PRACTICE SET 2.1 PAGE NO: 34

1. Write any two quadratic equations.

Solution:
Two quadratic equations are
$a^2 + 16 = 0$ and $x^2 + 2x + 6 = 0$

2. Decide which of the following are quadratic equations.
(1) $x^2 + 5x – 2 = 0$

Solution:
$x^2 + 5x – 2 = 0$ is a quadratic equation because it is the form of $ax^2 + bx + c = 0$ and it has degree 2.

(2) $y^2 = 5y - 10$

Solution:
$y^2 = 5y – 10$ is a quadratic equation because it is the form of $ax^2 + bx + c = 0$ and it has degree 2.

(3) $y^2 + 1/y = 2$

Solution:
$y^2 + 1/y = 2$ is a quadratic equation because it is the form of $ax^2 + bx + c = 0$ and it has degree 2.

(4) $x + 1/x = -2$

Solution:
Given equation can be written as
$x^2 + 1 = -2x$
$x^2 + 2x + 1 = 0$
It is a quadratic equation because it is the form of $ax^2 + bx + c = 0$ and it has degree 2.

(5) $(m + 2) (m–5) = 0$

Solution:

Given equation can be written as

m (m – 5) + 2 (m – 5)

= $m^2 - 5m + 2m - 10$

= $m^2 - 3m + 10 = 0$

It is a quadratic equation because it is the form of $ax^2 + bx + c = 0$ and it has degree 2.

(6) $m^3 + 3m^2 - 2 = 3m^3$

Solution:

Given $m^3 + 3m^2 - 2 = 3m^3$

It is not a quadratic equation because it is not in the form of $ax^2 + bx + c = 0$ and it has degree 3.

3. Write the following equations in the form $ax^2 + bx + c = 0$, then write the values of a, b, c for each equation.

(1) $2y = 10 - y^2$

Given

$2y = 10 - y^2$

$2y + y^2 - 10 = 0$

$y^2 + 2y - 10 = 0$;

a = 1, b = 2, c = -10

(2) $(x-1)^2 = 2x + 3$

Solution:

Given

$(x - 1)^2 = 2x + 3$

$\Rightarrow x^2 - 2x + 1 = 2x + 3$

$\Rightarrow x^2 - 2x - 2x + 1 - 3 = 0$

$\Rightarrow x^2 - 4x - 2 = 0$;

Here

a = 1, b = -4, c = -2

(3) $x^2 + 5x = -(3-x)$

Solution:

Given

$x^2 + 5x = -(3 - x)$

$\Rightarrow x^2 + 5x = -3 + x$

$\Rightarrow x^2 + 5x - x + 3 = 0$

$\Rightarrow x^2 + 4x + 3 = 0$;

Here

a = 1, b = 4, c = 3

(4) $3m^2 = 2m^2 - 9$

Solution:

Given

$3m^2 = 2m^2 - 9$

$\Rightarrow 3m^2 - 2m^2 + 9 = 0$

$m^2 + 0m + 9 = 0$

Here,

a = 1, b = 0, c = 9

(5) P (3 + 6p) = - 5

Solution:

Given

$p(3 + 6p) = -5$

$\Rightarrow 3p + 6p^2 + 5 = 0$

$6p^2 + 3p + 5 = 0$

Here,

a = 6, b = 3, c = 5

(6) $x^2 - 9 = 13$

Solution:

Given

$x^2 - 9 = 13$

$\Rightarrow x^2 - 9 - 13 = 0$

$x^2 + 0x - 22 = 0$

Here,

a = 1, b = 0, c = -22

4. Determine whether the values given against each of the quadratic equation are the roots of the equation.

(1) $x^2 + 4x - 5 = 0$, x = 1, -1

Solution:

Given

$x^2 + 4x - 5 = 0$

Put $x = 1$

$\Rightarrow 1^2 + 4 \times 1 - 5$

$\Rightarrow 1 + 4 - 5 = 0$

Put $x = -1$

$\Rightarrow (-1)^2 + 4(-1) - 5$

$\Rightarrow 1 - 4 - 5 = -8$

∴ x = 1 is a root of the equation and x = -1 is not a root of the equation.

(2) $2m^2 - 5m = 0$, m = 2, 5/ 2

Solution:

Given

$2m^2 - 5m = 0$

Put, $m = 2$

$\Rightarrow 2(2)^2 - 5 \times 2$

$\Rightarrow 2 \times 4 - 10 \Rightarrow 8 - 10 \Rightarrow -2$

Put, $m = \frac{5}{2}$

$\Rightarrow 2\left(\frac{5}{2}\right)^2 - 5 \times \frac{5}{2} \Rightarrow 2 \times \frac{25}{4} - \frac{25}{2} \Rightarrow \frac{25}{2} - \frac{25}{2} = 0$

∴ m = 2 is not root of the equation and m = 5/2 is a root of the equation.

5. Find k if x = 3 is a root of equation $kx^2 - 10x + 3 = 0$.

Solution:

Given

$kx^2 - 10x + 3 = 0$

Put $x = 3$

$\Rightarrow k(3)^2 - 10 \times 3 + 3 = 0$

$\Rightarrow 9k - 30 + 3 = 0$

$\Rightarrow 9k = 30 - 3$

$\Rightarrow 9k = 27$

$\Rightarrow k = \frac{27}{9} = 3$

6. One of the roots of equation $5m^2 + 2m + k = 0$ is -7/5. Complete the following activity to find the value of 'k'.

Solution:

-7/5 is a root of quadratic equation $kx^2 - 10x + 3 = 0$

∴ Put m = -7/5 in the equation.

$\Rightarrow 5 \times \left(-\frac{7}{5}\right)^2 + 2 \times \left(-\frac{7}{5}\right) + k = 0$

$\Rightarrow 5 \times \frac{49}{25} - \frac{14}{5} + k = 0$

$\Rightarrow \frac{35}{5} + k = 0$

k = -7

PRACTICE SET 2.2 PAGE NO: 36

1. Solve the following quadratic equations by factorization.

(1) $x^2 - 15x + 54 = 0$

Solution:

Given

$x^2 - 15x + 54 = 0$

Taking factors of -15x and above equation can be written as

$\Rightarrow x^2 - 6x - 9x + 54 = 0$

$\Rightarrow x(x - 6) - 9(x - 6) = 0$

$\Rightarrow (x - 6)(x - 9) = 0$

$x - 6 = 0 \Rightarrow x = 6$

$x - 9 = 0 \Rightarrow x = 9$

Hence, x = 6 and x = 9 are roots of the equation.

(2) $x^2 + x - 20 = 0$

Solution:

Given

$x^2 + x - 20 = 0$

Taking factors of x and above equation can be written as

$\Rightarrow x^2 + 5x - 4x - 20 = 0$

$\Rightarrow x(x + 5) - 4(x + 5) = 0$

$\Rightarrow (x + 5)(x - 4) = 0$

$x + 5 = 0 \Rightarrow x = -5$

$x - 4 = 0 \Rightarrow x = 4$

Hence, x = -5 and x = 4 are roots of the equation.

(3) $2y^2 + 27y + 13 = 0$

Solution:

Given

$2y^2 + 27y + 13 = 0$

Taking factors for 27 y and above equation can be written as

$\Rightarrow 2y^2 + 26y + y + 13 = 0$

$\Rightarrow 2y(y + 13) + (y + 13) = 0$

$\Rightarrow (2y + 1) (y + 13) = 0$

$2y + 1 = 0 \Rightarrow 2y = -1 \Rightarrow y = -\frac{1}{2}$

$y + 13 = 0 \Rightarrow y = -13$

Hence, y = -13 and y = - ½ are roots of the equation.

(4) $5m^2 = 22m + 15$

Solution:

Given

$5m^2 - 22m - 15 = 0$

Taking the factors of – 22m and the above equation can be written as

$\Rightarrow 5m^2 - 3m + 25m - 15$

$\Rightarrow m(5m - 3) + 5(5m - 3)$

$\Rightarrow (m + 5)(5m - 3)$

$m + 5 = 0 \Rightarrow m = -5$

$5m - 3 = 0 \Rightarrow 5m = 3 \Rightarrow m = \frac{3}{5}$

∴ Hence, m = -5 and m = 3/5 are roots of the equation.

(5) $2x^2 - 2x + ½ = 0$

Solution:

Given

$2x^2 - 2x + \frac{1}{2} = 0$

$\Rightarrow 4x^2 - 4x + 1 = 0$

$\Rightarrow 4x^2 - 2x - 2x + 1$

$2x (2x - 1) - 1 (2x - 1)$

(2x – 1) (2x – 1)

2x – 1 = 0

x = ½, ½

Hence x = ½, ½ are the roots of the equation

(6) 6x -2/x = 1

Solution:

Given

$6x^2 - 2 = x$

Taking factors for -x and the above equation can be written as

$\Rightarrow 6x^2 - x - 2 = 0$

$\Rightarrow 6x^2 + 3x - 4x - 2 = 0$

$\Rightarrow 3x(2x + 1) - 2(2x + 1) = 0$

$\Rightarrow (3x - 2)(2x + 1) = 0$

$3x - 2 = 0 \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$

$2x + 1 = 0 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$

Hence, x = 2/3 and x = - ½ are roots of the equation.

(7) √2 x² + 7 x + 5 √2 = 0 to solve this quadratic equation by factorization, complete the following activity.

Given

$\sqrt{2x^2} + 7x + 5\sqrt{2} = 0$

Taking factors for 7x and above equation can be written as

$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$(x + \sqrt{2}) = 0$ or $(\sqrt{2}x + 5) = 0$

$x = -\frac{5}{\sqrt{2}}$ or $x = -\sqrt{2}$

$\therefore -\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$ are roots of the equation.

(8) $3x^2 – 2\sqrt{6}x + 2 = 0$

Solution:

Given

$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

The above equation can be written as

$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2}) = 0 \text{ or } (\sqrt{3}x - \sqrt{2}) = 0$

The roots are

$x = \frac{\sqrt{2}}{\sqrt{3}} \text{ or } x = \frac{\sqrt{2}}{\sqrt{3}}$

(9) 2m (m-24) = 50

Solution:

Given

$2m(m - 24) = 50$

The given equation can be written as

$2m^2 - 48m - 50 = 0$

$\Rightarrow 2m^2 - 50m + 2m - 50 = 0$

$\Rightarrow 2m(m - 25) + 2(m - 25) = 0$

$\Rightarrow (2m + 2)(m - 25) = 0$

$\Rightarrow 2m + 2 = 0 \text{ or } m - 25 = 0$

$\Rightarrow m = -1 \text{ or } m = 25$

Hence, m = -1 or m = 25 are roots of the equation.

(10) $25m^2 = 9$

Solution:

Given

$25m^2 = 9$

On rearranging we get

$\Rightarrow m^2 = \frac{9}{25}$

$\Rightarrow m = \sqrt{\frac{9}{25}}$

$\Rightarrow m = \pm\frac{3}{5}$

Hence, $m = \pm\frac{3}{5}$ are roots of the equation.

(11) 7m² = 21m

Solution:

Given

$7m^2 - 21m = 0$

Given equation can be written as

$\Rightarrow 7m(m - 3) = 0$

$\Rightarrow 7m = 0 \text{ or } m - 3 = 0$

$\Rightarrow m = 0 \text{ or } m = 3$

Hence, m = 0 or m = 3 are roots of the equation.

(12) m² - 11 = 0

Solution:

Given

$m^2 - 11 = 0$

$\Rightarrow m^2 = 11$

$\Rightarrow m = \sqrt{11}$

$\Rightarrow m = \pm 11$

Hence, m = ±11 are roots of the equation.

PRACTICE SET 2.3

1. Solve the following quadratic equations by completing the square method.

(1) $x^2 + x - 20 = 0$

Solution:

Given

$x^2 + x - 20 = 0$

Add and subtract ¼ to the given equation

$\Rightarrow x^2 + x + \frac{1}{4} - \frac{1}{4} - 20 = 0$

On grouping to make it perfect square

$\Rightarrow (x^2 + x + \frac{1}{4}) + \left(\frac{1}{4} - 20\right) = 0$

$\Rightarrow \left(x + \frac{1}{2}\right)^2 - \frac{1+80}{4} = 0$

On simplifying we get

$\Rightarrow \left(x + \frac{1}{2}\right)^2 = \frac{81}{4}$

Shifting the square, we get

$\Rightarrow x + \frac{1}{2} = \sqrt{\frac{81}{4}}$

$\Rightarrow x + \frac{1}{2} = \pm\frac{9}{2}$

$\Rightarrow x + \frac{1}{2} = \frac{9}{2} \text{ or } x + \frac{1}{2} = -\frac{9}{2}$

$\Rightarrow x = \frac{9}{2} - \frac{1}{2} \text{ or } x = -\frac{9}{2} - \frac{1}{2}$

$\Rightarrow x = \frac{8}{2} \text{ or } x = -\frac{10}{2}$

$\Rightarrow x = 4 \text{ or } x = -5$

(2) $x^2 + 2x - 5 = 0$

Solution:

Given

$x^2 + 2x - 5 = 0$

Add and subtract 1 to the above equation we get

$\Rightarrow x^2 + 2x + 1 - 1 - 5 = 0$

On grouping to make it perfect square we get

$\Rightarrow (x^2 + 2x + 1) - (1 + 5) = 0$

$\Rightarrow (x + 1)^2 - 6 = 0$

On rearranging we get

$\Rightarrow (x + 1)^2 = 6$

$\Rightarrow x + 1 = \sqrt{6}$

$\Rightarrow x + 1 = \pm\sqrt{6}$

$\Rightarrow x + 1 = \sqrt{6}$ or $x + 1 = -\sqrt{6}$

$\Rightarrow x = \sqrt{6} - 1$ or $x = -\sqrt{6} - 1$

(3) m^2 – 5m = -3

Solution:

Given

$m^2 - 5m + 3 = 0$

Add and subtract 25/4 to the above equation we get

$\Rightarrow m^2 - 5m + \frac{25}{4} - \frac{25}{4} + 3 = 0$

On rearranging we get

$\Rightarrow \left(m^2 - 5m + \frac{25}{4}\right) = \frac{25}{4} - 3$

$\Rightarrow \left(m - \frac{5}{2}\right)^2 = \frac{25-12}{4}$

$\Rightarrow \left(m - \frac{5}{2}\right)^2 = \frac{13}{4}$

Shifting the square, we get

$\Rightarrow m - \frac{5}{2} = \sqrt{\frac{13}{4}}$

$\Rightarrow m - \frac{5}{2} = \pm\frac{\sqrt{13}}{2}$

$\Rightarrow m - \frac{5}{2} = \frac{\sqrt{13}}{2}$ or $m - \frac{5}{2} = -\frac{\sqrt{13}}{2}$

$\Rightarrow m = \frac{\sqrt{13}}{2} + \frac{5}{2}$ or $m = -\frac{\sqrt{13}}{2} - \frac{5}{2}$

$\Rightarrow m = \frac{\sqrt{13}+5}{2}$ or $m = \frac{-\sqrt{13}-5}{2}$

(4) $9y^2 - 12y + 2 = 0$

Solution:

Given

$9y^2 - 12y + 2 = 0$

The above equation can be written as

$(3y)^2 - 2 \times 3y \times 4 + (4)^2 - (4)^2 + 2 = 0$

$(3y)^2 - 2 \times 3y \times 4 + (4)^2 - 16 + 2 = 0$

$(3y - 4)^2 - 14 = 0$

$(3y - 4)^2 = 14$

$3y - 14 = \pm\sqrt{14}3y = 14 \pm \sqrt{14}y = (14 \pm \sqrt{14})/3$

(5) $2y^2 + 9y + 10 = 0$

Solution:

Given

$2y^2 + 9y + 10 = 0$

Dividing by the coefficient of 2, we get,

$\Rightarrow y^2 + \frac{9}{2}y + 5 = 0$

Add and subtract 81/16 to the both sides

$\Rightarrow y^2 + \frac{9}{2}y + \frac{81}{16} - \frac{81}{16} + 5 = 0$

Take out the terms following the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow (y^2 + \frac{9}{2}y + \frac{81}{16}) - \left(\frac{81}{16} - 5\right) = 0$$

$$\Rightarrow \left(y + \frac{9}{4}\right)^2 = \frac{81}{16} - 5$$

$$\Rightarrow \left(y + \frac{9}{2}\right)^2 = \frac{81-80}{16}$$

On rearranging we get

$\Rightarrow \left(y + \frac{9}{2}\right)^2 = \frac{1}{16}$

$\Rightarrow y + \frac{9}{2} = \sqrt{\frac{1}{16}}$

$\Rightarrow y + \frac{9}{2} = \pm\frac{1}{4}$

$\Rightarrow y + \frac{9}{2} = \frac{1}{4}$ or $y + \frac{9}{2} = -\frac{1}{4}$

$\Rightarrow y = \frac{1}{4} - \frac{9}{2}$ or $y = -\frac{1}{4} - \frac{9}{2}$

$\Rightarrow y = \frac{1-18}{4}$ or $y = \frac{-1-18}{4}$

$\Rightarrow y = -\frac{17}{4}$ or $y = -\frac{19}{4}$

(6) $5x^2$ - 4x + 7 = 0

Solution:

Given

$5x^2 - 4x - 7 = 0$

Dividing above equation by 5 we get

$\Rightarrow x^2 - \frac{4}{5}x - \frac{7}{5} = 0$

Adding and subtracting 4/25 we get

$\Rightarrow x^2 - \frac{4}{5}x + \frac{4}{25} = \frac{7}{5} + \frac{4}{25}$

On grouping

$\Rightarrow \left(x + \frac{2}{5}\right)^2 = \frac{35+4}{25}$

$\Rightarrow \left(x + \frac{2}{5}\right)^2 = \frac{39}{25}$

Shifting the square, we get

$\Rightarrow x + \frac{2}{5} = \sqrt{\frac{39}{25}}$

$\Rightarrow x + \frac{2}{5} = \pm\frac{\sqrt{39}}{5}$

$$x = \frac{\sqrt{39}}{5} - \frac{2}{5} \text{ or } x = -\frac{\sqrt{39}}{5} - \frac{2}{5}$$

The roots are

$$x = \frac{\sqrt{39} - 2}{5} \text{ or } x = \frac{-\sqrt{39} - 2}{5}$$

PRACTICE SET 2.4 PAGE NO: 43

1. Compare the given quadratic equations to the general form and write values of a, b, c.
(1) $x^2 - 7x + 5 = 0$

Solution:
Given
$x^2 - 7x + 5 = 0$
comparing with $ax^2 + bx + c$
we get
a = 1, b = -7, c = 5

(2) $2m^2 = 5m - 5$

Solution:
Given
$2m^2 = 5m - 5$
comparing with $ax^2 + bx + c$
we get
a = 2, b = -5, c = 5

(3) $y^2 = 7y$

Solution:
Given
$y^2 = 7y$
comparing with $ax^2 + bx + c$
we get
a = 1, b = -7, c = 0

2. Solve using formula.
(1) $x^2 + 6x + 5 = 0$

Solution:

Given

$x^2 + 6x + 5 = 0$

$\Rightarrow x^2 + 6x + 5 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 1, b = 6$ and $c = 5$

$\therefore b^2 - 4ac = 6^2 - 4(1)(5)$

$= 36 - 20$

$= 16$

We have the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values in formula we get

$$x = \frac{-6 \pm \sqrt{16}}{2 \times 1} = \frac{-6 \pm 4}{2}$$

$\Rightarrow x = \frac{-6+4}{2}$ or $x = \frac{-6-4}{2}$

$\Rightarrow x = -\frac{2}{2}$ or $x = -\frac{10}{2}$

The roots are

$\Rightarrow x = -1$ or $x = -5$

(2) $x^2 - 3x - 2 = 0$

Solution:

Given

$\Rightarrow x^2 + 3x - 2 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = 3$ and $c = -2$

$\therefore b^2 - 4ac = 3^2 - 4(1)(-2)$

$= 9 + 8$

$= 17$

We have the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values in formula we get

$\Rightarrow x = \frac{-3 \pm \sqrt{17}}{2 \times 1}$

$\Rightarrow x = \dfrac{-3 \pm \sqrt{17}}{2}$

The roots are

$\Rightarrow x = \dfrac{-3 + \sqrt{17}}{2}$ or $x = \dfrac{-3 - \sqrt{17}}{2}$

(3) $3m^2 + 2m - 7 = 0$

Solution:

Given

$\Rightarrow 3m^2 + 2m - 7 = 0$ compare with $ax^2 + bx + c = 0$

we get

$\Rightarrow a = 3, b = 2$ and $c = -7$

$\therefore b^2 - 4ac = 2^2 - 4(3)(-7)$

$= 4 + 84$

$= 88$

We have the formula

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substituting the values in formula we get

$\Rightarrow m = \dfrac{-2 \pm \sqrt{88}}{2 \times 3}$

$\Rightarrow m = \dfrac{-2 \pm \sqrt{88}}{6}$

$\Rightarrow m = \dfrac{-2 + 2\sqrt{22}}{6}$ or $m = \dfrac{-2 - 2\sqrt{22}}{6}$

The roots are

$\Rightarrow m = \dfrac{-1 + \sqrt{22}}{3}$ or $m = \dfrac{-1 - \sqrt{22}}{3}$

(4) $5m^2 - 4m - 2 = 0$

Solution:

Given

$\Rightarrow 5m^2 - 4m - 2 = 0$ compare with $ax^2 + bx + c = 0$

we get

$\Rightarrow a = 5, b = -4$ and $c = -2$

$\therefore b^2 - 4ac = (-4)^2 - 4(5)(-2)$

$= 16 + 40$

$= 56$

We have the formula

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values in formula we get

$\Rightarrow m = \frac{-(-4) \pm \sqrt{56}}{2 \times 5}$

$\Rightarrow m = \frac{4 \pm 2\sqrt{14}}{10}$

The roots are

$\Rightarrow m = \frac{4 + 2\sqrt{14}}{10}$ or $m = \frac{4 - 2\sqrt{14}}{10}$

$\Rightarrow m = \frac{2 + \sqrt{14}}{5}$ or $m = \frac{2 - \sqrt{14}}{5}$

(5) $y^2 + 1/3\ y = 2$

Solution:

Given

$3y^2 + y - 6 = 0$

$\Rightarrow 3y^2 + y - 6 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 3, b = 1$ and $c = -6$

$\therefore b^2 - 4ac = 1^2 - 4(3)(-6)$

$= 1 + 72$

$= 73$

We have the formula

$Y = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Substituting the values in formula we get

$\Rightarrow y = \frac{-1\pm\sqrt{73}}{2\times3}$

$\Rightarrow y = \frac{-1 \pm \sqrt{73}}{6}$

The roots are

$\Rightarrow y = \frac{-1+\sqrt{73}}{6}$ or $y = \frac{-1-\sqrt{73}}{6}$

(6) $5x^2 + 13x + 8 = 0$

Solution:

Given

$\Rightarrow 5x^2 + 13x + 8 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 5, b = 13$ and $c = 8$

$\therefore b^2 - 4ac = 13^2 - 4(5)(8)$

$= 169 - 160$

$= 9$

We have the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substituting the values in formula we get

$\Rightarrow x = \frac{-13\pm\sqrt{9}}{2\times5}$

$\Rightarrow x = \frac{-13 \pm 3}{10}$

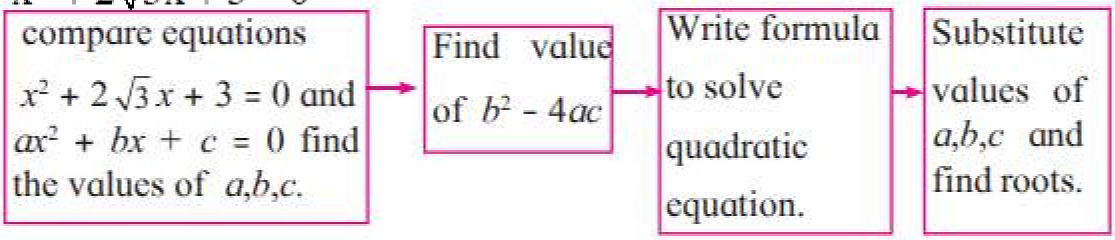
$\Rightarrow x = \frac{-13+3}{10}$ or $x = \frac{-13-3}{10}$

$\Rightarrow x = \frac{-10}{10}$ or $x = \frac{-16}{10}$

The roots are

$\Rightarrow x = -1$ or $x = -\frac{8}{5}$

3. With the help of the flow chart given below solve the equation $x^2 + 2\sqrt{3}x + 3 = 0$ using the formula.



Solution:

Given

$\Rightarrow x^2 + 2\sqrt{3}x + 3 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = 2\sqrt{3}$ and $c = 3$

$\therefore b^2 - 4ac = (2\sqrt{3})^2 - 4(1)(3)$

$= 12 - 12$

$= 0$

We have the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

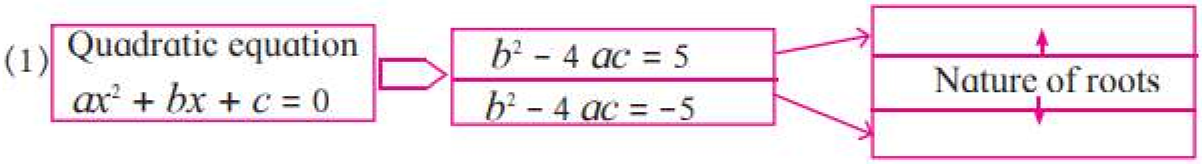
Substituting the values in formula we get

$$\Rightarrow x = \frac{-2\sqrt{3} \pm \sqrt{0}}{2 \times 1}$$

$$\Rightarrow x = \frac{-2\sqrt{3}}{2}$$

PRACTICE SET 2.5 PAGE NO: 49

1. Fill in the gaps and complete.



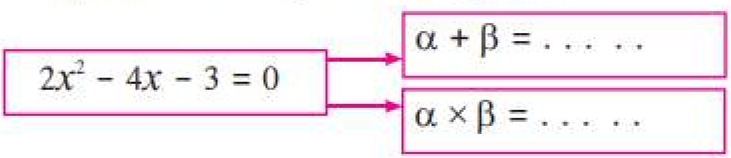
Solution:

Roots are distinct and real when $b^2 - 4ac = 5$, not real when $b^2 - 4ac = -5$.

Solution:

$x^2 + 7x + 5 = 0$

(3) If α, β are roots of quadratic equation,



Solution:

$$\alpha + \beta = 2, \alpha \times \beta = -\frac{3}{2}$$

2. Find the value of discriminant.

(1) $x^2 + 7x - 1 = 0$

Solution:

Given

$\Rightarrow x^2 + 7x - 1 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = 7$ and $c = -1$

The discriminant value is

$\therefore b^2 - 4ac = 7^2 - 4(1)(-1)$

$= 49 + 4$

$= 53$

(2) $2y^2 - 5y + 10 = 0$

Solution:

Given

$\Rightarrow 2y^2 - 5y + 10 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 2, b = -5$ and $c = 10$

The discriminant value is

$\therefore b^2 - 4ac = -5^2 - 4(2)(10)$

$= 25 - 80$

$= -55$

(3) $\sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$

Solution:

Given

$\Rightarrow \sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = \sqrt{2}, b = 4$ and $c = 2\sqrt{2}$

The discriminant value is

$\therefore b^2 - 4ac = 4^2 - 4(\sqrt{2})(2\sqrt{2})$

$= 16 - 16$

$= 0$

3. Determine the nature of roots of the following quadratic equations.

(1) $x^2 - 4x + 4 = 0$

Solution:

Given

$\Rightarrow x^2 - 4x + 4 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = -4$ and $c = 4$

The discriminant value is

$\therefore b^2 - 4ac = -4^2 - 4(1)(4)$

$= 16 - 16$

$= 0$

$\therefore b^2 - 4ac = 0$. hence, roots are real and equal

(2) $2y^2 - 7y + 2 = 0$

Solution:

Given

$\Rightarrow 2y^2 - 7y + 2 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 2, b = -7$ and $c = 2$

The discriminant value is

$\therefore b^2 - 4ac = -7^2 - 4(2)(2)$

$= 49 - 16$

$= 23$

$\therefore b^2 - 4ac > 0$. Hence, roots are real and unequal

(3) $m^2 + 2m + 9 = 0$

Solution:

Given

$\Rightarrow m^2 + 2m + 9 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = 2$ and $c = 9$

The discriminant value is

$\therefore\ b^2 - 4ac = 2^2 - 4(1)(9)$
$= 4 - 36$
$= -32$
$\therefore\ b^2 - 4ac < 0$ .hence, roots are not real.

4. Form the quadratic equation from the roots given below.
(1) 0 and 4

Solution:
Given 0, 4
Let $\alpha = 0$ and $\beta = 4$
$\therefore\ \alpha + \beta = 0 + 4 = 4$ and $\alpha\beta = 0 \times 4 = 0$
$\therefore$ quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
$\therefore\ x^2 - (4)x + (0) = 0$
$\therefore\ x^2 - 4x = 0$

(2) 3 and -10

Solution:
Given 3, 10
Let $\alpha = 3$ and $\beta = -10$
$\therefore\ \alpha + \beta = 3 - 10 = -7$ and $\alpha\beta = 3 \times -10 = -30$
$\therefore$ quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
$\therefore\ x^2 - (-7)x + (-30) = 0$
$\therefore\ x^2 + 7x - 30 = 0$

(3) ½, - ½

Solution:
Given ½, - ½
Let $\alpha = \frac{1}{2}$ and $\beta = -\frac{1}{2}$
$\therefore\ \alpha + \beta = \frac{1}{2} - \frac{1}{2} = 0$ and $\alpha\beta = \frac{1}{2} \times -\frac{1}{2} = -\frac{1}{4}$

$\therefore$ quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (0)x + \left(-\frac{1}{4}\right) = 0$

$\therefore x^2 - \frac{1}{4} = 0$

$\therefore 4x^2 - 1 = 0$

(4) 2 - √5, 2 + √5

Solution:

Given 2 - √5, 2 + √5

Let $\alpha = 2 - \sqrt{5}$ and $\beta = 2 + \sqrt{5}$

$\alpha + \beta = 2 - \sqrt{5} + 2 + \sqrt{5} = 4$ and $\alpha\beta = (2 - \sqrt{5})(2 + \sqrt{5}) = 4 - 5 = 1$

$\therefore$ quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (4)x + (1) = 0$

$\therefore x^2 - 4x + 1 = 0$

5. Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$

Solution:

According to question

$\alpha + \beta = 2\alpha\beta$

$\Rightarrow 4k = 2(k + 3)$

$\Rightarrow 4k = 2k + 6$

$\Rightarrow 4k - 2k = 6$

$\Rightarrow 2k = 6$

$\Rightarrow k = 3$

6. a, b are roots of y2 - 2y - 7 = 0 find,

(1) $\alpha^2 + \beta^2$

Solution:

Given

$y^2 - 2y - 7 = 0$

$\alpha + \beta = 2$ and $\alpha\beta = -7$

We know that

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow (2)^2 = \alpha^2 + \beta^2 + 2(-7)$

$\Rightarrow 4 + 14 = \alpha^2 + \beta^2$

$\Rightarrow \alpha^2 + \beta^2 = 18$

(2) $\alpha^3 + \beta^3$

Solution:

Given

$y^2 - 2y - 7 = 0$

$\alpha + \beta = 2$ and $\alpha\beta = -7$

We know that

$(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$

$\Rightarrow (2)^3 = \alpha^3 + \beta^3 + 3(-7)(2)$

$\Rightarrow 8 + 42 = \alpha^3 + \beta^3$

$\Rightarrow \alpha^3 + \beta^3 = 50$

7. The roots of each of the following quadratic equation are real and equal, find k.
(1) $3y^2 + ky + 12 = 0$

Solution:

Given

$\Rightarrow 3y^2 - ky + 12 = 0$ compare with $ax^2 + bx + c = 0$

we get

$\Rightarrow a = 3, b = -k$ and $c = 12$

We know that

$\therefore b^2 - 4ac = -k^2 - 4(3)(12)$

$= k^2 - 144$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

$k^2 - 144 = 0$

$\Rightarrow k^2 = 144$

$\Rightarrow k = \pm 12$

$\therefore k = 12$ and $k = -12$

(2) kx (x-2) + 6 = 0

Solution:

Given

$kx(x-2) + 6 = 0 \Rightarrow kx^2 - 2kx + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$ compare with $ax^2 + bx + c = 0$

we get

$\Rightarrow a = k, b = -2k$ and $c = 6$

$\therefore b^2 - 4ac = (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$\Rightarrow 4k(k-6) = 0$

$\Rightarrow 4k = 0$ and $k - 6 = 0$

$\therefore k = 0$ and $k = 6$

PRACTICE SET 2.6 PAGE NO: 52

1. Product of Pragati's age 2 years ago and 3 years hence is 84. Find her present age.

Solution:

Let her present age be x

According to question,

$(x-2)(x+3) = 84$

On multiplying we get

$\Rightarrow x^2 + x - 6 = 84$

On rearranging we get

$\Rightarrow x^2 + x - 90 = 0$

Taking the factors

$\Rightarrow x^2 + 10x - 9x - 90 = 0$

$\Rightarrow x(x + 10) - 9(x + 10) = 0$

$\Rightarrow (x - 9)(x + 10) = 0$

$\Rightarrow x - 9 = 0$ or $x + 10 = 0$

$\Rightarrow x = 9$ or $x = -10$

As age cannot be in negative, ∴ Pragati′sage is 9 years.

2. The sum of squares of two consecutive natural numbers is 244; find the numbers.

Solution:

Given

Let the two consecutive natural numbers be x and x + 2. Then,

$x^2 + (x + 2)^2 = 244$

The above equation can be written as

$\Rightarrow x^2 + x^2 + 4x + 4 = 244$

$\Rightarrow 2x^2 + 4x - 240 = 0$

Taking 2 common

$\Rightarrow x^2 + 2x - 120 = 0$

$\Rightarrow x^2 + 12x - 10x - 120 = 0$

$\Rightarrow x(x + 12) - 10(x + 12) = 0$

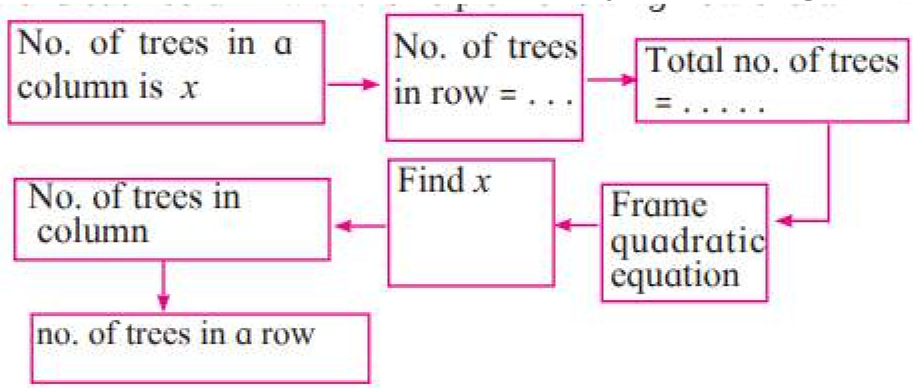
$\Rightarrow (x + 12)(x - 10) = 0$

$x + 12 = 0$ or $x - 10 = 0$

$x = -12$ or $x = 10$

Numbers cannot be negative, ∴ numbers are 10 and 12

3. In the orange garden of Mr. Madhusudan there are 150 orange trees. The number of trees in each row is 5 more than that in each column. Find the number of trees in each row and each column with the help of following flow chart.



Solution:

Let the number of columns be x

∴ rows = x + 5

$x(x + 5) = 150$

On multiplying we get

$\Rightarrow x^2 + 5x - 150 = 0$

Taking factors,

$\Rightarrow x^2 + 15x - 10x - 150 = 0$

The above equation can be written as

$\Rightarrow x(x + 15) - 10(x + 15) = 0$

$\Rightarrow (x + 15)(x - 10) = 0$

$x + 15 = 0$ or $x - 10 = 0$

$x = -15$ or $x = 10$

Hence, columns cannot be negative. ∴ columns are 10 and rows are 15.

4. Vivek is older than Kishor by 5 years. The sum of the reciprocals of their ages is 1/6. Find their present ages.

Solution:

Let Kishor's present age be x. Then, Vivek's age = x + 5

$\therefore \frac{1}{x} + \frac{1}{x+5} = \frac{1}{6}$

$\Rightarrow \frac{x+5+x}{x(x+5)} = \frac{1}{6} \Rightarrow 6(5 + 2x) = x^2 + 5x$

$\Rightarrow 30 + 12x = x^2 + 5x$

$\Rightarrow x^2 + 5x - 12x - 30 = 0$

$\Rightarrow x^2 - 7x - 30 = 0$

$\Rightarrow x^2 - 10x + 3x - 30 = 0$

$\Rightarrow x(x - 10) + 3(x - 10) = 0$

$\Rightarrow (x - 10)(x + 3) = 0$

$x - 10 = 0$ or $x + 3 = 0$

$x = 10$ or $x = -3$

Hence, age cannot be negative.

∴ age of Kishor is 10 and age of Vivek is 15.

5. Suyash scored 10 marks more in second test than that in the first. 5 times the score of the second test is the same as square of the score in the first test. Find his score in the first test.

Solution:

Let the score of first test be x. Then, second test score = x + 10.

$\therefore 5(x + 10) = x^2$

$\Rightarrow 5x + 50 = x^2$

On rearranging we get

$\Rightarrow x^2 - 5x - 50 = 0$

$\Rightarrow x^2 - 10x + 5x - 50 = 0$

$\Rightarrow x(x - 10) + 5(x - 10) = 0$

$\Rightarrow (x - 10)(x + 5) = 0$

$x - 10 = 0$ or $x + 5 = 0$

$x = 10$ or $x = -5$

Hence, score of first test is 10 as marks are not negative.

6. Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ` 600, find production cost of one pot and number of pots he makes per day.

Solution:

Let the number of pots made by Mr. Kasam each day be x. Then, production cost of each pot = ₹$40 + 10(x)$

∴ total cost = $(40 + 10x)x = 40x + 10x^2$

$10x^2 + 40x = 600$

$\Rightarrow 10x^2 + 40x - 600 = 0$

$\Rightarrow x^2 + 4x - 60 = 0$

$\Rightarrow x^2 - 6x + 10x - 60 = 0$

$\Rightarrow x(x - 6) + 10(x - 6) = 0$

$\Rightarrow (x - 6)(x + 10) = 0$

$x - 6 = 0$ or $x + 10 = 0$

$x = 6$ or $x = -10$

Hence number of pots made cannot be negative.

∴ number of pots he made each day = 6

Cost of one pot = $40 + 10(6) = 40 + 60 =$ ₹100

7. Pratik takes 8 hours to travel 36 km downstream and return to the same spot. The speed of boat in still water is 12 km. per hour. Find the speed of water current.

Solution:

Let the speed of water current be x.

$\therefore T_1 = \frac{D_1}{S_1} = \frac{36}{12 + x}\text{hr}$

$T_2 = \frac{D_2}{S_2} = \frac{36}{12 - x}\text{hr}$

$8\text{hr} = \frac{36}{12 + x} + \frac{36}{12 - x}$

$8 = \frac{[36(12 - x) + 36(12 + x)]}{144 - x^2}$

$8 = \frac{36(12 - x + 12 + x)}{144 - x^2}$

$144 - x^2 = \frac{36 \times 24}{8}$

$144 - x^2 = 108$

$144 - 108 = x^2$

$\Rightarrow 36 = x^2$

$\Rightarrow x = \pm 6$

Speed of water current is 6km/hr

8. Pintu takes 6 days more than those of Nishu to complete certain work. If they work together, they finish it in 4 days. How many days would it take to complete the work if they work alone.

Solution:

Suppose Nishu alone takes x days to finish work. Then, Pintu alone can finish in (x + 6) days.

⇒ Nishu's one day work + Pintu's one day work = $\frac{1}{x} + \frac{1}{x+6}$

(Nishu + Pintu)'s one day work = $\frac{1}{4}$

$\therefore \frac{1}{x} + \frac{1}{x+6} = \frac{1}{4}$

On rearranging we get

$\Rightarrow 4(x + 6 + x) = x(x + 6)$

$\Rightarrow 4x + 24 + 4x = x^2 + 6x$

$\Rightarrow x^2 + 6x - 8x - 24 = 0$

$\Rightarrow x^2 - 2x - 24 = 0$

$\Rightarrow x^2 - 6x + 4x - 24 = 0$

$\Rightarrow x(x - 6) + 4(x - 6) = 0$

$\Rightarrow (x - 6)(x + 4) = 0$

$x - 6 = 0$ or $x + 4 = 0$

$x = 6$ or $x = -4$

x = -4 is not possible, as no of days can't be negative.
Nishu will take 6 days alone and Pintu takes 12 days alone

9. If 460 is divided by a natural number, quotient is 6 more than five times the divisor and remainder is 1. Find quotient and divisor.

Solution:

Let the divisor be x. Then, Quotient be 6 + 5x
Now according to question,

dividend = divisor × quotient + remainder.

$\Rightarrow 460 = x \times (6 + 5x) + 1$

$\Rightarrow 459 = 5x^2 + 6x$

$\Rightarrow 5x^2 + 6x - 459 = 0$

$\Rightarrow 5x^2 - 45x + 51x - 459 = 0$

$\Rightarrow 5x(x-9) + 51(x-9) = 0$

$\Rightarrow (5x-51)(x-9) = 0$

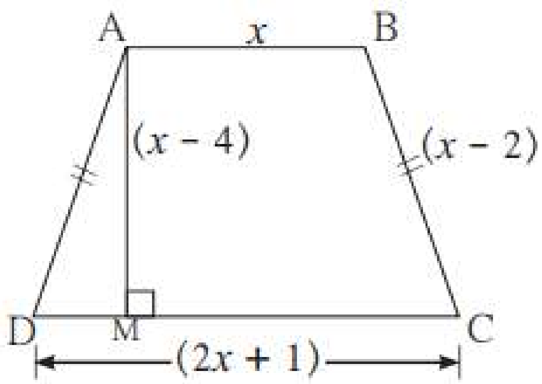
$5x - 51 = 0 \text{ or } x - 9 = 0$

$x = \frac{51}{5} \text{ or } x = 9$

∴ divisor = 9 and quotient = $6 + 5 \times 9 = 6 + 45 = 51$

∴Divisor = 9, quotient = 51

10. In the adjoining fig. □ABCD is a trapezium AB||CD and its area is 33 cm^2. From the information given in the figure find the lengths of all sides of the □ABCD. Fill in the empty boxes to get the solution.



Solution:

Given

ABCD is a trapezium.

And also given that AB||CD

A(□ABCD) = $\frac{1}{2}(AB + CD) \times AM$

$33 = \frac{1}{2}(x + 2x + 1) \times (x - 4)$

$\therefore 3x(x - 7) + 10(x - 7) = 0$

$\therefore (3x + 10)(x - 7) = 0$

$\therefore 3x + 10 = 0 \text{ or } x - 7 = 0$

$\therefore x = -\frac{10}{3} \text{ or } x = 7$

But length is never negative.

$\therefore x \neq \frac{10}{3}$

$\therefore x = 7$

AB = 7 cm, CD = 15 cm, AD = BC = 5 cm.

PROBLEM SET 2 — PAGE NO: 53

1. Choose the correct answers for the following questions.

(1) Which one is the quadratic equation?

(A) $\frac{5}{x} - 3 = x^2$ (B) $x(x + 5) = 2$ (C) $n - 1 = 2n$ (D) $\frac{1}{x^2}(x + 2) = x$

Solution:

B. $x (x + 5) = 2$

Explanation:

It is in the form of $ax^2 + bx + c$

(2) Out of the following equations which one is not a quadratic equation?

A. $x^2 + 4x = 11 + x^2$

B. $x^2 = 4x$

C. $5x^2 = 90$

D. $2x – x^2 = x^2 + 5$

Solution:

A. $x^2 + 4x = 11 + x^2$

Explanation:

In all other options highest degree of equation is 2, which also the degree of quadratic equation. But in Option A, degree of polynomial is 1

(3) The roots of $x^2 + kx + k = 0$ are real and equal, find k.

A. 0

B. 4

C. 0 or 4

D. 2

Solution:

C. 0 or 4

Explanation:

Given

$x^2 + kx + k = 0$, equation has real and equal roots.

$\therefore b^2 - 4ac = 0$

$\Rightarrow k^2 - 4(1)k = 0$

$\Rightarrow k(k - 4) = 0$

k = 0 or $k - 4 = 0 \Rightarrow k = 4$

∴ k = 0 or 4

4. For √2x² – 5x + √ 2 = 0 find the value of the discriminant.

A. -5

B. 17

C. 2

D. 2 √2 - 5

Solution:

B. 17

Explanation:

Given

$\Rightarrow \sqrt{2}x^2 + 5x + \sqrt{2} = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = \sqrt{2}, b = 5$ and $c = \sqrt{2}$

$\therefore b^2 - 4ac = 5^2 - 4(\sqrt{2})(\sqrt{2})$

$= 25 - 8$

$= 17$

5. Which of the following quadratic equations has roots 3, 5?

A. $x^2 - 15x + 8 = 0$

B. $x^2 - 8x + 15 = 0$

C. $x^2 + 3x + 5 = 0$

D. $x^2 + 8x - 15 = 0$

Solution:

B. $x^2 - 8x + 15 = 0$

Explanation:

Given

$x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x-5) - 3(x-5) = 0$

$\Rightarrow (x-5)(x-3) = 0$

$\Rightarrow x - 5 = 0 \text{ or } x - 3 = 0$

$x = 5 \text{ and } x = 3$

6. Out of the following equations, find the equation having the sum of its roots -5.

A. $3x^2 - 15x + 3 = 0$

B. $x^2 - 5x + 3 = 0$

C. $x^2 + 3x - 5 = 0$

D. $3x^2 + 15x + 3 = 0$

Solution:

A. $3x^2 - 15x + 3 = 0$

Explanation:

Sum of the roots i.e. $\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta = -\frac{15}{3} = -5$

7. $\sqrt{5}\, m^2 - \sqrt{5}m + \sqrt{5} = 0$ which of the following statement is true for this given equation?

A. Real and unequal roots

B. Real and equal roots

C. Roots are not real

D. Three roots.

Solution:

C. Roots are not real

Explanation:

$\Rightarrow \sqrt{5}m^2 + \sqrt{5}m + \sqrt{5} = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = \sqrt{5}, b = \sqrt{5}$ and $c = \sqrt{5}$

$\therefore b^2 - 4ac = \sqrt{5}^2 - 4(\sqrt{5})(\sqrt{5})$

$= 5 - 20$

$= -15$

$\therefore b^2 - 4ac < 0$ .hence, roots are not real.

8. One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.
A. -2
B. – ½
C. ½
D. 2

Solution:
C. ½

Explanation:

Given

$x^2 + mx - 5 = 0$,

Put value of x = 2

$2^2 + 2m = 5 \Rightarrow 2m = 5 - 4 \Rightarrow m = \frac{1}{2}$

2. Which of the following equations is quadratic?
(1) $x^2 + 2x + 11 = 0$
(2) $x^2 - 2x + 5 = x^2$
(3) $(x + 2)^2 = 2x^2$

Solution:

1. Given

$x^2 + 2x - 11 = 0$ is a quadratic equation because it is the form of $ax^2 + bc + c = 0$ and it has degree 2.

2. Given
$x^2 - 2x + 5 = x^2$
$-2x + 5 = 0$
∴ it is not a quadratic equation because it is not in the form of
$ax^2 + bc + c = 0$ and it doesn't have degree 2.

3. Given
$(x + 2)^2 = 2x^2 \Rightarrow x^2 + 4x + 4 = 2x^2$
$x^2 - 4x - 4 = 0$ is a quadratic equation because it is the form of
$ax^2 + bc + c = 0$ and it has degree 2.

3. Find the value of discriminant for each of the following equation.
(1) $2y^2 - y + 2 = 0$

Solution:
Given
$\Rightarrow 2y^2 - y + 2 = 0$ compare with $ax^2 + bx + c = 0$
We get
$\Rightarrow a = 2, b = -1$ and $c = 2$
The value of discriminant is
$\therefore b^2 - 4ac = -1^2 - 4(2)(2)$
$= 1 - 16$
$= -15$

(2) $5m^2 - m = 0$

Solution:
Given
$\Rightarrow 5m^2 - m = 0$ compare with $ax^2 + bx + c = 0$
We get
$\Rightarrow a = 5, b = -1$ and $c = 0$
The value of discriminant is
$\therefore b^2 - 4ac = -1^2 - 4(5)(0)$
$= 1$

(3) $\sqrt{5}x^2 - x - \sqrt{5} = 0$

Solution:

Given

$\Rightarrow \sqrt{5}x^2 - x - \sqrt{5} = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = \sqrt{5}, b = -1$ and $c = -\sqrt{5}$

The value of discriminant is

$\therefore b^2 - 4ac = -1^2 - 4(\sqrt{5})(-\sqrt{5})$

$= 1 + 20$

$= 21$

4. One of the roots of quadratic equation $2x^2 + kx - 2 = 0$ is -2, find k.

Solution:

Given

$2x^2 + kx - 2 = 0$

Substitute x = -2 in above equation

$\Rightarrow 2 \times -2^2 - 2k - 2 = 0$

$\Rightarrow 8 - 2 - 2k = 0$

$\Rightarrow 6 = 2k$

$k = 3$

5. Two roots of quadratic equations are given; frame the equation.
(1) 10 and -10

Solution:

Let $\alpha = 10$ and $\beta = -10$

$\therefore \alpha + \beta = 10 - 10 = 0$ $\alpha \beta = 10(-10) = -100$

Quadratic equation is

$x^2 - (\alpha + \beta) x + \alpha \beta = 0$

$\Rightarrow x^2 - 0(x) - 100 = 0$

$\Rightarrow x^2 - 100 = 0$

(2) 1–3√5 and 1 + 3√5

Solution:

Let $\alpha = 1 - 3\sqrt{5}$ and $\beta = 1 + 3\sqrt{5}$

$\therefore \alpha + \beta = 1 - 3\sqrt{5} + 1 + 3\sqrt{5} = 2$ and $\alpha\beta = (1 - 3\sqrt{5}) \times (1 + 3\sqrt{5})$

$= 1 - 45 = -44$

$\therefore$ and quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (2)x + (-44) = 0$

$\therefore x^2 - 2x - 44 = 0$

(3) 0 and 7

Solution:

Let $\alpha = 0$ and $\beta = 7$

$\therefore \alpha + \beta = 0 + 7 = 7$ and $\alpha\beta = 0 \times 7 = 0$

$\therefore$ and quadratic equation is, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore x^2 - (27)x + (0) = 0$

$\therefore x^2 - 7x = 0$

6. Determine the nature of roots for each of the quadratic equation.

(1) $3x^2 - 5x + 7 = 0$

Solution:

Given

$\Rightarrow 3x^2 - 5x + 7 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 3, b = -5$ and $c = 7$

The value of discriminant is

$\therefore b^2 - 4ac = -5^2 - 4(3)(7)$

$= 25 - 147$

$= -122$

$\therefore b^2 - 4ac < 0$ .hence, roots are not real.

(2) $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$

Solution:

Given

$\Rightarrow \sqrt{3}x^2 + \sqrt{2}x + 2\sqrt{3} = 0$ compare with $ax^2 + bx + c = 0$

we get

$\Rightarrow a = \sqrt{3}, b = \sqrt{2}$ and $c = -2\sqrt{3}$

The value of discriminant is

$\therefore b^2 - 4ac = \sqrt{2}^2 - 4(\sqrt{3})(-2\sqrt{3})$

$= 2 + 24 = 26$

$\therefore b^2 - 4ac > 0$. hence, roots are real and umequal .

(3) $m^2 - 2m + 1 = 0$

Solution:

Given

$\Rightarrow m^2 - 2m + 1 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = -2$ and $c = 1$

The value of discriminant is

$\therefore b^2 - 4ac = -2^2 - 4(1)(1)$

$= 4 - 4$

$= 0$

$\therefore b^2 - 4ac = 0$. hence, roots are real and equal.

7. Solve the following quadratic equation.

(1) $\frac{1}{x+5} = \frac{1}{x^2}$

Solution:

Given equation can be written as

$x^2 = x + 5$

$\Rightarrow x^2 - x - 5 = 0$

$\Rightarrow x^2 - x - 5 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 1, b = -1$ and $c = -5$

$\therefore\ b^2 - 4ac = -1^2 - 4(1)(-5)$

$= 1 + 20$

$= 21$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow x = \frac{1 \pm \sqrt{21}}{2 \times 1}$

$\Rightarrow x = \frac{1 \pm \sqrt{21}}{2}$

The roots are

$\Rightarrow x = \frac{1 + \sqrt{21}}{2}$ or $x = \frac{1 + \sqrt{21}}{2}$

(2) $x^2 - \frac{3x}{10} - \frac{1}{10} = 0$

Solution:

Given equation can be written as

$10x^2 - 3x - 1 = 0$

$\Rightarrow 10x^2 - 3x - 1 = 0$ compare with $ax^2 + bx + c = 0$

We get

$\Rightarrow a = 10, b = -3$ and $c = -1$

$\therefore\ b^2 - 4ac = -3^2 - 4(10)(-1)$

$= 9 + 40$

$= 49$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow x = \frac{3 \pm \sqrt{49}}{2 \times 10}$

$\Rightarrow x = \frac{3 \pm 7}{20}$

$\Rightarrow x = \frac{3 + 7}{20}$ or $x = \frac{3 - 7}{20}$

$\Rightarrow x = \frac{10}{20}$ or $x = \frac{-4}{20}$

The roots are

$\Rightarrow x = \frac{1}{2} \text{ or } x = -\frac{1}{5}$

(3) $(2x + 3)^2 = 25$

Solution:

Given equation can be written as

$4x^2 + 12x + 9 - 25 = 0 \Rightarrow 4x^2 + 12x - 16 = 0$

$\Rightarrow x^2 + 3x - 4 = 0$ compare with $ax^2 + bx + c = 0$

we get

$\Rightarrow a = 1, b = 3 \text{ and } c = -4$

$\therefore b^2 - 4ac = 3^2 - 4(1)(-4)$

$= 9 + 16$

$= 25$

we know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow x = \frac{-3 \pm \sqrt{25}}{2 \times 1}$

$\Rightarrow x = \frac{-3 \pm 5}{2}$

$\Rightarrow x = \frac{-3 + 5}{2} \text{ or } x = \frac{-3-5}{2}$

$\Rightarrow x = \frac{2}{2} \text{ or } x = \frac{-8}{2}$

the roots are

$\Rightarrow x = 1 \text{ or } x = -4$

(4) $m^2 + 5m + 5 = 0$

Solution:

Given

$\Rightarrow m^2 + 5m + 5 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = 1, b = 5 \text{ and } c = 5$

$\therefore b^2 - 4ac = 5^2 - 4(1)(5)$

$= 25 - 20$
$= 5$
We know that
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$\Rightarrow x = \frac{-5 \pm \sqrt{5}}{2 \times 1}$
$$\Rightarrow x = \frac{-5 \pm \sqrt{5}}{2}$$
The roots are
$\Rightarrow x = \frac{-5 + \sqrt{5}}{2}$ or $x = \frac{-5 - \sqrt{5}}{2}$

(5) $5m^2 + 2m + 1 = 0$

Solution:

Given
$\Rightarrow 5m^2 + 2m + 1 = 0$ compare with $ax^2 + bx + c = 0$
$\Rightarrow a = 5, b = 2$ and $c = 1$
$\therefore b^2 - 4ac = 2^2 - 4(5)(1)$
$= 4 - 20$
$= -16$
We know that
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
x = (-2 – 16)/10 Or x = (-2 + 16)/10

(6) $x^2 – 4x – 3 = 0$

Solution:

Given
$\Rightarrow x^2 - 4x - 3 = 0$ compare with $ax^2 + bx + c = 0$
$\Rightarrow a = 1, b = -4$ and $c = -3$
$\therefore b^2 - 4ac = -4^2 - 4(1)(-3)$
$= 16 + 12$

$= 28$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow x = \frac{4 \pm \sqrt{28}}{2 \times 1}$

$\Rightarrow x = \frac{4 \pm 2\sqrt{7}}{2}$

$\Rightarrow x = \frac{4 + 2\sqrt{7}}{2}$ or $x = \frac{4 - 2\sqrt{7}}{2}$

$\Rightarrow x = \frac{2(2 + \sqrt{7})}{2}$ or $x = \frac{2(2 - \sqrt{7})}{2}$

$\Rightarrow x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$

8. Find m if (m - 12) x^2 + 2 (m - 12) x + 2 = 0 has real and equal roots.

Solution:

Given

$\Rightarrow (m - 12)x^2 - (2m - 24)x + 2 = 0$ compare with $ax^2 + bx + c = 0$

$\Rightarrow a = m - 12, b = -2m + 24$ and $c = 2$

We know that

$\therefore b^2 - 4ac = (-2m + 24)^2 - 4(m - 12)(2)$

$= 4m^2 - 96m + 576 - 8m + 96$

$= 4m^2 - 104m + 672$

$= m^2 - 26m + 168$

If roots are equal and real then, $\therefore b^2 - 4ac = 0$

$m^2 - 26m + 168 = 0$

$\Rightarrow m^2 - 12m - 14m + 168 = 0$

$\Rightarrow m(m - 12) - 14(m - 12) = 0$

$\Rightarrow (m - 12)(m - 14) = 0$

m = 12 or m = 14

9. The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

Solution:

According to the question

$\alpha + \beta = 5$

$\Rightarrow \alpha^3 + \beta^3 = 35$

We know that

$\because \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$\Rightarrow 35 = 5(\alpha^2 + \beta^2 + 2\alpha\beta - 3\alpha\beta)$

$\Rightarrow 35 = 5\{(\alpha + \beta)^2 - 3\alpha\beta\}$

$\Rightarrow 7 = 25 - 3\alpha\beta$

$\Rightarrow 3\alpha\beta = 18$

$\Rightarrow \alpha\beta = 6$

$x^2 - (\alpha + \beta)x + \alpha\beta \Rightarrow x^2 - 5x + 6 = 0$

10. Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation
$2x^2 + 2(p + q)x + p^2 + q^2 = 0$

Solution:

Let's assume roots are m and n.

So, we want the equation whose roots would be $(m + n)^2$ and $(m - n)^2$

So, the sum of the roots of our desired equation would be $2(m + n)^2$ and product of the roots would be $(m + n)^2(m - n)^2$

What we know from given equation are:

$m + n = -(p + q)$

and $mn = \frac{p^2 + q^2}{2}$

the sum and product are:

$s = 2(m^2 + n^2) = 2(m + n)^2 - 2mn$

$= 2(p + q)^2 - (p^2 + q^2) = 2 \times 2pq = 4pq$

and

$P = (m + n)^2(m - n)^2$

$= (p + q)^2 (m + n)^2 - 4mn$

$= (p + q)^2(p + q)^2 - 2(p^2 + q^2)$

$= (p + q)^2(2pq - p^2 - q^2)$
$= -(p + q)^2(p - q)^2$
$= -(p^2 - q^2)^2$
Our desired equation would be $x^2 - sx + P = 0$
So, $x^2 - 4pqx - (p^2 - q^2)^2 = 0$ is our desired equation

11. Mukund possesses ₹50 more than what Sagar possesses. The product of the amount they have is 15,000. Find the amount each one has.

Solution:
Let Sagar has x amount
Then
Mukund's amount = $x + 50$
$x(x + 50) = 15000$
$\Rightarrow x^2 + 50x - 15000 = 0$
Splitting the middle term, we get
$\Rightarrow x^2 - 100x + 150x - 15000 = 0$
$\Rightarrow x (x - 100) + 150(x - 100) \Rightarrow (x - 100) (x+150)$
$\therefore x = (-150)$,
100x = 100 as money can't be negative
therefore, we ignore (-150)
∴ Sagar has 100Rs and Mukund has 150Rs

12. The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.

Solution:
The difference of the square of the two numbers is 120.
$a^2 - b^2 = 120 \ldots I$
The square of smaller number is 2 times the larger number.
$b^2 = 2a \ldots II$
Put the value of b^2 from equation II in equation I, it gives
$a^2 - 2a = 120$

$a^2 - 2a - 120 = 0$

$\Rightarrow a^2 + 10a - 12a - 120 = 0$

$\Rightarrow a(a + 10) - 12(a + 10) = 0$

$\Rightarrow (a + 10)(a - 12) = 0$

$a + 10 = 0$ or $a - 12 = 0$

$a = -10$ or $a = 12$

$b = \sqrt{2a} \Rightarrow b = \sqrt{2(12)} \Rightarrow b = \sqrt{24}$

$b = \pm\sqrt{24}$

12 and $\sqrt{24}$ or

12 and - $\sqrt{24}$

13. Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.

Solution:

Total oranges = 540

Initial student = x

Initial orange for 1 student = n

$nx = 540$

$(n - 3)(x + 30) = 540$

$nx = (n - 3)(x + 30)$

$nx = nx + 30n - 3x - 90$

$30n = 3x + 90$

$x = \frac{30n - 90}{3}$

$x = 10n - 30$

$\because nx = 540$

$n(10n - 30) = 540$

$n(n - 3) = 54$

$n^2 - 3n - 54 = 0$

$n^2 - 9n + 6n - 54 = 0$

$n(n - 9) + 6(n - 9) = 0$

$(n - 9)(n + 6) = 0$

$(n-9)(n+6) = 0$

$\Rightarrow n-9 = 0$ or $n+6 = 0$

$\Rightarrow n = 9$ or $n = -6$ ($\because$

$nx = 540 \Rightarrow x = \frac{540}{9} \Rightarrow x = 60$

$\therefore$ number of students = 60 students.

14. Mr. Dinesh owns an agricultural farm at village Talvel. The length of the farm is 10 meters more than twice the breadth. In order to harvest rain water, he dug a square shaped pond inside the farm. The side of pond is 1/3 of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and of the pond.

Solution:

Let the breadth of the farm be x.

$\therefore$ length of the farm $= 2x + 10$

side of the pond $= \frac{x}{3}$

According to the question,

area of farm $= 20$(area of pond)

$\Rightarrow x(2x+10) = 20\left(\frac{x}{3}\right)^2$

$\Rightarrow 2x^2 + 10x = \frac{20x^2}{9}$

$\Rightarrow 10x = \frac{20x^2}{9} - 2x^2$

$\Rightarrow 10x = \frac{20x^2 - 18x^2}{9}$

$\Rightarrow 90x = 2x^2 \Rightarrow 2x^2 - 90x$

$\Rightarrow x(2x-90) = 0$

$\Rightarrow x = 0$ or $2x - 90 = 0$

$x = \frac{90}{2} = 45$

$\therefore$ length of the farm $= 2x + 10 = 2(45) + 10 = 100$

side of the pond $= \frac{x}{3} = \frac{45}{3} = 15$

Breadth 45 m. length 100 m, side of the pond 15 m.

15. A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely?

Solution:

Let the time taken by larger tap alone be x hr.

Then,

Time taken by smaller tap be x + 3 hr

In an hour, the larger tap can fill $\frac{1}{x}$ tank.

∴ In an hour, the larger tap can fill $\frac{1}{x+3}$ tank.

Two taps together can fill a tank in 2 hr.

But in an hour, taps fill in ½ hr of the tank.

$\therefore \frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$

$\Rightarrow 2(x + 3 + x) = x(x + 3)$

$\Rightarrow 4x + 6 = x^2 + 3x$

$\Rightarrow x^2 + 3x - 4x - 6 = 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x - 3) + 2(x - 3) = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$x - 3 = 0$ or $x + 2 = 0$

$x = 3$ or $x = -2$

$x = 3$ because time taken cannot be negative

For larger tap 3 hours and for smaller tap 6 hours.