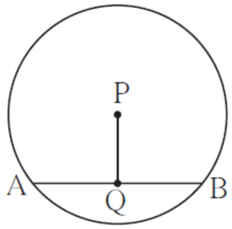
Practice Set 17.1

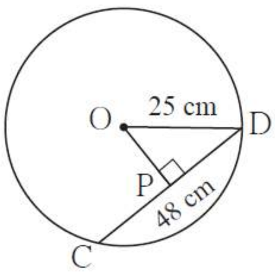
1. In a circle with centre P, chord AB is drawn of length 13 cm, seg PQ ⊥ chord AB, then find *l*(QB).



Solution:

Given,
seg PQ ⊥ chord AB and l(AB) = 13 cm
Now,
l(QB) = ½ l(AB
[As the perpendicular drawn from the centre of a circle to its chord bisects the chord]
⇒ l(QB) = ½ x 13
Thus, l(QB) = 6.5 cm

2. Radius of a circle with centre O is 25 cm. Find the distance of a chord from the centre if length of the chord is 48 cm.

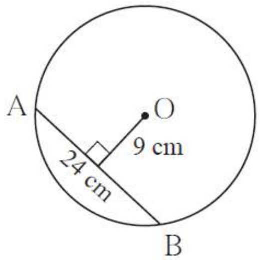


Solution:

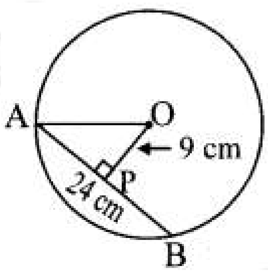
Given,
seg OP ⊥ chord CD and l(CD) = 48 cm
Radius of circle = 25 cm, so OD = 25 cm
Now,
l(PD) = ½ l(CD)
[Perpendicular drawn from the centre of a circle to its chord bisects the chord]
l(PD) = ½ x 48
⇒ l(PD) = 24 cm … (i)
In ΔOPD, we have
m ∠OPD = 90°
So, by Pythagoras theorem
$[l(OD)]^2 = [l(OP)]^2 + [l(PD)]^2$
$(25)^2 = [l(OP)]^2 + (24)^2$ [From (i)]
$(25)^2 – (24)^2 = [l(OP)]^2$
$(25 + 24) (25 – 24) = [l(OP)]^2$ [Since, $a^2 – b^2 = (a + b) (a – b)$]

$49 \times 1 = [l(OP)]^2$
$[l(OP)]^2 = 49$
$\therefore l(OP) = \sqrt{49} = 7$ cm [Taking square root of both sides]
Thus, the distance of the chord from the centre of the circle is 7 cm.

3. O is centre of the circle. Find the length of radius, if the chord of length 24 cm is at a distance of 9 cm from the centre of the circle.

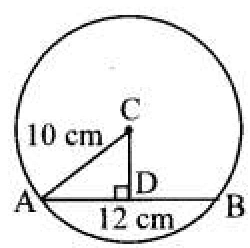


Solution:



Let seg OP ⊥ chord AB
l(AB) = 24 cm and l(OP) = 9 cm
Now,
l(AP) = ½ l(AB)
[Perpendicular drawn from the centre of a circle to its chord bisects the chord]
l(AP) = ½ x 24
⇒ l(AP) = 12 cm …(i)
In ΔOPA, we have
m ∠OPA = 90°
So, by Pythagoras theorem
$[l(AO)]^2 = [l(OP)]^2 + [l(AP)]^2$
$[l(AO)]^2 = (9)^2 + (12)^2$ [From (i)]
$= 81 + 144 = 225$
⇒ $l(AO) = \sqrt{225} = 15$ cm [Taking square root of both sides]
Thus, the length of radius of the circle is 15 cm.

4. C is the centre of the circle whose radius is 10 cm. Find the distance of the chord from the centre if the length of the chord is 12 cm.
Solution:



Let's consider a circle of radius 10 cm and centre C.
And, seg AB be the chord of the circle
Now, draw seg CD ⊥ chord AB, l(AB) = 12 cm and l(AC) = 10 cm.
l(AD) = ½ l(AB)
[Perpendicular drawn from the centre of a circle to its chord bisects the chord]
l(AD) = ½ x 12
⇒ l(AD) = 6 cm …(i)
So,

In ΔACD, m $\angle ADC = 90°$
By Pythagoras theorem,
$[l(AC)]^2 = [l(AD)]^2 + [l(CD)]^2$
$(10)^2 = (6)^2 + [l(CD)]^2$ [From (i) and l(AC) = 10 cm]
$(10)^2 - (6)^2 = [l(CD)]^2$
$100 - 36 = [l(CD)]^2$
$64 = [l(CD)]^2$
$\Rightarrow l(CD) = \sqrt{64} = 8$ cm [Taking square root of both sides]
Thus, the distance of the chord from the centre of the circle is 8 cm.

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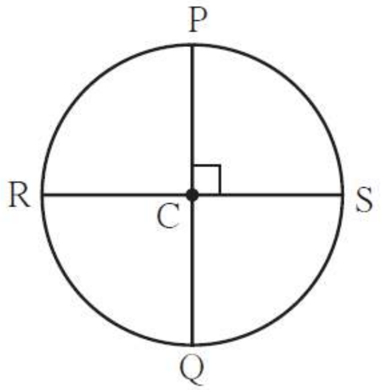
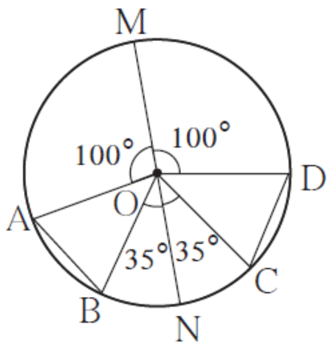
1. The diameters PQ and RS of the circle with centre C are perpendicular to each other at C. State, why arc PS and arc SQ are congruent. Write the other arcs which are congruent to arc PS.



Solution:

Given, diameter PQ ⊥ diameter RS
So, we have
m ∠PCS = m ∠SCQ = m ∠PCR = m ∠RCQ = 90°
We know that, the measure of the angle subtended at the centre by an arc is the measure of the arc.
So,
m (arc PS) = m ∠PCS = 90° … (i)
m (arc SQ) = m ∠SCQ = 90° … (ii)
From (i) and (ii), we see that
m (arc PS) = m (arc SQ)
Hence, arc PS ≅ arc SQ
[If the measures of two arcs of a circle are same, then the two arcs are congruent]
Similarly,
m (arc PR) = m ∠PCR = 90° … (iii)
m (arc RQ) = m ∠RCQ = 90° … (iv)
Hence, from (i), (iii) and (iv) we have
m (arc PS) = m (arc PR) = m (arc RQ)
∴ arc PS ≅ arc PR ≅ arc RQ
[If the measures of two arcs of a circle are same, then the two arcs are congruent]
Thus, arc PR and arc RQ are congruent to arc PS.

2. In the adjoining figure O is the centre of the circle whose diameter is MN. Measures of some central angles are given in the figure. Hence find the following
(1) *m* ∠ AOB and *m* ∠ COD
(2) Show that arc AB ≅ arc CD.
(3) Show that chord AB ≅ chord CD



Solution:

(1) Given,
Seg MN is the diameter of the circle and m ∠AOM = 100°, m ∠BON = 35°, m ∠DOM = 100° and m ∠CON = 35°
Now,
m ∠AOM + m ∠AON = 180° [Angles in a linear pair]
m ∠AOM + (m ∠AOB + m ∠BON) = 180° [Angle addition property]
100° + m ∠AOB + 35° = 180°
m ∠AOB + 135° = 180°
m ∠AOB = 180°- 135°
⇒ m ∠AOB = 45° … (i)
Also, we have
m ∠DOM + m ∠DON = 180° [Angles in a linear pair]
m ∠DOM + (m ∠COD + m ∠CON) = 180° [Angle addition property]
100° + m ∠COD + 35°= 180°
m ∠COD + 135° = 180°
m ∠COD = 180°- 135°
⇒ m ∠COD = 45° … (ii)

(2) Now, m (arc AB) = m ∠AOB = 45° [From (i)]
And, m (arc DC) = m ∠DOC = 45° [From (ii)]
So, from (i) and (ii)
m (arc AB) = m (arc DC)
Hence,
arc AB ≅ arc CD
[If the measures of two arcs of a circle are same, then the two arcs are congruent]

(3) We have, arc AB ≅ arc CD [Proved above]
Hence, as the chords corresponding to congruent arcs are congruent
chord AB ≅ chord CD.