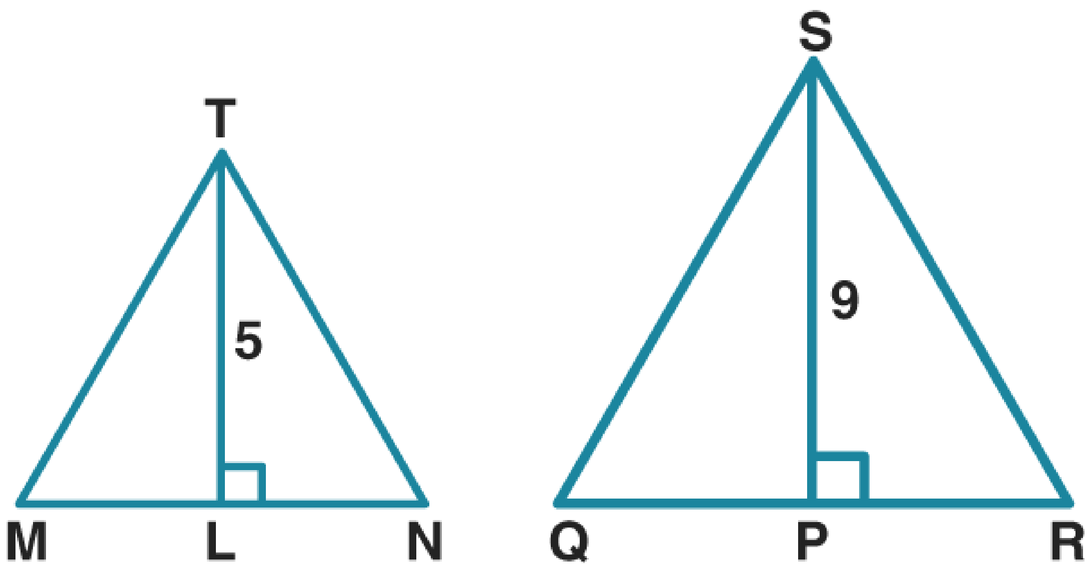


Fig. 1.74

Solution:
Given PM = 10 cm
A(△PQS) = 100 sq.cm
A(△QRS) = 110 sq.cm
△PQS and △QRS have common base QS
$\therefore$ A(△PQS)/ A(△QRS) = PM/NR
$\therefore$ 100/110= 10/NR
$\Rightarrow$ NR = 110×10/100
$\Rightarrow$ NR = 11
Hence NR = 11 cm.

6. △ MNT ~ △ QRS. Length of altitude drawn from point T is 5 and length of altitude drawn from point S is 9. Find the ratio A(△ MNT) /A(△ QRS) .

Solution:



Given $\triangle MNT \sim \triangle QRS$

$\therefore \angle TMN \cong \angle SQR$ [corresponding angles of similar triangles]

Construction:

Draw altitude from T to MN meeting at L.

Draw altitude from S to QR meeting at P.

$\angle TLM = \angle SPQ = 90^\circ$

In $\triangle MLT$ and $\triangle QPS$

$\angle TMN \cong \angle SQR$

$\angle TLM \cong \angle SPQ$

$\therefore \triangle MLT \sim \triangle QPS$ [AA test of similarity]

$\therefore MT/QS = TL/SP$

$\therefore MT/QS = 5/9$

$\triangle MNT \sim \triangle QRS$ [Given]

$\therefore A(\triangle MNT)/A(\triangle QRS) = MT^2/QS^2$ [Theorem of areas of similar triangles]

$\therefore A(\triangle MNT)/A(\triangle QRS) = 5^2/9^2$

$\therefore A(\triangle MNT)/A(\triangle QRS) = 25/81$

Hence $A(\triangle MNT):A(\triangle QRS) = 25:81$

7. In figure 1.75, A – D – C and B – E – C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

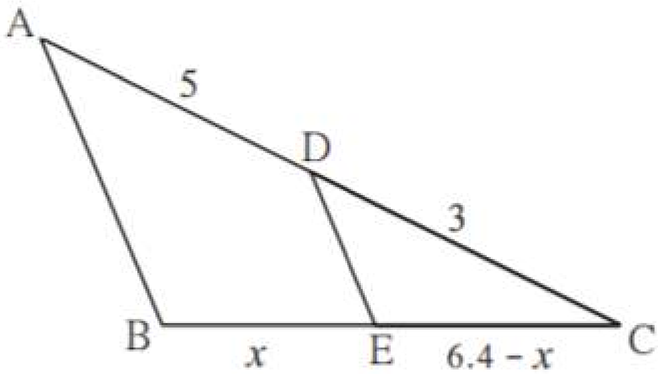


Fig. 1.75

Solution:

Given DE ∥ AB.

$\therefore$ AD/DC = BE/EC [Basic proportionality theorem]

AD = 5, DC = 3, BC = 6.4 [Given]

BE = x , EC = 6.4-x [Given]

$\therefore$ 5/3 = x/(6.4-x)

Cross multiplying we get

5×(6.4-x) = 3× x

32-5x = 3x

32 = 8x

$\therefore$ x = 32/8 = 4

Hence BE = 4 units.