

PRACTICE SET 7.1

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1. Write the following statements using the symbol of variation.

(1) Circumference (c) of a circle is directly proportional to its radius (r).

(2) Consumption of petrol (I) in a car and distance traveled by that car (D) are in direct variation.

Solution:

(1) Circumference = c and radius = r

So, $c \propto r$ or $c = kr$, where $k = \text{constant}$

(2) Consumption of petrol in a car = I

Distance traveled by that car = D

So, $I \propto D$ or $I = kD$, where $k = \text{constant}$

2. Complete the following table considering that the cost of apples and their number are in direct variation.

Number of apples (x)	1	4	...	12	...
Cost of apples (y)	8	32	56	...	160

Solution:

Number of apples (x) and the cost of apples (y) are in direct variation.

$y \propto x$

$y = kx$... (i) where k is constant of variation

Now let us consider the conditions,

When, $x = 1, y = 8$

Substitute the value of $x = 1$ and $y = 8$ in (i), we get

$8 = k \times 1$

$k = 8$

Substituting the value of $k = 8$ back in (i), we get

$y = kx$

$\therefore y = 8x$... (ii)

This the equation of variation

When, $y = 56, x = ?$

Substituting $y = 56$ in (ii), we get

$$y = 8x$$
$$56 = 8x$$
$$x = 568$$
$$\therefore x = 7$$

When, $x = 12$, $y = ?$

Substituting $x = 12$ in (ii), we get

$$y = 8x$$
$$y = 8 \times 12$$
$$\therefore y = 96$$

When, $y = 160$, $x = ?$

Substituting $y = 160$ in (ii), we get

$$y = 8x$$
$$160 = 8x$$
$$x = 1608$$
$$\therefore x = 20$$

Number of apples (x)	1	4	7	12	20
Cost of apples (y)	8	32	56	96	160

3. If $m \propto n$ and when $m = 154$, $n = 7$. Find the value of m , when $n = 14$.

Solution:

Given:

$$m \propto n$$

$m = kn \dots$ (i) where k is constant of variation.

When $m = 154$, $n = 7$

Substitute the value of $m = 154$ and $n = 7$ in (i), we get

$$m = kn$$
$$154 = k \times 7$$
$$k = 1547$$
$$k = 22$$

Now, substitute the value of $k = 22$ back in (i), we get

$$m = kn$$

$$\therefore m = 22n \dots (ii)$$

This is the equation of variation.

When, $n = 14$, $m = ?$

Substitute $n = 14$ in (ii), we get

$$m = 22n$$

$$m = 22 \times 14$$

$$m = 308$$

\therefore The value of m is 308.

4. If n varies directly as m , complete the following table.

m	3	5	6.5	...	1.25
n	12	20	...	28	...

Solution:

Given:

n varies directly as m

So, $n \propto m$

$n = km \dots (i)$ where, k is the constant of variation

Now let us consider the conditions,

When $m = 3$, $n = 12$

Substitute the value of $m = 3$ and $n = 12$ in (i), we get

$$n = km$$

$$12 = k \times 3$$

$$k = 12/3$$

$$k = 4$$

Substitute the value of $k = 4$ back in (i), we get

$$n = km$$

$$\therefore n = 4m \dots (ii)$$

This is the equation of variation.

When $m = 6.5$, $n = ?$

Substituting, $m = 6.5$ in (ii), we get

$$n = 4m$$

$$n = 4 \times 6.5$$

$$\therefore n = 26$$

When $n = 28$, $m = ?$

Substituting, $n = 28$ in (ii), we get

$$n = 4m$$

$$28 = 4m$$

$$28 = 4m$$

$$m = 284$$

$$\therefore m = 7$$

When $m = 1.25$, $n = ?$

Substituting $m = 1.25$ in (ii), we get

$$n = 4m$$

$$n = 4 \times 1.25$$

$$\therefore n = 5$$

m	3	5	6.5	7	1.25
n	12	20	26	28	5

5. y varies directly as the square root of x . When $x = 16$, $y = 24$. Find the constant of variation and equation of variation.

Solution:

Given:

y varies directly as square root of x .

$$\text{So, } y \propto \sqrt{4x}$$

$$y = k \sqrt{x} \dots (i) \text{ where, } k \text{ is the constant of variation.}$$

When $x = 16$, $y = 24$.

Substituting, $x = 16$ and $y = 24$ in (i), we get

$$y = k\sqrt{x}$$

$$24 = k\sqrt{16}$$

$$24 = 4k$$

$$k = 244$$

$$k = 6$$

Substitute the value of $k = 6$ back in (i), we get

$$y = k\sqrt{x}$$

$$y = 6\sqrt{x}$$

This is the equation of variation.

\therefore The constant of variation is 6 and the equation of variation is $y = 6\sqrt{x}$.

6. The total remuneration paid to laborers, employed to harvest soybeans is indirect variation with the number of laborers. If remuneration of 4 laborers is Rs 1000, find the remuneration of 17 laborers.

Solution:

Let total remuneration paid to laborers be = 'm' and

Number of laborers employed to harvest soybean be = 'n'.

Since, the total remuneration paid to laborers, is in direct variation with the number of laborers.

So, $m \propto n$

$\therefore m = kn \dots (i)$ where, k = constant of variation

Remuneration of 4 laborers is Rs 1000.

When $n = 4$, $m = \text{Rs } 1000$

So, substitute the value of $n = 4$ and $m = 1000$ in (i), we get

$$m = kn$$

$$1000 = k \times 4$$

$$k = 1000/4$$

$$k = 250$$

Now, substitute the value of $k = 250$ back in (i), we get

$$m = kn$$

$$\therefore m = 250 n \dots (ii)$$

This is the equation of variation

Now, let us find the remuneration of 17 laborers.

When $n = 17$, $m = ?$

Substituting $n = 17$ in (ii), we get

$$m = 250 n$$

$$m = 250 \times 17$$

$$m = 4250$$

\therefore The remuneration of 17 laborers is Rs 4250.

PRACTICE SET 7.2

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1. The information about numbers of workers and the number of days to complete work is given in the following table. Complete the table.

Number of workers	30	20		10	
Days	6	9	12		36

Solution:

Let the number of workers be = 'n'

Number of days required to complete a work be = 'd'

Since, number of workers and number of days to complete a work are in inverse proportion.

$$n \propto (1/d)$$

$n = k \times (1/d)$ where k, is the constant of variation.

$$\therefore n \times d = k \dots (i)$$

When $n = 30$, $d = 6$

Substitute the value of $n = 30$ and $d = 6$ in (i), we get

$$n \times d = k$$

$$30 \times 6 = k$$

$$k = 180$$

Now, substitute the value of $k = 180$ back in (i), we get

$$n \times d = k$$

$$\therefore n \times d = 180 \dots (ii)$$

This is the equation of variation

When $d = 12$, $n = ?$

Substituting $d = 12$ in (ii), we get

$$n \times d = 180$$

$$n \times 12 = 180$$

$$n = 180/12$$

$$\therefore n = 15$$

When $n = 10$, $d = ?$

Substituting $n = 10$ in (ii), we get

$$n \times d = 180$$

$$10 \times d = 180$$

$$d = 180/10$$

$$\therefore d = 18$$

When $d = 36$, $n = ?$

Substituting $d = 36$ in (ii), we get

$$n \times d = 180$$

$$n \times 36 = 180$$

$$n = 180/36$$

$$\therefore n = 5$$

Number of workers	30	20	15	10	5
Days	6	9	12	18	36

2. Find constant of variation and write equation of variation for every example given below:

(1) $p \propto \frac{1}{q}$; if $p = 15$ then $q = 4$

(2) $z \propto \frac{1}{w}$; when $z = 2.5$ then $w = 24$

(3) $s \propto \frac{1}{t^2}$; if $s = 4$ then $t = 5$

(4) $x \propto \frac{1}{\sqrt{y}}$; if $x = 15$ then $y = 9$

Solution:

(1) $p \propto \frac{1}{q}$; if $p = 15$ then $q = 4$

It is given that, $p \propto 1/q$

$p = k \times 1/q$ where, k is the constant of variation.

$$\therefore p \times q = k \dots (i)$$

When $p = 15$, $q = 4$

Substitute the value of $p = 15$ and $q = 4$ in (i), we get

$$p \times q = k$$

$$15 \times 4 = k$$

$$k = 60$$

Now, substitute the value of $k = 60$ back in (i), we get

$$p \times q = k$$

$$\therefore p \times q = 60$$

This is the equation of variation.

\therefore The constant of variation is 60 and the equation of variation is $pq = 60$.

$$(2) z \propto \frac{1}{w} ; \text{ when } z = 2.5 \text{ then } w = 24$$

It is given that, $z \propto 1/w$

$z = k \times 1/w$ where, k is the constant of variation,

$$\therefore z \times w = k \dots (i)$$

When $z = 2.5$, $w = 24$

Substitute the value of $z = 2.5$ and $w = 24$ in (i), we get

$$z \times w = k$$

$$2.5 \times 24 = k$$

$$k = 60$$

Now, substitute the value of $k = 60$ back in (i), we get

$$z \times w = k$$

$$\therefore z \times w = 60$$

This is the equation of variation.

\therefore The constant of variation is 60 and the equation of variation is $zw = 60$.

$$(3) s \propto \frac{1}{t^2} ; \text{ if } s = 4 \text{ then } t = 5$$

It is given that, $s \propto 1/t^2$

$s = k \times (1/t^2)$ where, k is the constant of variation,

$$\therefore s \times t^2 = k \dots (i)$$

When $s = 4$, $t = 5$

Substitute the value of $s = 4$ and $t = 5$ in (i), we get

$$s \times t^2 = k$$

$$4 \times (5)^2 = k$$

$$k = 4 \times 25$$

$$k = 100$$

Substitute the value of $k = 100$ back in (i), we get

$$s \times t^2 = k$$

$$\therefore s \times t^2 = 100$$

This is the equation of variation.

\therefore The constant of variation is 100 and the equation of variation is $st^2 = 100$.

$$(4) x \propto \frac{1}{\sqrt{y}} ; \text{ if } x = 15 \text{ then } y = 9$$

It is given that, $x \propto 1/\sqrt{y}$

$x = k \times (1/\sqrt{y})$ where, k is the constant of variation,

$$\therefore x \times \sqrt{y} = k \dots (i)$$

When $x = 15$, $y = 9$

Substitute the value of $x = 15$ and $y = 9$ in (i), we get

$$x \times \sqrt{y} = k$$

$$15 \times \sqrt{9} = k$$

$$k = 15 \times 3$$

$$k = 45$$

Now, substitute the value of $k = 45$ back in (i), we get

$$x \times \sqrt{y} = k$$

$$\therefore x \times \sqrt{y} = 45.$$

This is the equation of variation.

\therefore The constant of variation is $k = 45$ and the equation of variation is $x\sqrt{y} = 45$.

3. The boxes are to be filled with apples in a heap. If 24 apples are put in a box then 27 boxes are needed. If 36 apples are filled in a box how many boxes will be needed?

Solution:

Let the number of apples in each box be = 'x'

Total number of boxes required be = 'y'

The number of apples in each box are varying inversely with the total number of boxes.

So, $x \propto 1/y$

$x = k \times (1/y)$ where, k is the constant of variation,

$$\therefore x \times y = k \dots (i)$$

If 24 apples are put in a box then 27 boxes are needed.

When $x = 24$, $y = 27$

Substitute the value of $x = 24$ and $y = 27$ in (i), we get

$$x \times y = k$$

$$24 \times 27 = k$$

$$k = 648$$

Now, substitute the value of $k = 648$ back in (i), we get

$$x \times y = k$$

$$\therefore x \times y = 648 \dots (ii)$$

This is the equation of variation.

Now, let us find number of boxes needed when 36 apples are filled in each box.

So, when $x = 36$, $y = ?$

Substituting $x = 36$ in (ii), we get

$$x \times y = 648$$

$$36 \times y = 648$$

$$y = 648/36$$

$$y = 18$$

\therefore If 36 apples are filled in a box then 18 boxes are required.

4. Write the following statements using symbol of variation.

(1) The wavelength of sound (l) and its frequency (f) are in inverse variation.

(2) The intensity (I) of light varies inversely with the square of the distance (d) of a screen from the lamp.

Solution:

(1) Wavelength of sound (l) and frequency (f) are in inverse proportion.

$$l \propto 1/f$$

(2) Intensity (I) of light varies inversely

With the square of the distance (d)

$$I \propto 1/d^2$$

5. $x \propto 1/\sqrt{y}$ and when $x = 40$ then $y = 16$. If $x = 10$, find y .

Solution:

It is given that, $x \propto 1/\sqrt{y}$

$x = k \times (1/\sqrt{y})$ where, k is the constant of variation.

$$\therefore x \times \sqrt{y} = k \dots (i)$$

When $x = 40$, $y = 16$

Substitute the value of $x = 40$ and $y = 16$ in (i), we get

$$x \times \sqrt{y} = k$$

$$40 \times \sqrt{16} = k$$

$$k = 40 \times 4$$

$$k = 160$$

Now, substitute the value of $k = 160$ back in (i), we get

$$x \times \sqrt{y} = k$$

$$\therefore x \times \sqrt{y} = 160 \dots (ii)$$

This is the equation of variation.

When $x = 10$, $y = ?$

Substitute the value of $x = 10$ in (ii), we get

$$x \times \sqrt{y} = 160$$

$$10 \times \sqrt{y} = 160$$

$$\sqrt{y} = 16010$$

$$\sqrt{y} = 16$$

Square on both the sides, we get

$$y = 256$$

\therefore Value of y is 256.

6. x varies inversely as y , when $x = 15$ then $y = 10$, if $x = 20$ then $y = ?$

Solution:

Given:

$$x \propto 1/\sqrt{y}$$

$x = k \times (1/\sqrt{y})$ where, k is the constant of variation.

$$\therefore x \times y = k \dots (i)$$

When $x = 15$, $y = 10$

Substitute the value of $x = 15$ and $y = 10$ in (i), we get

$$x \times y = k$$

$$15 \times 10 = k$$

$$k = 150$$

Now, substitute the value of $k = 150$ back in (i), we get

$$x \times y = k$$

$$\therefore x \times y = 150 \dots (ii)$$

This is the equation of variation.

When $x = 20$, $y = ?$

Substitute the value of $x = 20$ in (ii), we get

$$x \times y = 150$$

$$20 \times y = 150$$

$$y = 15020$$

$$y = 7.5$$

\therefore Value of y is 7.5

PRACTICE SET 7.3

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1. Which of the following statements is of inverse variation?

- (1) The number of workers on a job and time taken by them to complete the job.
- (2) The number of pipes of the same size to fill a tank and the time taken by them to fill the tank.
- (3) Petrol filled in the tank of a vehicle and its cost.
- (4) Area of the circle and its radius.

Solution:

(1) As the number of workers increases, the time required to complete the job decreases.
Hence, it is of inverse variation.

(2) As the number of pipes increases, the time required to fill the tank decreases.
Hence, it is of inverse variation.

(3) As the quantity of petrol in the tank increases, its cost increases.
Hence, it is of direct variation.

(4) As the area of circle increases, its radius increases.
Hence, it is of direct variation.

2. If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Solution:

Let the number of workers building the wall be = 'n'

The time required be = 't'

Since, the number of workers varies inversely with the time required to build the wall.

$$n \propto 1/t$$

$$n = k \times (1/t) \text{ where, } k \text{ is the constant of variation}$$

$$\therefore n \times t = k \dots(i)$$

15 workers can build a wall in 48 hours,

when $n = 15$, $t = 48$

Substitute the value of $n = 15$ and $t = 48$ in (i), we get

$$n \times t = k$$

$$15 \times 48 = k$$

$$k = 720$$

Now, substitute the value of $k = 720$ back in (i), we get

$$n \times t = k$$

$$\therefore n \times t = 720 \dots (ii)$$

This is the equation of variation.

Now, let us find number of workers required to do the same work in 30 hours.

When $t = 30$, $n = ?$

Substitute the value of $t = 30$ in (ii), we get

$$n \times t = 720$$

$$n \times 30 = 720$$

$$n = 720/30$$

$$n = 24$$

$\therefore 24$ workers are required to build the wall in 30 hours.

3. 120 bags of half liter milk can be filled by a machine within 3 minutes find the time to fill such 1800 bags?

Solution:

Let the number of bags of half liter milk be = 'b'

The time required to fill the bags = 't'

Since, the number of bags and time required to fill the bags varies directly.

$$b \propto t$$

$$\therefore b = kt \dots (i) \text{ where } k \text{ is the constant of variation.}$$

Since, 120 bags can be filled in 3 minutes

When $b = 120$, $t = 3$

Substitute the value of $b = 120$ and $t = 3$ in (i), we get

$$b = kt$$

$$120 = k \times 3$$

$$k = 120/3$$

$$k = 40$$

Now, substitute the value of $k = 40$ back in (i), we get

$$b = kt$$

$$\therefore b = 40 t \dots (ii)$$

This is the equation of variation.

Now, let us find the time required to fill 1800 bags

When $b = 1800$, $t = ?$

Substitute the value of $b = 1800$ in (ii), we get

$$b = 40 t$$

$$1800 = 40 t$$

$$t = 1800/40$$

$$t = 45$$

∴ 1800 bags of half liter milk can be filled by the machine in 45 minutes.

4. A car with a speed of 60 km/hr takes 8 hours to travel some distance. What should be the increase in the speed if the same distance is to be covered in $7\frac{1}{2}$ hours?

Solution:

Let the speed of car in km/hr be = 'v'

The time required be = 't'

Since, speed of a car varies inversely as the time required to cover a distance.

$$v \propto 1/t$$

$v = k \times (1/t)$ where, k is the constant of variation.

$$\therefore v \times t = k \dots (i)$$

Since, a car with speed 60 km/hr takes 8 hours to travel some distance.

When $v = 60$, $t = 8$

Substitute the value of $v = 60$ and $t = 8$ in (i), we get

$$v \times t = k$$

$$60 \times 8 = k$$

$$k = 480$$

Now, substitute the value of $k = 480$ back in (i), we get

$$v \times t = k$$

$$\therefore v \times t = 480 \dots (ii)$$

This is the equation of variation.

Now, let us find the speed of car if the same distance is to be covered in $7\frac{1}{2}$ hours.

When $t = 7\frac{1}{2} = 7.5$, $v = ?$

Substitute the value of $t = 7.5$ in (ii), we get

$$v \times t = 480$$

$$v \times 7.5 = 480$$

$$v = 480/7.5$$

$$v = 64$$

The speed of vehicle should be 64 km/hr to cover the same distance in 7.5 hours.

The increase in speed = $64 - 60$

$$= 4 \text{ km/hr}$$

∴ The increase in speed of the car is 4 km/hr.