

Practice set 2.1

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1. Identify, with reason, which of the following are Pythagorean triplets.

(i)(3, 5, 4)

(ii)(4, 9, 12)

(iii)(5, 12, 13)

(iv) (24, 70, 74)

(v)(10, 24, 27)

(vi)(11, 60, 61)

Solution:

(i)(3, 5, 4)

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$\text{Here } 9+16 = 25$$

$$\therefore 5^2 = 3^2+4^2$$

The square of largest number is equal to sum of squares of the other two numbers.

\therefore (3, 5, 4) is a Pythagorean triplet.

(ii)(4, 9, 12)

$$4^2 = 16$$

$$9^2 = 81$$

$$12^2 = 144$$

$$\text{Here } 4^2+9^2 \neq 12^2$$

The square of largest number is not equal to sum of squares of the other two numbers.

\therefore (4, 9, 12) is not a Pythagorean triplet.

(iii)(5, 12, 13)

$$5^2 = 25$$

$$12^2 = 144$$

$$13^2 = 169$$

$$\text{Here } 5^2+ 12^2 = 13^2$$

The square of largest number is equal to sum of squares of the other two numbers.

\therefore (5, 12, 13) is a Pythagorean triplet.

(iv) (24, 70, 74)

$$24^2 = 576$$

$$70^2 = 4900$$

$$74^2 = 5476$$

$$\text{Here } 24^2+ 70^2 = 74^2$$

The square of largest number is equal to sum of squares of the other two numbers.

\therefore (24, 70, 74) is a Pythagorean triplet.

(v)(10, 24, 27)

$$10^2 = 100$$

$$24^2 = 576$$

$$27^2 = 729$$

Here $10^2 + 24^2 \neq 27^2$

The square of largest number is not equal to sum of squares of the other two numbers.

\therefore (10, 24, 27) is not a Pythagorean triplet.

(vi)(11, 60, 61)

$$11^2 = 121$$

$$60^2 = 3600$$

$$61^2 = 3721$$

Here $11^2 + 60^2 = 61^2$

The square of largest number is equal to sum of squares of the other two numbers.

\therefore (11, 60, 61) is a Pythagorean triplet.

2. In figure 2.17, $\angle MNP = 90^\circ$, seg $NQ \perp$ seg MP , $MQ = 9$, $QP = 4$, find NQ .

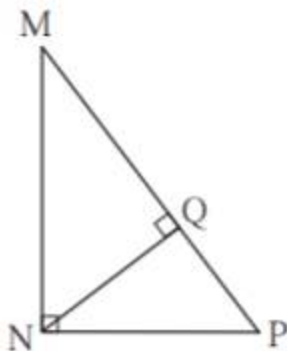


Fig. 2.17

Solution:

In $\triangle MNP$, $\angle MNP = 90^\circ$, seg $NQ \perp$ seg MP

$$MQ = 9, QP = 4$$

[Given]

$$NQ = \sqrt{(MQ \times QP)}$$

[Theorem of geometric mean]

$$\therefore NQ = \sqrt{(9 \times 4)} = \sqrt{36} = 6$$

Hence $NQ = 6$ units.

3. In figure 2.18, $\angle QPR = 90^\circ$, seg $PM \perp$ seg QR and $Q-M-R$, $PM = 10$, $QM = 8$, find QR .

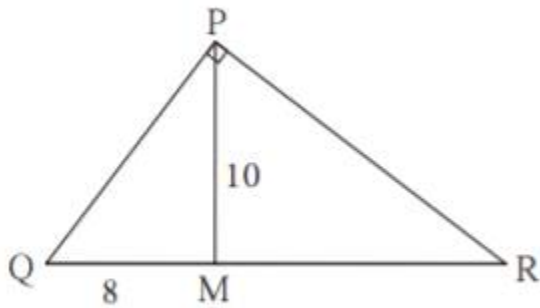


Fig. 2.18

Solution:

In $\triangle PQR$, $\angle QPR = 90^\circ$

seg $PM \perp$ seg QR [Given]

$PM^2 = QM \times MR$ [Theorem of geometric mean]

$$10^2 = 8 \times MR$$

$$\therefore MR = 100/8 = 12.5$$

$$\therefore QR = QM + MR$$

$$\therefore QR = 8 + 12.5 = 20.5$$

Hence measure of QR is 20.5 units.

4. See figure 2.19. Find RP and PS using the information given in $\triangle PSR$.

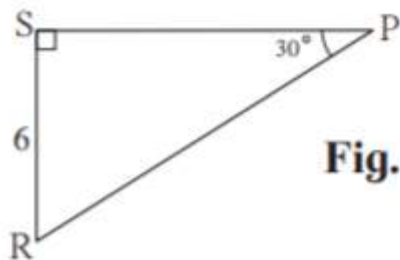


Fig. 2.19

Solution:

In $\triangle PSR$, $\angle P = 30^\circ$, $\angle S = 90^\circ$

$\therefore \angle R = 180 - (90 + 30) = 60^\circ$ [Angle Sum property of triangle]

$\therefore \triangle PSR$ is a $30^\circ - 60^\circ - 90^\circ$ triangle.

$\therefore SR = (\frac{1}{2}) \times PR$ [side opposite to 30°]

$$6 = (\frac{1}{2}) \times PR$$

$$\Rightarrow PR = 12$$

$PS = \sqrt{PR^2 - SR^2}$ [Pythagoras theorem]

$$\Rightarrow PS = \sqrt{12^2 - 6^2}$$

$$\Rightarrow PS = \sqrt{144 - 36}$$

$$\Rightarrow PS = \sqrt{108}$$

$$\Rightarrow PS = 6\sqrt{3}$$

Hence $RP = 12$ units and $PS = 6\sqrt{3}$ units.

5. For finding AB and BC with the help of information given in figure 2.20, complete following activity.

$$\begin{aligned} AB &= BC \dots \underline{\hspace{2cm}} \\ \therefore \angle BAC &= \underline{\hspace{2cm}} \\ \therefore AB = BC &= \underline{\hspace{2cm}} \times AC \\ &= \underline{\hspace{2cm}} \times \sqrt{8} \\ &= \underline{\hspace{2cm}} \times 2\sqrt{2} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

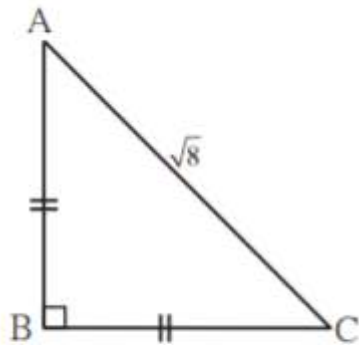


Fig. 2.20

Solution:

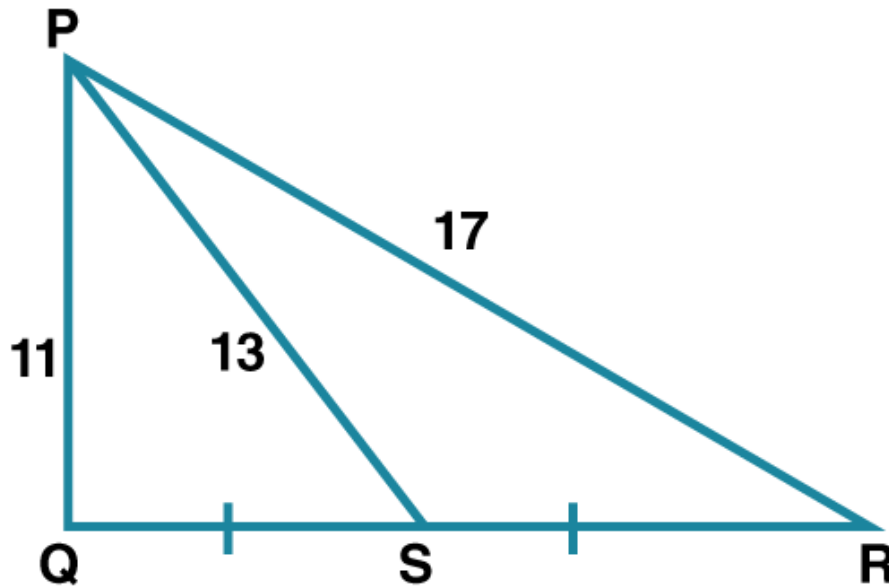
$$\begin{aligned} AB &= AC && \text{[Given]} \\ \therefore \angle BAC &= \angle BCA && \text{[Angles opposite to equal sides of an isosceles triangle are equal]} \\ \therefore AB = BC &= \frac{1}{\sqrt{2}} \times AC && \text{[By } 45^\circ - 45^\circ - 90^\circ \text{ theorem]} \\ &= \frac{1}{\sqrt{2}} \times \sqrt{8} \\ &= \frac{1}{\sqrt{2}} \times 2\sqrt{2} \\ &= 2 \end{aligned}$$

Practice set 2.2

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1. In $\triangle PQR$, point S is the midpoint of side QR. If $PQ = 11, PR = 17, PS = 13$, find QR.

Solution:



Given, S is the midpoint of QR.

\therefore PS is the median.

$\therefore PQ^2 + PR^2 = 2PS^2 + 2SR^2$ [By Apollonius theorem]

$\therefore 11^2 + 17^2 = 2 \times 13^2 + 2 \times SR^2$

$\therefore 121 + 289 = 2 \times 169 + 2 \times SR^2$

$\Rightarrow 2SR^2 = 121 + 289 - 338$

$\Rightarrow 2SR^2 = 72$

$\Rightarrow SR^2 = 72/2 = 36$

$\Rightarrow SR = 6$

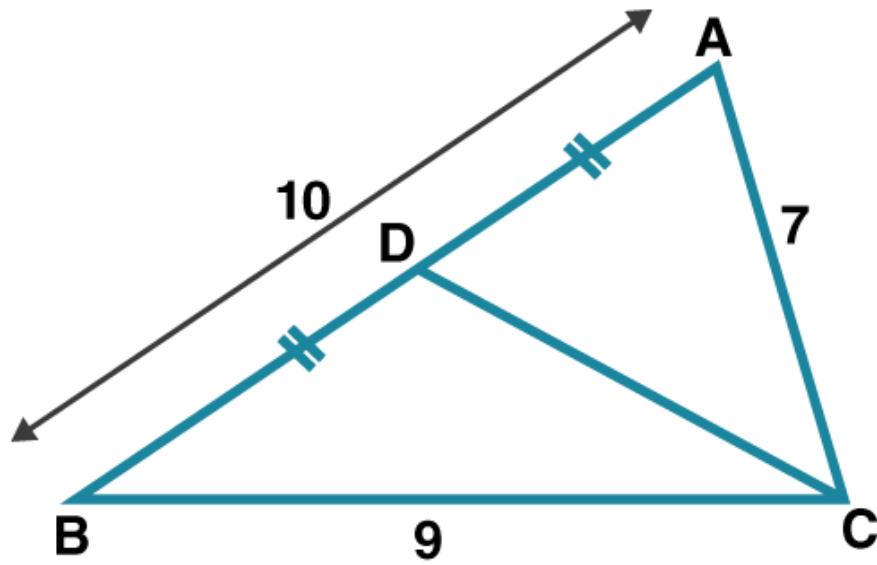
Since S is the midpoint of QR, $QR = 2SR$

$\therefore QR = 2 \times 6 = 12$

Hence QR = 12 units.

2. In $\triangle ABC$, $AB = 10, AC = 7, BC = 9$ then find the length of the median drawn from point C to side AB

Solution:



Let CD is the median drawn from C to AB.

Given AB = 10

AD = $(1/2) \times AB$ [D is the midpoint of side AB]

AD = $10/2 = 5$

Since CD is the median

$AC^2 + BC^2 = 2CD^2 + 2AD^2$ [Apollonius theorem]

$\therefore 7^2 + 9^2 = 2CD^2 + 2 \times 5^2$

$\Rightarrow 2CD^2 = 7^2 + 9^2 - 2 \times 5^2$

$\Rightarrow 2CD^2 = 80$

$\Rightarrow CD^2 = 40$

Taking square roots on both sides

CD = $2\sqrt{10}$

Hence the length of median drawn from point C to side AB is $2\sqrt{10}$ units.

3. In the figure 2.28 seg PS is the median of $\triangle PQR$ and $PT \perp QR$. Prove that,

(i) $PR^2 = PS^2 + QR \times ST + (QR/2)^2$

(ii) $PQ^2 = PS^2 - QR \times ST + (QR/2)^2$

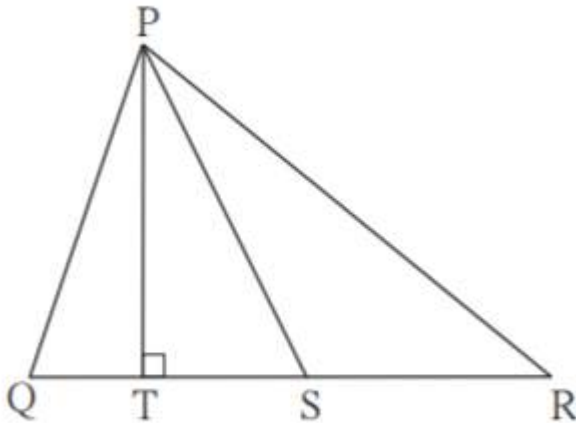
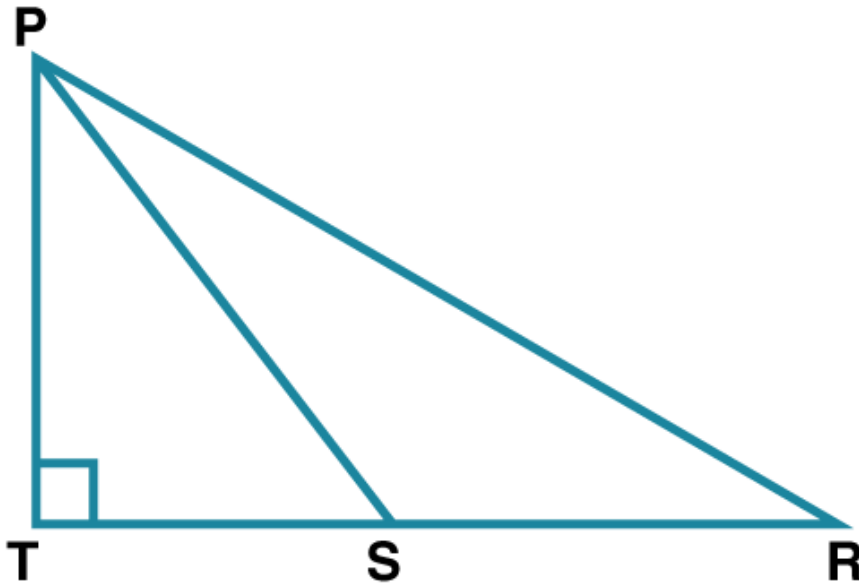


Fig. 2.28

Solution:



(1) $QS = \frac{1}{2} QR$ (i)

$SR = \frac{1}{2} QR$ (ii)

$\therefore QS = SR$

$PT \perp QR$

$\angle PSR$ is an obtuse angle.

$\therefore PR^2 = SR^2 + PS^2 + 2SR \times ST$ (iii)

Substitute $SR = \frac{1}{2} QR$ in (iii)

$\therefore PR^2 = \left[\frac{1}{2}QR\right]^2 + PS^2 + 2\left(\frac{1}{2}QR\right) \times ST$

$\therefore PR^2 = \left[\frac{1}{2}QR\right]^2 + PS^2 + QR \times ST$

$\therefore PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

Hence proved.

[S is the midpoint of QR]

[From (i) and (ii)]

[Given]

[From figure]

[Application of Pythagoras theorem]

(ii) $PT \perp QS$

[Given]



$\angle PSQ$ is an acute angle

[From figure]

$$\therefore PQ^2 = QS^2 + PS^2 - 2QS \times ST$$

[Application of Pythagoras theorem]

$$\therefore PR^2 = [(1/2)QR]^2 + PS^2 - 2(1/2)QR \times ST$$

$$\therefore PR^2 = (QR/2)^2 + PS^2 - QR \times ST$$

$$\therefore PR^2 = PS^2 - QR \times ST + (QR/2)^2$$

Hence proved.

Problem set 2

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1. Some questions and their alternative answers are given. Select the correct alternative.

(1) Out of the following which is the Pythagorean triplet?

(A) (1, 5, 10)

(B) (3, 4, 5)

(C) (2, 2, 2)

(D) (5, 5, 2)

Solution:

(A) (1, 5, 10)

Here $1^2 + 5^2 \neq 10^2$

The square of largest number is not equal to sum of squares of the other two numbers.

So (1, 5, 10) is not a Pythagorean triplet.

(B) (3, 4, 5)

Here $3^2 + 4^2 = 5^2$

The square of largest number is equal to sum of squares of the other two numbers.

So (3, 4, 5) is a Pythagorean triplet.

(C) (2, 2, 2)

Here $2^2 + 2^2 \neq 2^2$

The square of largest number is not equal to sum of squares of the other two numbers.

So (2, 2, 2) is not a Pythagorean triplet.

(D) (5, 5, 2)

Here $2^2 + 5^2 \neq 5^2$

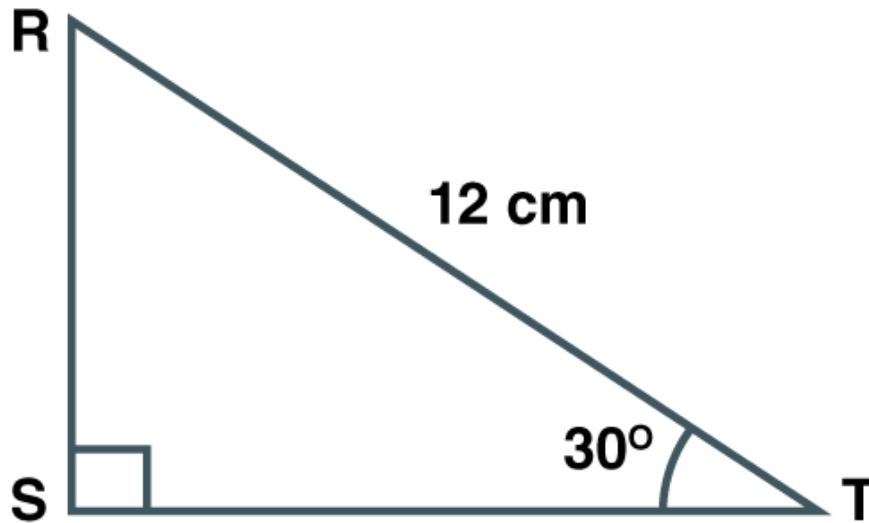
The square of largest number is not equal to sum of squares of the other two numbers.

So (5, 5, 2) is not a Pythagorean triplet.

Hence option B is the correct answer.

3. In $\triangle RST$, $\angle S = 90^\circ$, $\angle T = 30^\circ$, $RT = 12$ cm then find RS and ST .

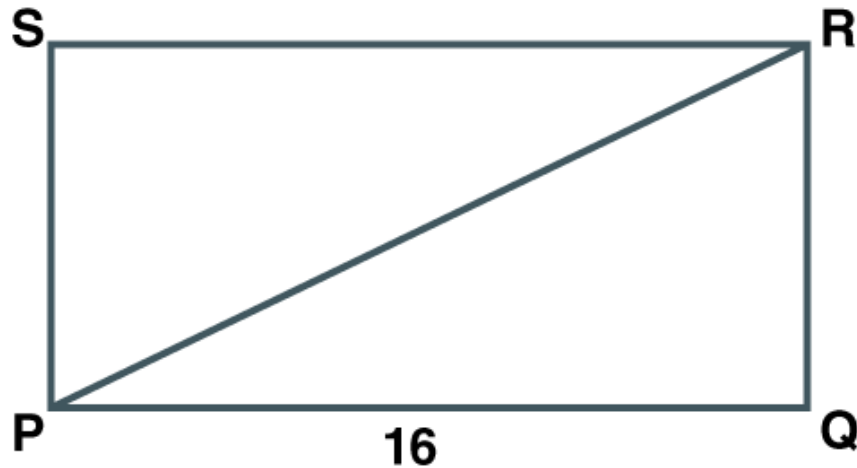
Solution:



Given $\angle S = 90^\circ$, $\angle T = 30^\circ$
 $\therefore \angle R = 180 - (90 + 30) = 60^\circ$ [Sum of angles of triangle is equal to 180°]
 $\triangle RST$ is a $30^\circ - 60^\circ - 90^\circ$ triangle
 $\therefore RS = \frac{1}{2} RT$ [Side opposite to 30°]
 $\Rightarrow RS = \frac{1}{2} \times 12 = 6$
 $ST = \frac{\sqrt{3}}{2} RT$ [side opposite to 60°]
 $\therefore ST = (\frac{\sqrt{3}}{2}) \times 12$
 $\Rightarrow ST = 6\sqrt{3}$
Hence $RS = 6$ cm and $ST = 6\sqrt{3}$ cm.

4. Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm.

Solution:



Let PQRS be the rectangle.

Let length be $PQ = 16$ cm

Area of rectangle = Length \times Breadth

Area of rectangle PQRS = $PQ \times QR$

$$\therefore 192 = 16 \times QR$$

$$\Rightarrow QR = 192/16 = 12 \text{ cm}$$

Now in $\triangle PQR$, $\angle Q = 90^\circ$ [Angles of a rectangle are 90°]

$$\therefore PR^2 = PQ^2 + QR^2 \quad [\text{Pythagoras theorem}]$$

$$\therefore PR^2 = 16^2 + 12^2$$

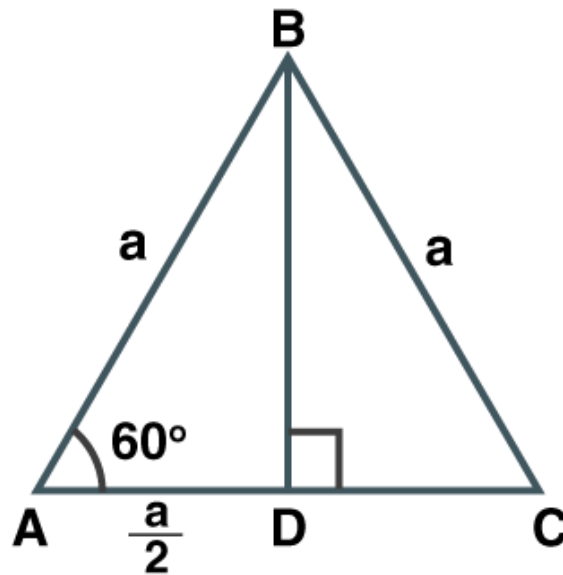
$$\therefore PR^2 = 256 + 144 = 400$$

$$\Rightarrow PR = 20$$

Hence the diagonal of the rectangle is 20cm long.

5*. Find the length of the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.

Solution:



Let ABC be the equilateral triangle with side a.

Let BD be height of the triangle.

Since $\triangle ABC$ is equilateral, BD is a perpendicular bisector.

$$\therefore AD = a/2$$

$$BD = \sqrt{3} \quad [\text{given height} = \sqrt{3}]$$

$$AB = a$$

Applying Pythagoras theorem in $\triangle ABD$

$$AB^2 = AD^2 + BD^2$$

$$\therefore a^2 = (a/2)^2 + (\sqrt{3})^2$$

$$a^2 = (a^2/4) + 3$$

$$(3/4)a^2 = 3$$

$$\Rightarrow a^2 = 4$$

$$\Rightarrow a = 2$$

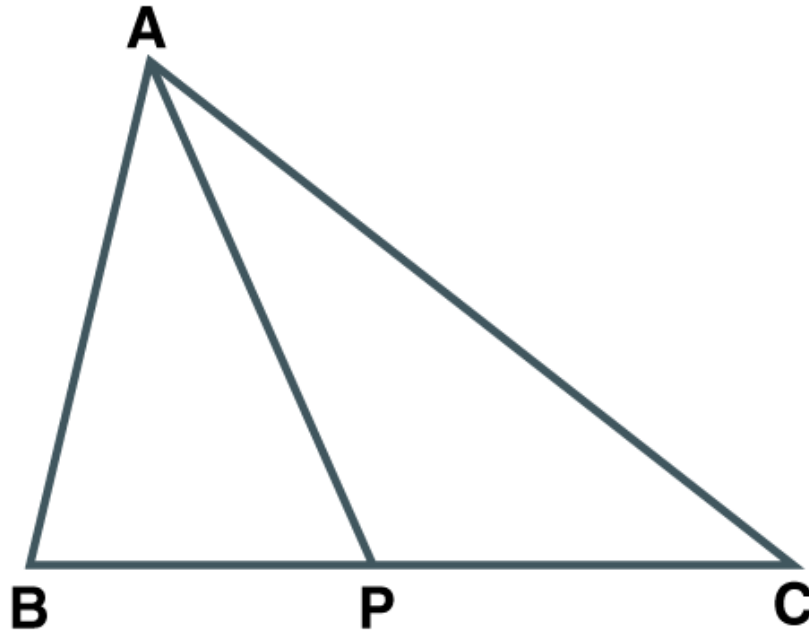
Hence length of side of equilateral triangle is 2 cm.

$$\therefore \text{Perimeter} = 3 \times 2 = 6 \quad [\text{Perimeter of equilateral triangle} = 3 \times \text{side}]$$

Hence perimeter of equilateral triangle is 6 cm.

6. In $\triangle ABC$ seg AP is a median. If $BC = 18$, $AB^2 + AC^2 = 260$ Find AP.

Solution:



Given AP is the median.

$$\therefore PC = BC/2$$

$$PC = 18/2 = 9$$

$$AB^2 + AC^2 = 2AP^2 + 2PC^2$$

[Apollonius theorem]

$$\therefore 260 = 2AP^2 + 2 \times 9^2$$

$$\Rightarrow 2AP^2 = 260 - 2 \times 9^2$$

$$\Rightarrow 2AP^2 = 260 - 162$$

$$\Rightarrow AP^2 = 68/2 = 49$$

Taking square roots on both sides

$$AP = 7$$

Hence AP = 7 units.

7*. $\triangle ABC$ is an equilateral triangle. Point P is on base BC such that $PC = 1/3 BC$, if $AB = 6$ cm find AP.

Solution:

Given $\triangle ABC$ is an equilateral triangle.

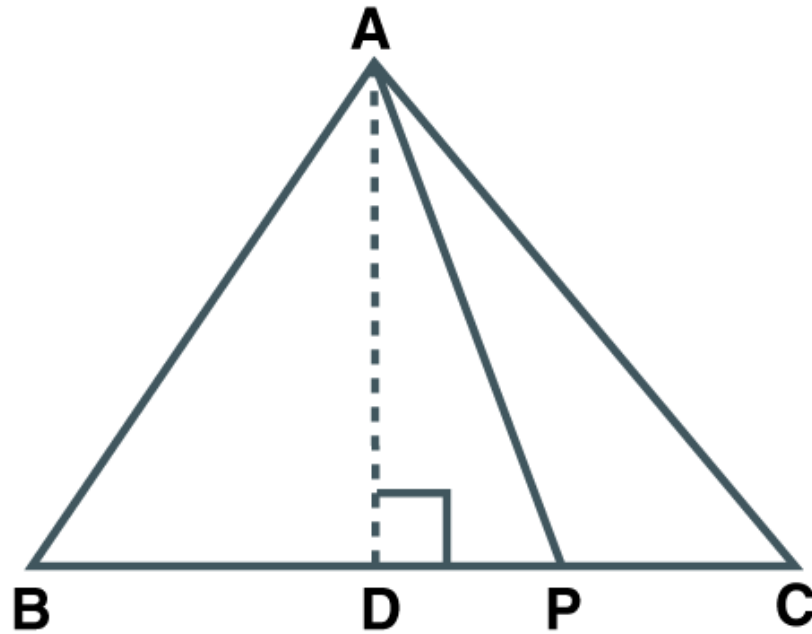
$$PC = (1/3) BC$$

$$\therefore PC = (1/3) \times 6 \quad [BC = 6, \text{ side of equilateral triangle}]$$

$$PC = 2$$

Construction:

Draw segment $AD \perp BC$



In $\triangle ADC$, $\angle C = 60^\circ$

$\angle D = 90^\circ$

$\angle CAD = 30^\circ$

$\triangle ADC$ is a $30^\circ - 60^\circ - 90^\circ$ triangle

$\therefore AD = (\sqrt{3}/2) \times AC$ [Side opposite to 60°]

$\Rightarrow AD = (\sqrt{3}/2) \times 6$

$\Rightarrow AD = 3\sqrt{3} \text{ cm}$

$DC = (1/2)BC$ [$AD \perp BC$]

$\therefore DC = (1/2) \times 6 = 3 \text{ cm}$

$DC = DP + PC$ [D-P-C]

$3 = DP + 2$

$\Rightarrow DP = 1$

In $\triangle ADP$, $\angle D = 90^\circ$

Applying Pythagoras theorem

$AP^2 = AD^2 + DP^2$

$\therefore AP^2 = (3\sqrt{3})^2 + 1^2$

$AP^2 = 28$

$\Rightarrow AP = 2\sqrt{7} \text{ cm}$

Hence $AP = 2\sqrt{7} \text{ cm}$.

8. From the information given in the figure 2.31, prove that $PM = PN = \sqrt{3} \times a$

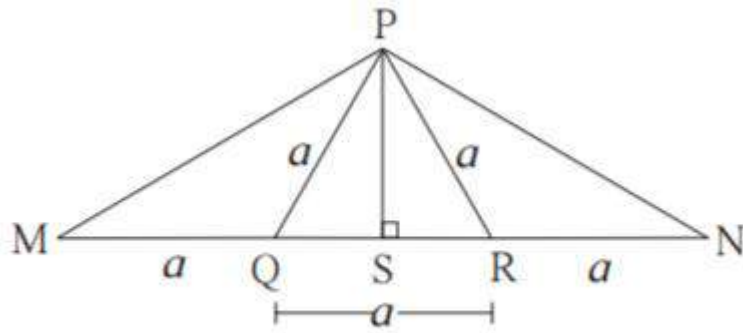
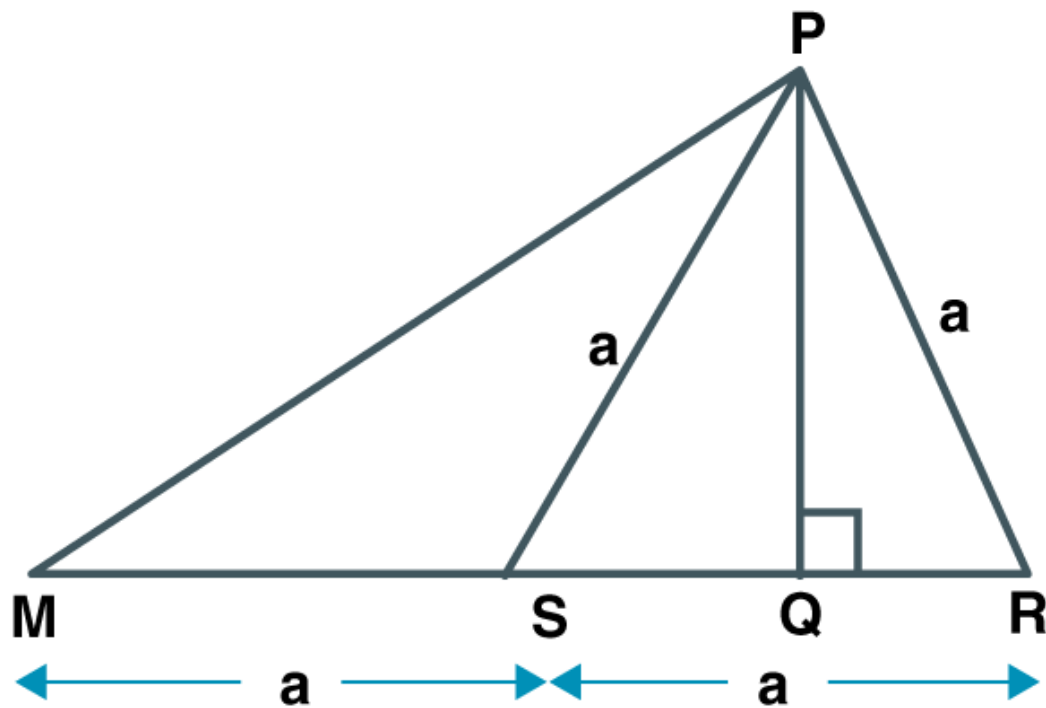


Fig. 2.31

Solution:

Proof:



In $\triangle PRM$

Given $MQ = QR = a$

$\therefore Q$ is the midpoint of MR .

$\therefore PQ$ is the median.

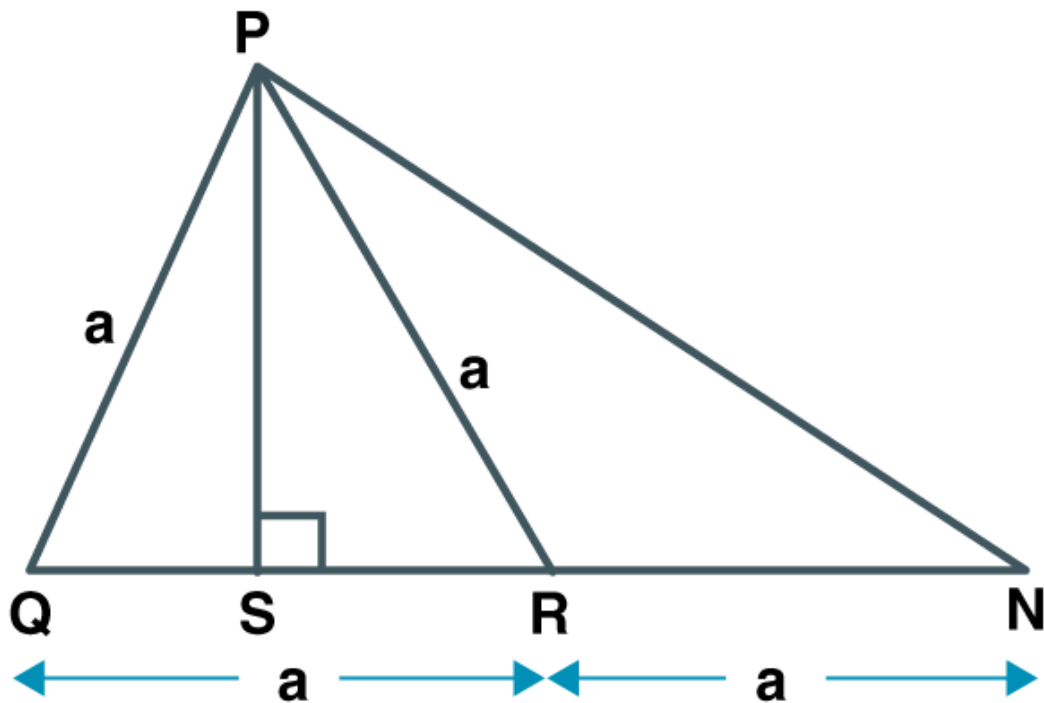
$\therefore PR^2 + PM^2 = 2PQ^2 + 2QM^2$

[Apollonius theorem]

$\therefore a^2 + PM^2 = 2a^2 + 2a^2$

$\Rightarrow PM^2 = 3a^2$

$\Rightarrow PM = \sqrt{3}a \dots \dots \dots (i)$



In $\triangle PQN$

Given $NR = QR = a$

$\therefore R$ is the midpoint of QN .

$\therefore PR$ is the median.

$\therefore PN^2 + PQ^2 = 2PR^2 + 2RN^2$ [Apollonius theorem]

$\therefore PN^2 + a^2 = 2a^2 + 2a^2$

$\Rightarrow PN^2 = 3a^2$

$\Rightarrow PN = \sqrt{3}a \dots \dots \dots (ii)$

From (i) and (ii) $PM = PN = \sqrt{3} \times a$

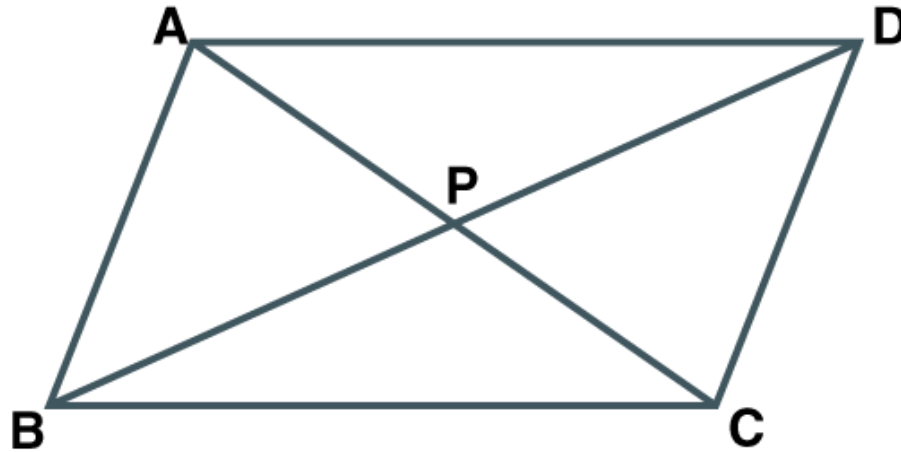
Hence proved.

9. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Solution:

Construction:

Draw a parallelogram $ABCD$. Let diagonals AC and BD meet at P .



To prove : $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

$AB = CD$ [Opposite sides of parallelogram are equal]

$BC = DA$ [Opposite sides of parallelogram are equal]

Since diagonals of a parallelogram bisect each other,

$AP = \frac{1}{2} AC$

$BP = \frac{1}{2} BD$

P is the midpoint of diagonals AC and BD.

In $\triangle ABC$, BP is the median.

$AB^2 + BC^2 = 2AP^2 + 2BP^2$ [Apollonius theorem]

$\therefore AB^2 + BC^2 = 2\left[\left(\frac{1}{2}AC\right)^2\right] + 2\left[\left(\frac{1}{2}BD\right)^2\right]$

$\therefore AB^2 + BC^2 = AC^2/2 + BD^2/2$

$2(AB^2 + BC^2) = AC^2 + BD^2$

$2AB^2 + 2BC^2 = AC^2 + BD^2$

$AB^2 + AB^2 + BC^2 + BC^2 = AC^2 + BD^2$

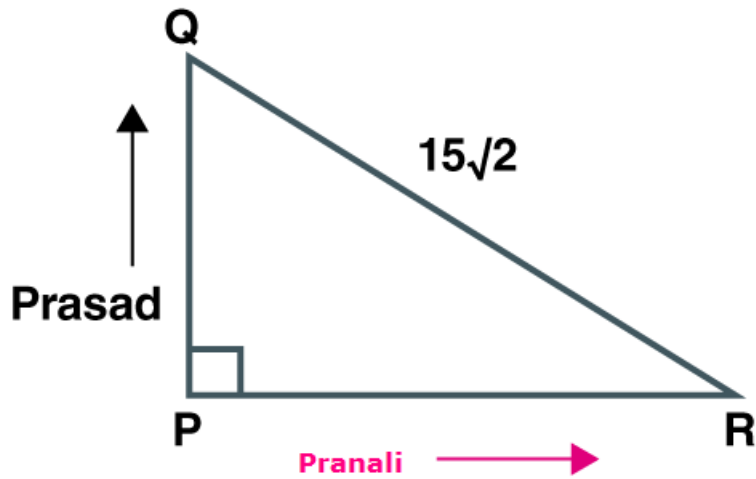
$\therefore AB^2 + CD^2 + BC^2 + DA^2 = AC^2 + BD^2$

$\therefore AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Hence proved.

10. Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was $15\sqrt{2}$ km. Find their speed per hour.

Solution:



Distance between Pranali and Prasad after 2 hours = $15\sqrt{2}$ km

Since they travel at same speed, they have covered same distance.

Construction: Draw a triangle PQR such that $PQ = PR = x$ and $QR = 15\sqrt{2}$

$\angle P = 90^\circ$

In $\triangle PQR$, $PQ^2 + PR^2 = QR^2$ [Pythagoras theorem]

$$\therefore x^2 + x^2 = (15\sqrt{2})^2$$

$$2x^2 = 2 \times 225$$

$$x^2 = 225$$

$$\Rightarrow x = 15$$

\therefore Distance covered by them is 15 km.

Given time = 2 hours

Speed = Distance / time

$$\text{Speed} = 15/2 = 7.5 \text{ km/hr}$$

\therefore Speed of Pranali and Prasad is 7.5 km/hr.