PRACTICE SET 3.1 PAGE NO: 61

1. Which of the following sequences are A.P.? If they are A.P. find the common difference.
(1) 2, 4, 6, 8, . . .

Solution:
Given 2, 4, 6, 8, . . .
Here, the first term, $a_1 = 2$
Second term, $a_2 = 4$
And $a_3 = 6$
Now, common difference = $a_2 - a_1 = 4 - 2 = 2$
Also, $a_3 - a_2 = 6 - 4 = 2$
Since, the common difference is same.
Hence the terms are in Arithmetic progression with common difference, d = 2.

(2) 2, 5/2, 3, 7/3, . . .

Solution:
Given
$2, \frac{5}{2}, 3, \frac{7}{3}, \ldots..$
Here, the first term, $a_1 = 2$
Second term, $a_2 = \frac{5}{2}$
Third Term, $a_3 = 3$
Now, common difference = $a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$
Also, $a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$
Since, the common difference is same.
Hence the terms are in Arithmetic progression with common difference, $d = \frac{1}{2}$.

(3) – 10, – 6, – 2, 2, . . .

Solution:

Given – 10, – 6, – 2,2, . . .
Here, the first term, $a_1 = -10$
Second term, $a_2 = -6$
$a_3 = -2$
Now, common difference = $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$
Also, $a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$
Since, the common difference is same.
Hence the terms are in Arithmetic progression with common difference, d = 4.

(4) 0.3, 0.33, .0333, . . .

Solution:
Given 0.3, 0.33, 0.333, . . .
Here, the first term, $a_1 = 0.3$
Second term, $a_2 = 0.33$
$a_3 = 0.333$
Now, common difference = $a_2 - a_1 = 0.33 - 0.3 = 0.03$
Also, $a_3 - a_2 = 0.333 - 0.33 = 0.003$
Since, the common difference is not same.
Hence the terms are not in Arithmetic progression

(5) 0, – 4, – 8, – 12, . . .

Solution:
Given 0, – 4, – 8, – 12, . . .
Here, the first term, $a_1 = 0$
Second term, $a_2 = -4$
$a_3 = -8$
Now, common difference = $a_2 - a_1 = -4 - 0 = -4$
Also, $a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$
Since, the common difference is same.
Hence the terms are in Arithmetic progression with common difference, d = – 4.

(6) -1/5, -1/5, - 1/5, . . .

Solution:

Given

$-\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}, ...$

Here, the first term, $a_1 = -\frac{1}{5}$

Second term, $a_2 = -\frac{1}{5}$

$a_3 = -\frac{1}{5}$

Now, common difference $= a_2 - a_1 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$

Also, $= a_3 - a_2 = -\frac{1}{5} - \left(-\frac{1}{5}\right) = -\frac{1}{5} + \frac{1}{5} = 0$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = 0.

(7) 3, 3 + √2, 3 + 2√2, 3 + 3√2, ….

Solution:

Given

3, 3 + √2, 3 + 2√2, 3 + 3√2, ….

Here, the first term, $a_1 = 3$

Second term, $a_2 = 3 + \sqrt{2}$

$a_3 = 3 + 2\sqrt{2}$

Now, common difference = $a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

Also, $a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = √2.

(8) 127, 132, 137, . . .

Solution:

Given 127, 132, 137, . . .

Here, the first term, $a_1 = 127$

Second term, $a_2 = 132$

$a_3 = 137$

Now, common difference = $a_2 - a_1 = 132 - 127 = 5$

Also, $a_3 - a_2 = 137 - 132 = 5$

Since, the common difference is same.

Hence the terms are in Arithmetic progression with common difference, d = 5.

2. Write an A.P. whose first term is a and common difference is d in each of the following.
(1) a = 10, d = 5

Solution:
Given a = 10, d = 5
Let $a_1 = a = 10$
Since, the common difference d = 5
Using formula $a_{n+1} = a_n + d$
Thus, $a_2 = a_1 + d = 10 + 5 = 15$
$a_3 = a_2 + d = 15 + 5 = 20$
$a_4 = a_3 + d = 20 + 5 = 25$
Hence, An A.P with common difference 5 is 10, 15, 20, 25, ….

(2) a = – 3, d = 0

Solution:
Given a = – 3, d = 0
Let $a_1 = a = -3$
Since, the common difference d = 0
Using formula $a_{n+1} = a_n + d$
Thus, $a_2 = a_1 + d = -3 + 0 = -3$
$a_3 = a_2 + d = -3 + 0 = -3$
$a_4 = a_3 + d = -3 + 0 = -3$
Hence, An A.P with common difference 0 is – 3, – 3, – 3, – 3, ….

(3) a = - 7, d = ½

Solution:
Given
$a = -7, d = \frac{1}{2}$
Let $a_1 = a = -7$
Since, the common difference $d = \frac{1}{2}$

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = -7 + \frac{1}{2} = \frac{-14+1}{2} = -\frac{13}{2}$

$a_3 = a_2 + d = -\frac{13}{2} + \frac{1}{2} = \frac{-13+1}{2} = -\frac{12}{2} = -6$

$a_4 = a_3 + d = -6 + \frac{1}{2} = \frac{-12+1}{2} = -\frac{11}{2}$

Hence, An A.P with common difference $\frac{1}{2}$ is $-7, -\frac{13}{2}, -6, -\frac{11}{2}, \ldots..$

(4) a = – 1.25, d = 3

Solution:

Given a = – 1.25, d = 3

Let $a_1 = a = -1.25$

Since, the common difference d = 3

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = -1.25 + 3 = 1.75$

$a_3 = a_2 + d = 1.75 + 3 = 4.75$

$a_4 = a_3 + d = 4.75 + 3 = 7.75$

Hence, An A.P with common difference 3 is – 1.25, 1.75, 4.75, 7.75

(5) a = 6, d = – 3

Solution:

Given a = 6, d = – 3

Let $a_1 = a = 6$

Since, the common difference d = – 3

Using formula $a_{n+1} = a_n + d$

Thus, $a_2 = a_1 + d = 6 + (-3) = 6 - 3 = 3$

$a_3 = a_2 + d = 3 + (-3) = 3 - 3 = 0$

$a_4 = a_3 + d = 0 + (-3) = -3$

Hence, An A.P with common difference – 3 is 6, 3, 0, – 3…

(6) a = – 19, d = – 4

Solution:

Given a = – 19, d = – 4

Let $a_1 = a = -19$
Since, the common difference $d = -4$
Using formula $a_{n+1} = a_n + d$
Thus, $a_2 = a_1 + d = -19 + (-4) = -19 - 4 = -23$
$a_3 = a_2 + d = -23 + (-4) = -23 - 4 = -27$
$a_4 = a_3 + d = -27 + (-4) = -27 - 4 = -31$
Hence, An A.P with common difference – 4 is – 19, – 23, – 27, – 31, ….

3. Find the first term and common difference for each of the A.P.
(1) 5, 1, – 3, – 7, . . .

Solution:
Given 5, 1, – 3, – 7, . . .
First term $a_1 = 5$
Second term $a_2 = 1$
Third term $a_3 = -3$
We know that $d = a_{n+1} - a_n$
Thus, $d = a_2 - a_1 = 1 - 5 = -4$
Hence, the common difference d = – 4 and first term is 5

(2) 0.6, 0.9, 1.2, 1.5, . . .

Solution:
Given 0.6, 0.9, 1.2, 1.5, . . .
First term $a_1 = 0.6$
Second term $a_2 = 0.9$
Third term $a_3 = 1.2$
We know that $d = a_{n+1} - a_n$
Thus, $d = a_2 - a_1 = 0.9 - 0.6 = 0.3$
Hence, the common difference d = 0.3 and first term is 0.6

(3) 127, 135, 143, 151, . . .

Solution:
Given 127, 135, 143, 151, . . .
First term $a_1 = 127$
Second term $a_2 = 135$

Third term a_3 = 143

We know that $d = a_{n+1} - a_n$

Thus, $d = a_2 - a_1 = 135 - 127 = 8$

Hence, the common difference d = 8 and first term is 127

(4) 1/4, ¾, 5/4, 7/4, . . .

Solution:

Given

$\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, ...$

First term $a_1 = \frac{1}{4}$

Second term $a_2 = \frac{3}{4}$

Third term $a_3 = \frac{5}{4}$

We know that $d = a_{n+1} - a_n$

Thus, $d = a_2 - a_1 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the common difference d = ½ and first term is $\frac{1}{4}$

PRACTICE SET 3.2

1. Write the correct number in the given boxes from the following A. P.

(1) 1, 8, 15, 22, . . .

Here $a = \square$, $t_1 = \square$, $t_2 = \square$, $t_3 = \square$,

$t_2 - t_1 = \square - \square = \square$

$t_3 - t_2 = \square - \square = \square \therefore d = \square$

Solution:

Given 1, 8, 15, 22, . . .

First term a = 1

Second term $t_1 = 8$

Third term $t_2 = 15$

Fourth term $t_3 = 22$

We know that $d = t_{n+1} - t_n$

Thus, $t_2 - t_1 = 15 - 8 = 7$

$t_3 - t_2 = 22 - 15 = 7$

Thus, d = 7

(2) 3, 6, 9, 12, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, $t_4 = \square$,

$t_2 - t_1 = \square$, $t_3 - t_2 = \square$ $\therefore d = \square$

Solution:

Given 3,6,9,12, . . .

First term a = 3

Second term $t_1 = 6$

Third term $t_2 = 9$

Fourth term $t_3 = 12$

We know that $d = t_{n+1} - t_n$

Thus, $t_2 - t_1 = 9 - 6 = 3$

$t_3 - t_2 = 12 - 9 = 3$

Thus, d = 3

(3) – 3, – 8, – 13, – 18, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, $t_4 = \square$,
$t_2 - t_1 = \square$, $t_3 - t_2 = \square$ $\therefore d = \square$

Solution:
Given – 3, – 8, – 13, – 18, . . .
First term a = – 3
Second term t_1 = – 8
Third term t_2 = – 13
Fourth term t_3 = – 18
We know that $d = t_{n+1} - t_n$
Thus, $t_2 - t_1 = -13 - (-8) = -13 + 8 = -5$
$t_3 - t_2 = -18 - (-13) = -18 + 13 = -5$
Thus, d = – 5

(4) 70, 60, 50, 40, . . .

Here $t_1 = \square$, $t_2 = \square$, $t_3 = \square$, . . .
$\therefore a = \square$, $d = \square$

Solution:
Given 70, 60, 50, 40, . . .
First term a = 70
Second term t_1 = 60
Third term t_2 = 50
Fourth term t_3 = 40
We know that $d = t_{n+1} - t_n$
Thus, $t_2 - t_1 = 50 - 60 = -10$
$t_3 - t_2 = 40 - 50 = -10$
Thus, d = – 10

2. – 12, – 5, 2, 9, 16, 23, 30, . . .

Solution:
Given A.P. is – 12, – 5, 2, 9, 16, 23, 30, . . .
Here first term a = – 12
Second term t_1 = – 5

Third term $t_2 = 2$
Common Difference $d = t_2 - t_1 = 2 - (-5) = 2 + 5 = 7$
We know that, n^{th} term of an A.P. is
$t_n = a + (n - 1) d$
We need to find the 20^{th} term,
Here n = 20
Thus, $t_{20} = -12 + (20 - 1) \times 7$
$t_{20} = -12 + (19) \times 7 = -12 + 133 = 121$
Thus, $t_{20} = 121$

3. Given Arithmetic Progression 12, 16, 20, 24, . . . Find the 24th term of this progression.

Solution:
Given A.P. is 12, 16, 20, 24, . . .
Where first term a = 12
Second term $t_1 = 16$
Third term $t_2 = 20$
Common Difference $d = t_2 - t_1 = 20 - 16 = 4$
We know that, n^{th} term of an A.P. is $t_n = a + (n - 1) d$
We need to find the 24^{th} term,
Here n = 24
Thus, $t_{24} = 12 + (24 - 1) \times 4$
$t_{24} = 12 + (23) \times 4 = 12 + 92 = 104$
Thus, $t_{24} = 104$

4. Find the 19th term of the following A.P.
7, 13, 19, 25, . . .

Solution:
Given A.P. is 7, 13, 19, 25, . . .
Where first term a = 7
Second term $t_1 = 13$
Third term $t_2 = 19$
Common Difference $d = t_2 - t_1 = 19 - 13 = 6$
We know that, n^{th} term of an A.P. is
$t_n = a + (n - 1) d$

We need to find the 19th term,
Here n = 19
Thus, $t_{19} = 7 + (19 – 1) \times 6$
$t_{19} = 7 + (18) \times 6 = 7 + 108 = 115$
Thus, $t_{19} = 115$

5. Find the 27th term of the following A.P.
9, 4, – 1, – 6, – 11, . . .

Solution:
Given A.P. is 9, 4, – 1, – 6, – 11, . . .
Where first term a = 9
Second term $t_1 = 4$
Third term $t_2 = – 1$
Common Difference $d = t_2 – t_1 = – 1 – 4 = – 5$
We know that, nth term of an A.P. is $t_n = a + (n – 1) d$
We need to find the 27th term,
Here n = 27
Thus, $t_{27} = 9 + (27 – 1) \times (– 5)$
$t_{27} = 9 + (26) \times (– 5) = 9 – 130 = – 121$
Thus, $t_{27} = – 121$

6. Find how many three-digit natural numbers are divisible by 5.

Solution:
List of three-digit number divisible by 5 are
100, 105, 110, 115, ………. 995
Let us find how many such number are there?
From the above sequence, we know that
$t_n = 995$, a = 100
$t_1 = 105$, $t_2 = 110$
Thus, $d = t_2 – t_1 = 110 – 105 = 5$
Now, by using nth term of an A.P. formula that is $t_n = a + (n – 1) d$
we can find value of "n"
Thus, on substituting all the value in formula we get,
$995 = 100 + (n – 1) \times 5$
$\Rightarrow 995 – 100 = (n – 1) \times 5$

$\Rightarrow 895 = (n - 1) \times 5$
$\Rightarrow n - 1 = 895/5 = 179$
$\Rightarrow n = 179 + 1 = 180$

7. The 11th term and the 21st term of an A.P. are 16 and 29 respectively, then find the 41th term of that A.P.

Solution:

Given $t_{11} = 16$ and $t_{21} = 29$
Now we have to find t_{41}
Using n^{th} term of an A.P. formula $t_n = a + (n - 1)\, d$
we will find value of "a" and "d"
Let, $t_{11} = a + (11 - 1)\, d$
$\Rightarrow 16 = a + 10\, d$ .... (1)
$t_{21} = a + (21 - 1)\, d$
$\Rightarrow 29 = a + 20\, d$ .... (2)
Subtracting equation (1) from equation (2), we get,
$\Rightarrow 29 - 16 = (a - a) + (20\, d - 10\, d)$
$\Rightarrow 13 = 10\, d$
$\Rightarrow d = \frac{13}{10} = 1.3$
Substitute value of "d" in equation (1) to get value of "a"
$\Rightarrow 16 = a + 10 \times \frac{13}{10}$
$\Rightarrow 16 = a + 13$
$\Rightarrow a = 16 - 13 = 3$
Now, we will find the value of t_{41} using n^{th} term of an A.P. formula
$\Rightarrow t_{41} = 3 + (41 - 1) \times \frac{13}{10}$
$\Rightarrow t_{41} = 3 + 40 \times \frac{13}{10}$
$\Rightarrow t_{41} = 3 + 4 \times 13 = 3 + 52 = 55$
Thus, $t_{41} = 55$

8. 11, 8, 5, 2, . . . In this A.P. which term is number – 151?

Solution:
By, given A.P. 11, 8, 5, 2, . . .
we have $a = 11$, $t_1 = 8$, $t_2 = 5$
Thus, $d = t_2 - t_1 = 5 - 8 = -3$
Given $t_n = -151$
Now, by using n^{th} term of an A.P. formula $t_n = a + (n - 1) d$
we can find value of "n"
Thus, on substituting all the value in formula we get,
$-151 = 11 + (n - 1) \times (-3)$
$\Rightarrow -151 - 11 = (n - 1) \times (-3)$
$\Rightarrow -162 = (n - 1) \times (-3)$
$\Rightarrow n - 1 = -162/-3 = 54$
$\Rightarrow n = 54 + 1 = 55$

9. In the natural numbers from 10 to 250, how many are divisible by 4?

Solution:
The number divisible by 4 in between 10 to 250 are
12, 16, 20, 24, ………... 248
From the above sequence, we have
$t_n = 248$, $a = 12$
$t_1 = 16$, $t_2 = 20$
Thus, $d = t_2 - t_1 = 20 - 16 = 4$
Now, by using n^{th} term of an A.P. formula $t_n = a + (n - 1) d$
we can find value of "n"
Thus, on substituting all the value in formula we get,
$248 = 12 + (n - 1) \times 4$
$\Rightarrow 248 - 12 = (n - 1) \times 4$
$\Rightarrow 236 = (n - 1) \times 4$
$\Rightarrow n - 1 = 236/3 = 59$
$\Rightarrow n = 59 + 1 = 60$

10. In an A.P. 17^{th} term is 7 more than its 10^{th} term. Find the common difference.

Solution:
Given $t_{17} = 7 + t_{10}$ …… (1)
In t_{17}, $n = 17$

In t_{10}, n = 10

By using n^{th} term of an A.P. formula, $t_n = a + (n – 1) d$

where n = number of terms

a = first term

d = common difference

t_n = n^{th} term

Thus, on using formula in eq. (1) we get,

⇒ a + (17 – 1) d = 7 + (a + (10 – 1) d)

⇒ a + 16 d = 7 + (a + 9 d)

⇒ a + 16 d – a – 9 d = 7

⇒ 7 d = 7

⇒ d = 7/7 = 1

Thus, common difference "d" = 1

PRACTICE SET 3.3 PAGE NO: 72

1. First term and common difference of an A.P. are 6 and 3 respectively; find S_{27}.

$a = 6,\ d = 3,\ S_{27} = ?$

$$S_n = \frac{n}{2}[\square + (n-1)d]$$

$$S_{27} = \frac{27}{2}[12 + (27-1)\square]$$

$$= \frac{27}{2} \times \square$$

$$= 27 \times 45 = \square$$

Solution:

Given first term a = 6

And Common Difference d = 3

Now we have to find S_{27} where n = 27

By using sum of n^{th} term of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = number of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, substituting given value in formula we can find the value of S_{27}

$$\Rightarrow S_{27} = \frac{27}{2}[2 \times 6 + (27-1) \times 3]$$

$$\Rightarrow S_{27} = \frac{27}{2}[12 + 26 \times 3]$$

$$\Rightarrow S_{27} = \frac{27}{2}[12 + 78]$$

$$\Rightarrow S_{27} = \frac{27}{2} \times 90 = 27 \times 45 = 1215$$

Thus, S_{27} = 1215

2. Find the sum of first 123 even natural numbers.

Solution:

The first 123 even natural number is
2, 4, 6, …….
Where first term a = 2
Second term t_1 = 4
Third term t_2 = 6
Thus, common difference d = $t_2 - t_1$ = 6 – 4 = 2
n = 123
By using sum of n^{th} term of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = number of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, substituting given value in formula we can find the value of S_n

$$\Rightarrow S_n = \frac{123}{2}[2 \times 2 + (123 - 1) \times 2]$$

$$\Rightarrow S_n = \frac{123}{2}[4 + 122 \times 2]$$

$$\Rightarrow S_n = \frac{123}{2}[4 + 244]$$

$$\Rightarrow S_n = \frac{123}{2} \times 248 = 123 \times 122 = 15252$$

Thus, S_n = 15252

3. Find the sum of all even numbers from 1 to 350.

Solution:
The even natural number between 1 to 350 is
2,4, 6, …….348
Where first term a = 2
Second term t_1 = 4
Third term t_2 = 6
Thus, common difference d = $t_2 - t_1$ = 6 – 4 = 2
t_n = 348 (As we have to find the sum of even numbers between 1 and 350 therefore excluding 350)
Now, by using n^{th} term of an A.P. formula $t_n = a + (n-1)d$

where n = number of terms
a = first term
d = common difference
t_n = n^{th} terms
we can find value of "n" by substituting all the value in formula we get,
$\Rightarrow 348 = 2 + (n - 1) \times 2$
$\Rightarrow 348 - 2 = 2(n - 1)$
$\Rightarrow 346 = 2(n - 1)$
$$\Rightarrow n - 1 = \frac{346}{2} = 173$$
$\Rightarrow n = 173 + 1 = 174$
Now, by using sum of n^{th} term of an A.P. we have to find its sum
$$S_n = \frac{n}{2}[2a + (n - 1)d]$$
Where, n = number of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, substituting given value in formula we can find the value of S_n
$$\Rightarrow S_{174} = \frac{174}{2}[2 \times 2 + (174 - 1) \times 2]$$
$$\Rightarrow S_{174} = \frac{174}{2}[4 + 173 \times 2]$$
$$\Rightarrow S_{174} = \frac{174}{2}[4 + 346]$$
$$\Rightarrow S_{174} = \frac{174}{2} \times 350 = 174 \times 175 = 30{,}450$$
Thus, S_{174} = 30,450

4. In an A.P. 19th term is 52 and 38th term is 128, find sum of first 56 terms.

Solution:
Given $t_{19} = 52$ and $t_{38} = 128$
Now we have to find the value of "a" and "d"
Using n^{th} term of an A.P. formula $t_n = a + (n - 1)d$
where n = no. of terms
a = first term

d = common difference
t_n = n^{th} terms
we will find value of "a" and "d"
Let, $t_{19} = a + (19 – 1) d$
⇒ 52 = a + 18 d …. (1)
$t_{38} = a + (38 – 1) d$
⇒ 128 = a + 37 d …. (2)
Subtracting equation (1) from equation (2), we get,
⇒ 128 – 52 = (a – a) + (37 d – 18 d)
⇒ 76 = 19 d
⇒ d = 76/19 = 4
Substitute value of "d" in equation (1) to get value of "a"
⇒ 52 = a + 18 ×4
⇒ 52 = a + 72
⇒ a = 52 – 72 = – 20
Now, to find value of S_{56} we will using formula of sum of n terms

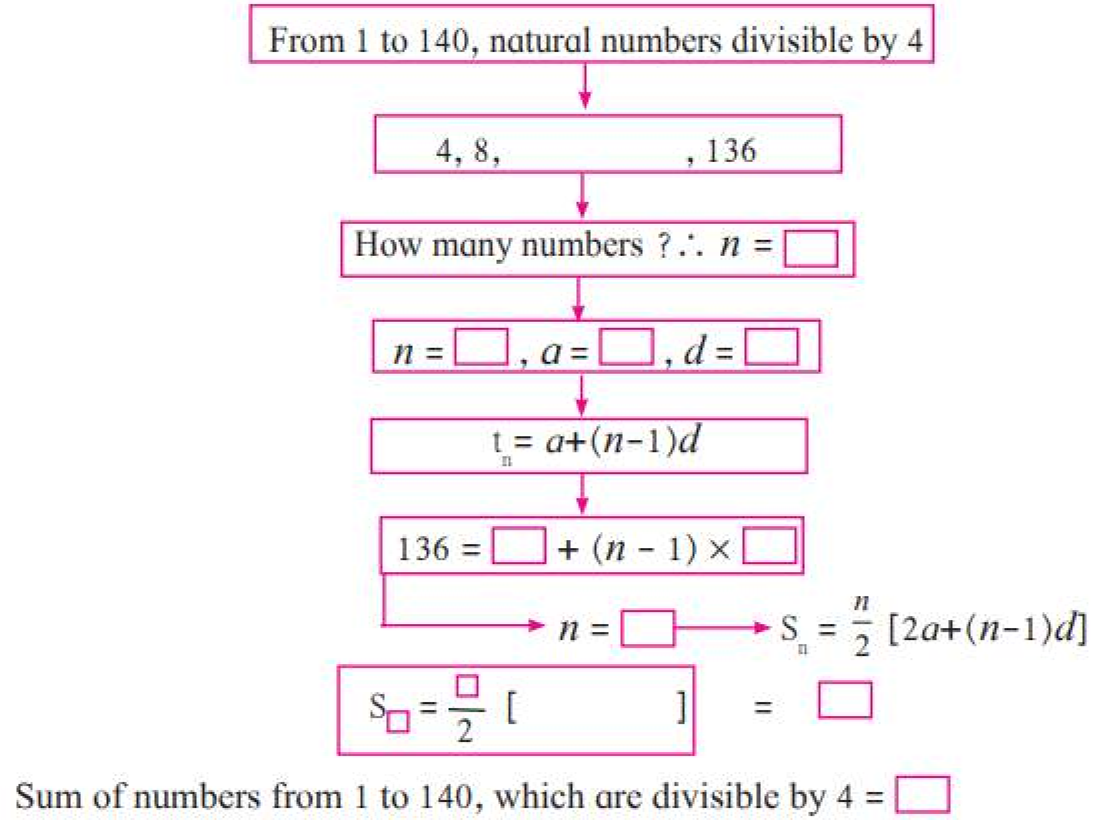
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, substituting given value in formula we can find the value of S_n

$$\Rightarrow S_{56} = \frac{56}{2}[2 \times (-20) + (56-1) \times 4]$$

⇒S_{56} = 28 × [– 40 + 55×4]
⇒S_{56} = 28 × [– 40 + 220]
⇒S_{56} = 28 × 180 = 5040
Thus, S_{56} = 5040

5. Complete the following activity to find the sum of natural numbers from 1 to 140 which are divisible by 4.



Solution:

The natural number divisible by 4 between 1 to 140 is

4, 8, 12, …….136

Where first term a = 4

Second term $t_1 = 8$

Third term $t_2 = 12$

Thus, common difference $d = t_2 - t_1 = 12 - 8 = 4$

$t_n = 136$

Now, by using n^{th} term of an A.P. formula

$t_n = a + (n - 1) d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

we can find value of "n" by substituting all the value in formula we get,

$\Rightarrow 136 = 4 + (n - 1) \times 4$

$\Rightarrow 136 - 4 = 4(n - 1)$

$\Rightarrow 132 = 4(n - 1)$

⇒ n – 1 = 132/4 = 33
⇒ n = 33 + 1 = 34
Now, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, substituting given value in formula we can find the value of S_{34}

$$\Rightarrow S_{34} = \frac{34}{2}[2 \times 4 + (34-1) \times 4]$$

$\Rightarrow S_{34} = 17 \times [8 + 33 \times 4]$
$\Rightarrow S_{34} = 17 \times [8 + 132]$
$\Rightarrow S_{34} = 17 \times 140 = 2380$
Thus, $S_{34} = 2380$

6. Sum of first 55 terms in an A.P. is 3300, find its 28th term.

Solution:
Given $S_{55} = 3300$ where n = 55
Now, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{55} = \frac{55}{2}[2a + (55-1)d]$$

$$\Rightarrow 3300 = \frac{55}{2}[2a + 54\,d]$$

$$\Rightarrow 3300 = \frac{55}{2} \times 2 \times [a + 27\,d]$$

$\Rightarrow 3300 = 55 \times [a + 27d]$

$$\Rightarrow \frac{3300}{55} = a + 27d$$

⇒ $a + 27d = 60$ …… (1)
We need to find value of 28^{th} term that is t_{28}
Now, by using n^{th} term of an A.P. formula
$t_n = a + (n – 1) d$
where n = number of terms
a = first term
d = common difference
$t_n = n^{th}$ terms
we can find value of t_{28} by substituting all the value in formula we get,
⇒ $t_{28} = a + (28 – 1) d$
⇒ $t_{28} = a + 27 d$
From equation (1) we get,
⇒ $t_{28} = a + 27 d = 60$
⇒ $t_{28} = 60$

7. In an A.P. sum of three consecutive terms is 27 and their product is 504 find the terms? (Assume that three consecutive terms in A.P. are a – d, a, a + d.)

Solution:
Let the first term be a – d
the second term be a
the third term be a + d
Given sum of consecutive three term is 27
⇒ $(a – d) + a + (a + d) = 27$
⇒ $3 a = 27$
⇒ $a = 27/ 3 = 9$
Also, given product of three consecutive term is 504
⇒ $(a – d) \times a \times (a + d) = 504$
⇒ $(9 – d) \times 9 \times (9 + d) = 504$ (since, a = 9)
⇒ $(9 – d) \times (9 + d) = 504/ 9 = 56$
⇒ $9^2 – d^2 = 56$ (since, $(a – b) (a + b) = a^2 – b^2$)
⇒ $81 – d^2 = 56$
⇒ $d^2 = 81 – 56 = 25$
⇒ $d = \sqrt{25} = \pm 5$
Case 1:
Thus, if a = 9 and d = 5
Then the three terms are,

First term a – d = 9 – 5 = 4
Second term a = 9
Third term a + d = 9 + 5 = 14
Thus, the A.P. is 4, 9, 14
Case 2:
Thus, if a = 9 and d = – 5
Then the three terms are,
First term a – d = 9 – (– 5) = 9 + 5 = 14
Second term a = 9
Third term a + d = 9 + (– 5) = 9 – 5 = 4
Thus, the A.P. is 14, 9, 4

8. Find four consecutive terms in an A.P. whose sum is 12 and sum of 3rd and 4th term is 14.
(Assume the four consecutive terms in A.P. are a – d, a, a + d, a + 2d.)

Solution:
Let the first term be a – d
the second term be a
the third term be a + d
the fourth term be a + 2 d
Given sum of consecutive four term is 12
⇒ (a – d) + a + (a + d) + (a + 2d) = 12
⇒ 4 a + 2d = 12
⇒ 2(2 a + d) = 12
⇒ 2a + d = 12/2 = 6
⇒ 2a + d = 6 …. (1)
Also, sum of third and fourth term is 14
⇒ (a + d) + (a + 2d) = 14
⇒ 2a + 3d = 14 …… (2)
Subtracting equation (1) from equation (2) we get,
⇒ (2a + 3d) – (2a + d) = 14 – 6
⇒ 2a + 3d – 2a – d = 8
⇒ 2d = 8
⇒ d = 8/2 = 4
⇒ d = 4
Substituting value of "d" in equation (1) we get,

⇒ $2a + 4 = 6$
⇒ $2a = 6 – 4 = 2$
⇒ $a = 2/2 = 1$
⇒ $a = 1$
Thus, $a = 1$ and $d = 4$
Hence, first term $a – d = 1 – 4 = – 3$
the second term $a = 1$
the third term $a + d = 1 + 4 = 5$
the fourth term $a + 2d = 1 + 2\times4 = 1 + 8 = 9$
Thus, the A.P. is – 3, 1, 5, 9

9. If the 9th term of an A.P. is zero then show that the 29^{th} term is twice the 19^{th} term.

Solution:
By using n^{th} term of an A.P. formula
$t_n = a + (n – 1) d$
where n = number of terms
a = first term
d = common difference
$t_n = n^{th}$ terms
Given: $t_9 = 0$
⇒ $t_9 = a + (9 – 1) d$
⇒ $0 = a + 8d$
⇒ $a = – 8d$
To Show: $t_{29} = 2\times t_{19}$
Now,
⇒ $t_{29} = a + (29 – 1) d$
⇒ $t_{29} = a + 28d$
⇒ $t_{29} = – 8d + 28d = 20 d$ (since, $a = – 8d$)
⇒ $t_{29} = 20 d$
⇒ $t_{29} = 2 \times 10 d$ …. (1)
Also,
⇒ $t_{19} = a + (19 – 1) d$
⇒ $t_{19} = a + 18d$
⇒ $t_{19} = – 8d + 18d = 10 d$ (since, $a = – 8d$)
⇒ $t_{19} = 10 d$ …. (2)
From equation (1) and equtaion (2) we get,

$t_{29} = 2 \times t_{19}$

PRACTICE SET 3.4 PAGE NO: 78

1. On 1st Jan 2016, Sanika decides to save ₹ 10, ₹ 11 on second day, ₹ 12 on third day. If she decides to save like this, then on 31st Dec 2016 what would be her total saving?

Solution:

According to the question we can form an A.P.

10, 11, 12, 13, ……

Hence, the first term a = 10

Second term $t_1 = 11$

Third term $t_2 = 12$

Thus, common difference $d = t_2 - t_1 = 12 - 11 = 1$

Here, number of terms from 1st Jan 2016 to 31st Dec 2016 is,

n = 366

We need to find S_{366}

Now, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{366} = \frac{366}{2}[2 \times 10 + (366-1) \times 1]$$

$\Rightarrow S_{366} = 183 [20 + 365]$

$\Rightarrow S_{366} = 183 \times 385$

$\Rightarrow S_{366}$ = Rs 70,455

2. A man borrows ₹ 8000 and agrees to repay with a total interest of ₹ 1360 in 12 monthly instalments. Each instalment being less than the preceding one by ₹ 40. Find the amount of the first and last instalment.

Solution:

Given A man borrows = Rs. 8000

Repay with total interest = Rs 1360

In 12 months, thus n = 12

Thus, S_{12} = 8000 + 1360 = 9360
Each installment being less than preceding one
Thus, d = – 40
We need to find "a"
Now, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = number of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{12} = \frac{12}{2}[2a + (12-1) \times (-40)]$$

⇒ 9360 = 6 [2a – 11 × 40]

$$\Rightarrow \frac{9360}{6} = 2a - 440$$

⇒ 1560 = 2a – 440
⇒ 1560 + 440 = 2a
⇒ 2a = 2000

$$\Rightarrow a = \frac{2000}{2} = 1000$$

Thus, first installment a = Rs. 1000
Now, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[\text{first term} + \text{last term}]$$

Where, n = number of terms
S_n = sum of n terms
Thus, on substituting the given value in formula we get,
Let a = first term, t_n = last term

$$\Rightarrow S_{12} = \frac{12}{2}[a + t_n]$$

⇒ 9360 = 6 [1000 + t_n]

$$\Rightarrow 1000 + t_n = \frac{9360}{6} = 1560$$

⇒ t_n = 1560 – 1000 = 560

Thus, last installment t_n = 560

3. Sachin invested ina national saving certificate scheme. In the first year he invested ₹ 5000, in the second year ₹ 7000, in the third year ₹ 9000 and so on. Find the total amount that he invested in 12 years.

Solution:

According to the question we can form an A.P.
5000, 7000, 9000, ……
Hence, the first term a = 5000
Second term t_1 = 7000
Third term t_2 = 9000
Thus, common difference d = $t_2 - t_1$ = 9000 – 7000 = 2000
Here, number of terms n = 12
We need to find S_{12}
Now, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{12} = \frac{12}{2}[2 \times 5000 + (12-1) \times 2000]$$

$\Rightarrow S_{12}$ = 6 × [10,000 + 11 × 2000]
$\Rightarrow S_{12}$ = 6 × [10,000 + 22,000]
$\Rightarrow S_{12}$ = 6 × 32,000
$\Rightarrow S_{12}$ = Rs. 192000

4. There is an auditorium with 27 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the 15th row and also find how many total seats are there in the auditorium?

Solution:

Given first term a = 20
Second term t_1 = 22
Third term t_2 = 24

Common difference $d = t_2 - t_1 = 24 - 22 = 2$
We need to find t_{15} thus $n = 15$
Now, by using n^{th} term of an A.P. formula
$t_n = a + (n - 1)\ d$
where n = number of terms
a = first term
d = common difference
$t_n = n^{th}$ terms
On substituting all value in n^{th} term of an A.P.
$\Rightarrow t_{15} = 20 + (15 - 1) \times 2$
$\Rightarrow t_{15} = 20 + 14 \times 2$
$\Rightarrow t_{15} = 20 + 28 = 48$
We have been given that, there are 27 rows in an auditorium
Thus, we need to find total seats in auditorium i.e. S_{27}
Now, by using sum of n^{th} term of an A.P. we will find its sum
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
Where, n = number of terms
a = first term
d = common difference
S_n = sum of n terms
Thus, on substituting the given value in formula we get,
$$\Rightarrow S_{27} = \frac{27}{2}[2 \times 20 + (27-1) \times 2]$$
$\Rightarrow S_{27} = 27 \times 46$
$\Rightarrow S_{27} = 1242$

5. Kargil's temperature was recorded in a week from Monday to Saturday. All readings were in A.P. The sum of temperatures of Monday and Saturday was 5° C more than sum of temperatures of Tuesday and Saturday. If temperature of Wednesday was – 30° Celsius then find the temperature on the other five days.

Solution:
Let Monday be the first term i.e. $a = t_1$
Let Tuesday be the second term that is t_2
Let Wednesday be the third term that is t_3
Let Thursday be the fourth term that is t_4
Let Friday be the fifth term that is t_5

Let Saturday be the sixth term that is t_6
Given: $t_1 + t_6 = 5 + (t_2 + t_6)$
$\Rightarrow a = 5 + (t_2 + t_6) - t_6$
$\Rightarrow a = 5 + t_2$ …. (1)
We know that,
Now, by using n^{th} term of an A.P. formula
$t_n = a + (n - 1) d$
where n = no. of terms
a = first term
d = common difference
$t_n = n^{th}$ terms
Thus, $t_2 = a + (2 - 1) d$
$\Rightarrow t_2 = a + d$
Now substitute value of t_2 in (1) we get,
$\Rightarrow a = 5 + (a + d)$
$\Rightarrow d = a - 5 - a = -5$
Given: $t_3 = -30°$
Thus, $t_3 = a + (3 - 1) \times (-5)$
$\Rightarrow -30 = a + 2 \times (-5)$
$\Rightarrow -30 = a - 10$
$\Rightarrow a = -30 + 10 = -20°$
Thus, Monday, $a = t_1 = -20°$
Using formula $t_{n+1} = t_n + d$
We can find the value of the other terms
Tuesday, $t_2 = t_1 + d = -20 - 5 = -25°$
Wednesday, $t_3 = t_2 + d = -25 - 5 = -30°$
Thursday, $t_4 = t_3 + d = -30 - 5 = -35°$
Friday, $t_5 = t_4 + d = -35 - 5 = 40°$
Saturday, $t_6 = t_5 + d = -40 - 5 = -45°$
Thus, we obtain an A.P.
$-20°, -25°, -30°, -35°, -40°, -45°$

6. On the world environment day tree plantation programme was arranged on a land which is triangular in shape. Trees are planted such that in the first row there is one tree, in the second row there are two trees, in the third row three trees and so on. Find the total number of trees in the 25 rows.

Solution:

First term a = 1

Second term $t_1 = 2$

Third term $t_3 = 3$

Common difference $d = t_3 - t_2 = 3 - 2 = 1$

We need to find total number of trees when n = 25

Thus, by using sum of n^{th} term of an A.P. we will find its sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Where, n = no. of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_{25}

Thus, on substituting the given value in formula we get,

$$\Rightarrow S_{25} = \frac{25}{2}[2 \times 1 + (25-1) \times 1]$$

$$\Rightarrow S_{25} = \frac{25}{2}[2 + 24]$$

$$\Rightarrow S_{25} = \frac{25}{2} \times 2 \times [1 + 12]$$

$$\Rightarrow S_{25} = 25 \times 13 = 325$$

PROBLEM SET 3 PAGE NO: 78

1. Choose the correct alternative answer for each of the following sub questions.
(1) Choose the correct alternative answer for each of the following sub questions.
The sequence – 10, – 6, – 2, 2, . . .
A. is an A.P., Reason d = – 16
B. is an A.P., Reason d = 4
C. is an A.P., Reason d = – 4
D. is not an A.P.

Solution:
B. is an A.P., Reason d = 4

Explanation:
First term a = – 10
Second term t_1 = – 6
Third term t_2 = – 2
Fourth term t_3 = 2
Common difference $d = t_1 - a = -6 - (-10) = -6 + 10 = 4$
Common difference $d = t_2 - t_1 = -2 - (-6) = -2 + 6 = 4$
Common difference $d = t_3 - t_2 = 2 - (-2) = 2 + 2 = 4$
Since, the common difference is same
∴ The given sequence is A.P. with common difference d = 4
Hence, correct answer is (B)

(2) First four terms of an A.P. are ....., whose first term is – 2 and common difference is – 2.
A. – 2, 0, 2, 4
B. – 2, 4, – 8, 16
C. – 2, – 4, – 6, – 8
D. – 2, – 4, – 8, – 16

Solution:
C. – 2, – 4, – 6, – 8

Explanation:
Given first term t_1 = – 2

Common difference d = – 2
By using formula $t_{n+1} = t_n + d$
$t_2 = t_1 + d = -2 + (-2) = -2 - 2 = -4$
$t_3 = t_2 + d = -4 + (-2) = -4 - 2 = -6$
$t_4 = t_3 + d = -6 + (-2) = -6 - 2 = -8$
Hence, the A.P. is – 2, – 4, – 6, – 8
∴ correct answer is (C)

(3) What is the sum of the first 30 natural numbers?
A. 464
B. 465
C. 462
D. 461

Solution:
B. 465

Explanation:
List of first 30 natural number is
1, 2, 3, ……..,30
First term a = 1
Second term $t_1 = 2$
Third term $t_2 = 3$
Common difference $d = t_3 - t_2 = 3 - 2 = 1$
number of terms n = 30
Thus, by using sum of n^{th} term of an A.P. we will find its sum
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
Where, n = number of terms
a = first term
d = common difference
S_n = sum of n terms
We need to find S_{30}
$$\Rightarrow S_{30} = \frac{30}{2}[2 \times 1 + (30-1) \times 1]$$
$\Rightarrow S_{30} = 15 [2 + 29]$
$\Rightarrow S_{30} = 15 \times 31$
$\Rightarrow S_{30} = 465$

Hence, Correct answer is (B)

(4) For a given A.P. $t_7 = 4$, $d = -4$ then a = . . .

A. 6
B. 7
C. 20
D. 28

Solution:
D. 28

Explanation:
By using n^{th} term of an A.P. formula
$t_n = a + (n - 1) d$
where n = number of terms
a = first term
d = common difference
$t_n = n^{th}$ terms
$\Rightarrow t_7 = a + (7 - 1) \times (-4)$
$\Rightarrow 4 = a + 6 \times (-4)$
$\Rightarrow 4 = a - 24$
$\Rightarrow a = 24 + 4 = 28$
Thus, the correct answer is (D)

(5) For a given A.P. a = 3.5, d = 0, n = 101, then t_n = . . .

A. 0
B. 3.5
C. 103.5
D. 104.5

Solution:
B. 3.5

Explanation:
Given: a = 3.5, d = 0, n = 101
By using n^{th} term of an A.P. formula
$t_n = a + (n - 1) d$

where n = number of terms
a = first term
d = common difference
t_n = n^{th} terms
Substituting all given value in the formulae we get,
⇒ $t_n = 3.5 + (101 – 1) \times 0$
⇒ $t_n = 3.5$
Thus, correct answer is (B)

(6) In an A.P. first two terms are – 3, 4 then 21^{st} term is . . .
A. – 143
B. 143
C. 137
D. 17

Solution:
C. 137

Explanation:
Given first term a = – 3
Second term $t_1 = 4$
Common difference $d = t_1 – a = 4 – (– 3) = 4 + 3 = 7$
We need to find t_{21} where n = 21
Now, by using n^{th} term of an A.P. formula
$t_n = a + (n – 1) d$
where n = no. of terms
a = first term
d = common difference
t_n = n^{th} terms
Substituting all given value in the formulae we get,
⇒ $t_{21} = – 3 + (21 – 1) \times 7$
⇒ $t_{21} = – 3 + 20 \times 7$
⇒ $t_{21} = – 3 + 140 = 137$
Hence, correct answer is (C)

(7) If for any A.P. d = 5 then $t_{18} – t_{13}$ = ...
A. 5

B. 20
C. 25
D. 30

Solution:
C. 25

Explanation:
Given d = 5
By using n^{th} term of an A.P. formula
$t_n = a + (n – 1) d$
where n = number of terms
a = first term
d = common difference
$t_n = n^{th}$ terms
Thus, $t_{18} – t_{13} = [a + (18 – 1) \times 5] – [a + (13 – 1) \times 5]$
$\Rightarrow t_{18} – t_{13} = [17 \times 5] – [12 \times 5]$
$\Rightarrow t_{18} – t_{13} = 85 – 60 = 25$
Thus, correct answer is (C)

(8) Sum of first five multiples of 3 is. . .
A. 45
B. 55
C. 15
D. 75

Solution:
A. 45

Explanation:
First five multiples of 3 are
3, 6, 9, 12, 15
First term a = 3
Second term $t_1 = 6$
Third term $t_2 = 9$
Common difference $d = t_2 – t_1 = 9 – 6 = 3$
Thus, by using sum of n^{th} term of an A.P. we will find its sum

$S_n = \frac{n}{2}[2a + (n-1)d]$

Where, n = number of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_5.

$\Rightarrow S_5 = \frac{5}{2}[2 \times 3 + (5-1) \times 3]$

$\Rightarrow S_5 = \frac{5}{2}[6 + 4 \times 3]$

$\Rightarrow S_5 = \frac{5}{2}[6 + 12]$

$\Rightarrow S_5 = \frac{5}{2} \times 18 = 5 \times 9 = 45$

Thus, correct answer is (A)

(9) 15, 10, 5, . . . In this A.P. sum of first 10 terms is . . .

A. – 75

B. – 125

C. 75

D. 125

Solution:

A. – 75

Explanation:

First term a = 15

Second term t_1 = 10

Third term t_2 = 5

Common difference d = $t_2 - t_1$ = 5 – 10 = – 5

Number of terms n = 10

Thus, by using sum of n^{th} term of an A.P. we will find its sum

$S_n = \frac{n}{2}[2a + (n-1)d]$

Where, n = number of terms

a = first term

d = common difference

S_n = sum of n terms

We need to find S_{10}

$$\Rightarrow S_{10} = \frac{10}{2}[2 \times 15 + (10-1) \times (-5)]$$

$\Rightarrow S_{10} = 5 [30 + 9 \times (-5)]$

$\Rightarrow S_{10} = 5 [30 - 45]$

$\Rightarrow S_{10} = 5 \times (-15) = -75$

Hence, correct answer is (A)

(10) In an A.P. 1st term is 1 and the last term is 20. The sum of all terms is = 399 then n = . . .

A. 42

B. 38

C. 21

D. 19

Solution:

B. 38

Explanation:

Given, first term = 1

Last term = 20

Sum of n terms, S_n = 399

We need to find number of terms n

Using Sum of n terms of an A.P. formula

$$S_n = \frac{n}{2}[\text{first term} + \text{last term}]$$

where n = number of terms

S_n = sum of n terms

Now, on substituting given value in formula we get,

$$\Rightarrow 399 = \frac{n}{2}[1 + 20]$$

$$\Rightarrow 399 = \frac{n}{2} \times 21$$

$$\Rightarrow n = \frac{399 \times 2}{21} = 19 \times 2 = 38$$

∴ correct answer is (B)

2. Find the fourth term from the end in an A.P. – 11, – 8, – 5, . . ., 49.

Solution:

First term from end a = 49

$t_n = -11$

$t_{n-1} = -8$

Common difference $d = t_n - t_{n-1}$

$= -11 - (-8)$

$= -11 + 8$

$= -3$

Now, by using n^{th} term of an A.P. formula

$t_n = a + (n - 1) d$

where n = no. of terms

a = first term

d = common difference

$t_n = n^{th}$ terms

number of terms n = 4

$\Rightarrow t_4 = 49 + (4 - 1) \times (-3)$

$\Rightarrow t_4 = 49 + 3 \times (-3)$

$\Rightarrow t_4 = 49 - 9 = 40$