Practice Set 2.1 Page 17

1. In figure 2.5, line RP || line MS and line DK is their transversal. ∠DHP = 85° Find the measures of following angles.
(i) ∠RHD (ii) ∠PHG (iii) ∠HGS (iv) ∠MGK

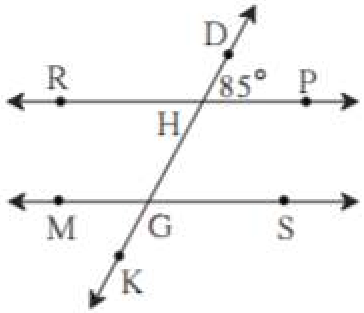


Fig. 2.5

Solution:
Given RP || MS.
DK is the transversal.
(i) ∠DHP = 85˚
∴ ∠RHD = 180-85 = 95˚ [Linear pair]
(ii) ∠PHG = ∠RHD = 95˚ [∠RHD and ∠PHG are vertically opposite angles]
(iii)∠HGS = 180-95 = 85˚ [∠PHG and ∠HGS are corresponding interior angles]
(iv)∠MGK = ∠HGS = 85˚ [Vertically opposite angles]

2. In figure 2.6, line p || line q and line *l* and line *m* are transversals. Measures of some angles are shown. Hence find the measures of ∠a, ∠b, ∠c, ∠d.

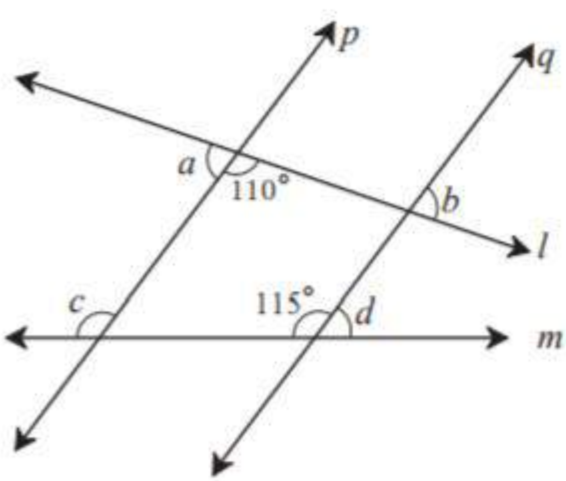
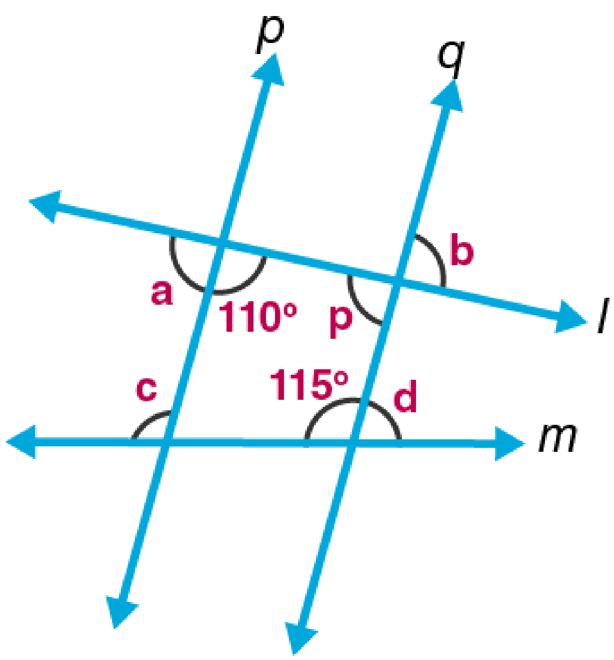


Fig. 2.6

Solution:



Given line p || line q.
l and m are transversals.
$\therefore \angle a = 180-110 = 70^\circ$ [Linear pair]
$\angle p$ and 110° are corresponding interior angles.
$\therefore \angle p = 180-110 = 70^\circ$
$\therefore \angle b = \angle p = 70^\circ$ [Vertically opposite angles]
line p || line q
m is the transversal.
$\therefore c = 115^\circ$ [Corresponding angles]
$\angle d + 115 = 180^\circ$ [Linear pair]
$\therefore \angle d = 180-115 = 65^\circ$
Hence measure of $\angle a$, $\angle b$, $\angle c$ and $\angle d$ are 70°, 70°, 115° and 65° respectively.

3. In figure 2.7, line *l* || line *m* and line *n* || line *p*. Find ∠a, ∠b, ∠c from the given measure of an angle.

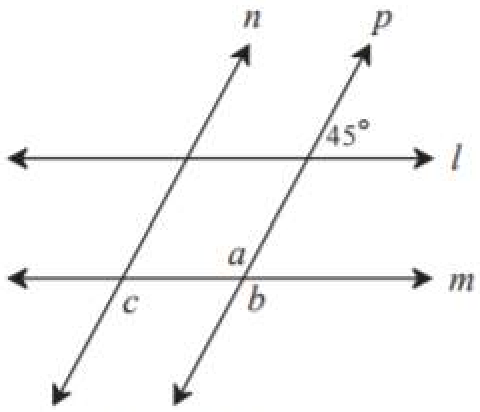
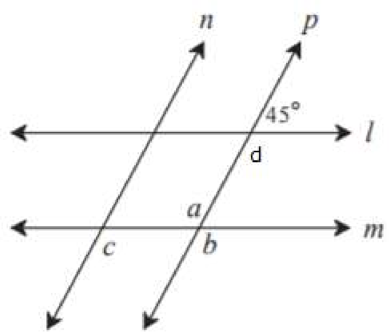


Fig. 2.7

Solution:



From figure , ∠d and 45˚ form a linear pair.

∴∠d = 180-45 = 135˚

∴∠a = ∠d = 135˚ [∠d and ∠a are alternate interior angles]

∴∠b = ∠a = 135˚ [∠ a and ∠b are Vertically opposite angles]

∴∠c = ∠a = 135˚ [∠c and ∠a are alternate interior angles]

4* . In figure 2.8, sides of ∠PQR and ∠XYZ are parallel to each other. Prove that, ∠PQR ≅ ∠XYZ

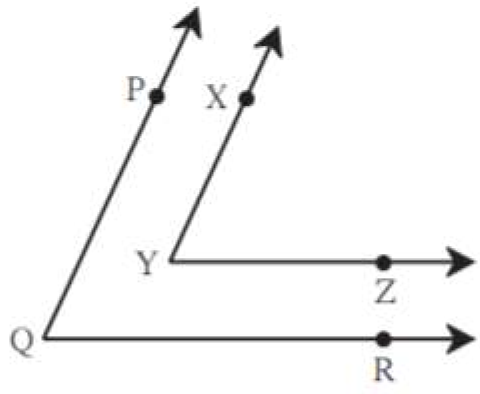
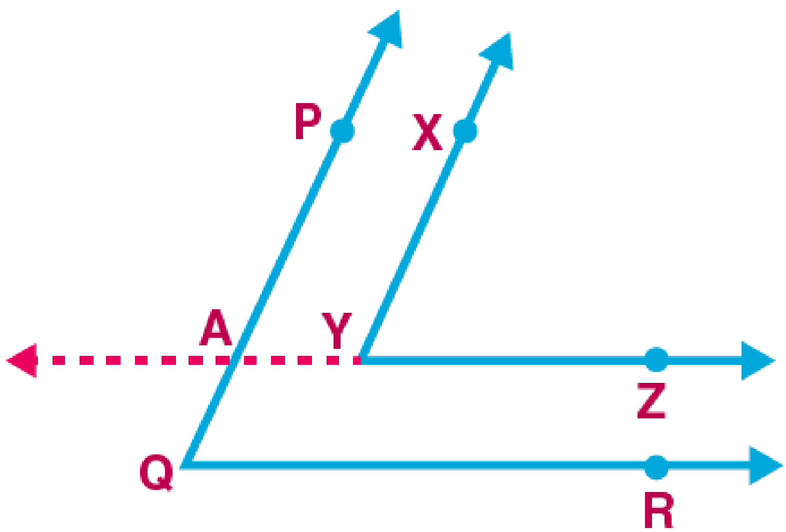


Fig. 2.8

Solution:
Construction: Produce ray YZ in opposite direction. It meets at point A.



To prove : ∠PQR ≅ ∠XYZ
Proof:
Given XY ∥ PQ
XY ∥ PA
AY is the transversal.
∴∠XYZ ≅ ∠PAY …..(i) [Corresponding angles]
AZ ∥ QR
PQ is the transversal.
∴∠ PAY ≅ ∠PQR …..(ii) [Corresponding angles]
∴∠XYZ ≅ ∠PQR [From (i) and (ii)]
∴∠PQR ≅ ∠XYZ

Hence proved.

5. In figure 2.9, line AB || line CD and line PQ is transversal. Measure of one of the angles is given. Hence find the measures of the following angles.
(i) ∠ART
(ii) ∠CTQ
(iii) ∠DTQ
(iv) ∠PRB

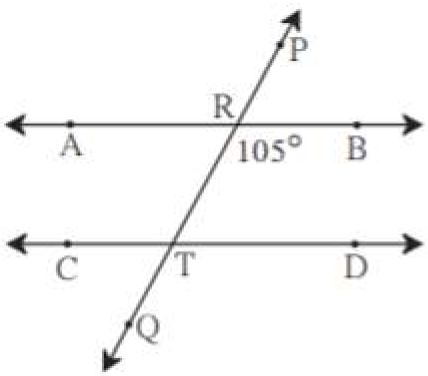


Fig. 2.9

Solution:
Given line AB || line CD.
PQ is the transversal.

(i)∠BRT = 105˚	[Given in figure]
∠ART = 180-105 = 75˚	[∠ART and ∠BRT form linear pair]
Hence ∠ART is 75˚.	
(ii)∠RTD = 75˚	[∠ART and ∠RTD are alternate interior angles]
∴∠CTQ = 75˚	[∠RTD and ∠CTQ are vertically opposite angles]
Hence ∠CTQ is 75˚.	
(iii)∴∠DTQ = 180- ∠CTQ	[∠DTQ and ∠CTQ form linear pair]
∴∠DTQ = 180-75 = 105˚	
Hence ∠DTQ is 105˚.	
(iv)∠PRB = 180-∠BRT	[∠PRB and ∠BRT form a linear pair]
∴∠PRB = 180-105 = 75˚	
Hence ∠PRB is 75˚.	

Practice Set 2.2

1. In figure 2.18, y = 108° and x = 71° Are the lines m and n parallel ? Justify ?

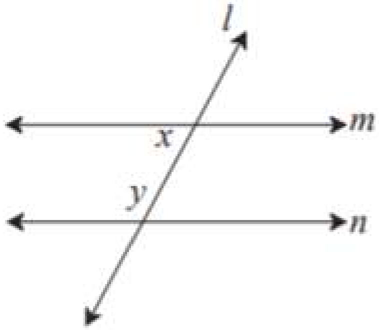


Fig. 2.18

Solution:
Given $y = 108^\circ$
$x = 71^\circ$
If lines m and n are parallel, then angles x and y are supplementary. [Interior angles test]
Here $x+y = 71+108 = 179 \neq 180$
So angles x and y are not supplementary.
Angles x and y fail the interior angle test for parallel lines.
Hence m and n are not parallel.

2. In figure 2.19, if ∠a ≅ ∠b then prove that line *l* || line m.

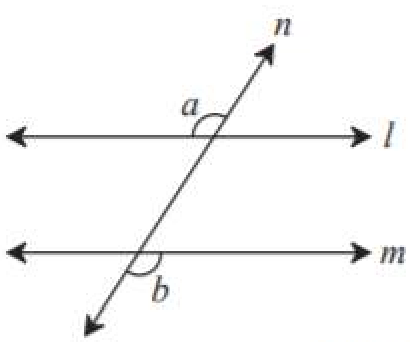
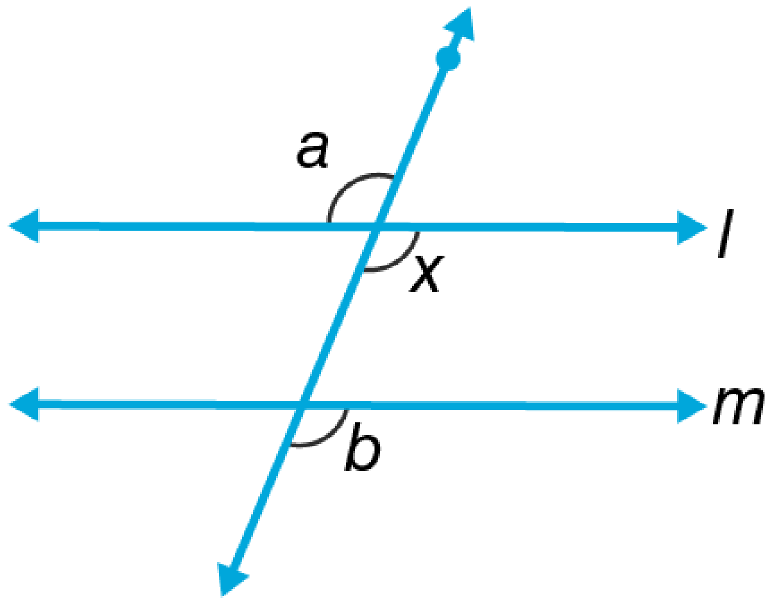


Fig. 2.19

Solution:
Given ∠a ≅ ∠b
To prove : line *l* ‖ line m.
Proof:



Consider $\angle x$ in figure.

$\angle a \cong \angle x$ ……..(i) [Vertically opposite angles]

Given $\angle a = \angle b$

$\therefore \angle x = \angle b$

$\angle b$ and $\angle x$ are corresponding angles on line *l* and m where n is the transversal.

$\therefore$ line *l* || line m. [Corresponding angle test]

Hence proved.

3. In figure 2.20, if $\angle a \cong \angle b$ and $\angle x \cong \angle y$ then prove that line l || line n.

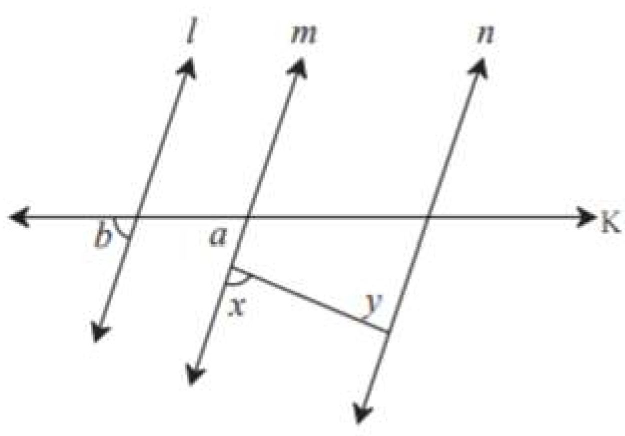
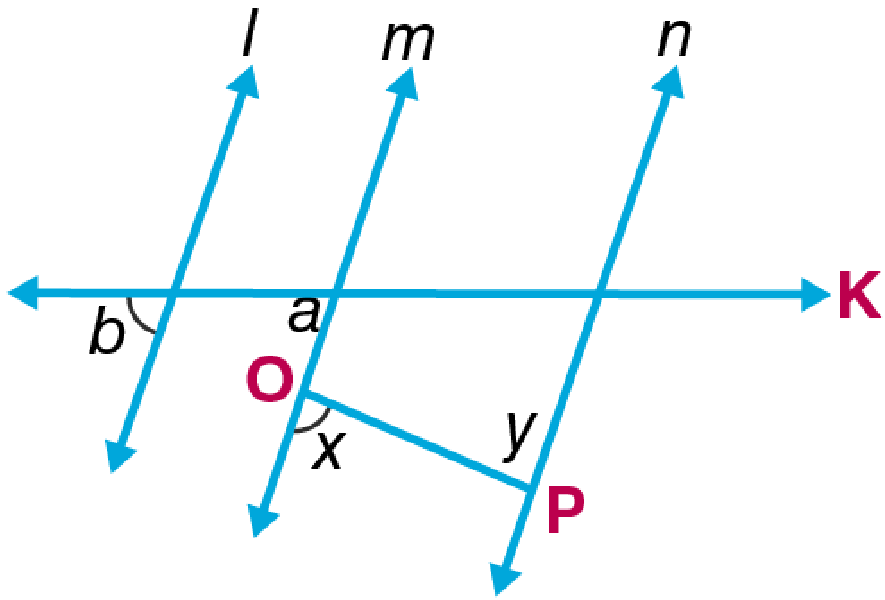


Fig. 2.20

Solution:



Given ∠a ≅ ∠b and ∠x ≅ ∠y
To prove: line l ∥ line n
Proof:
∠a ≅ ∠b [Given]
But ∠a and ∠b are corresponding angles formed by a transversal K of line *m* and line *l*.
∴ line *m* ∥ line *l* ….(i) [corresponding angles test]

Also we have,
∠x ≅ ∠y
∴m∠x = m∠y
But ∠x and ∠y are alternate interior angles formed by a transversal OP of line *m* and line *n*.
∴line *m* ∥ line *n*……(ii) [alternate angles test]
From (i) and (ii)
line *m* ∥ line *l* and line *m* || line *n*.
If two lines are both parallel to a third line, then they are also parallel to each other.
∴line *l* ∥ line *n*
Hence proved.

4. In figure 2.21, if ray BA || ray DE, ∠C = 50° and ∠D = 100°. Find the measure of ∠ABC. (Hint : Draw a line passing through point C and parallel to line AB.)

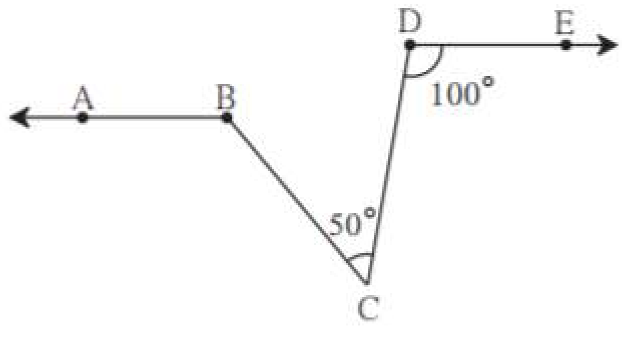
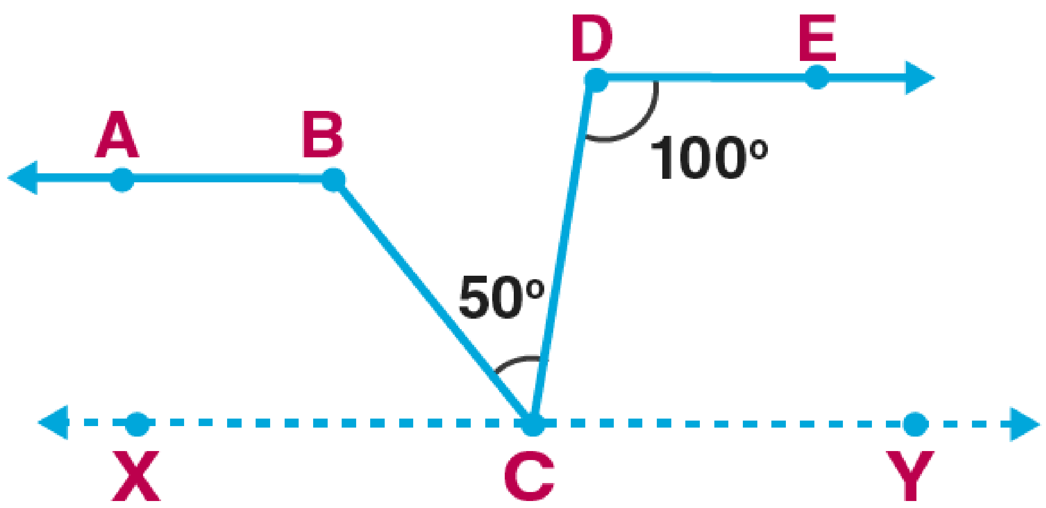


Fig. 2.21

Solution:
Draw a line passing XY through point C and parallel to line AB.



line XY ∥ ray AB
Given ray AB ∥ ray DE
∴ line XY ∥ ray DE
DC is the transversal.
$\angle DCX = \angle CDE = 100^\circ$ [Alternate interior angles]
$\angle DCX = \angle DCB + \angle BCX$ [Angle addition property]
$\therefore 100 = 50 + \angle BCX$ [$\angle DCB = 50^\circ$]
$\angle BCX = 100\text{-}50 = 50^\circ$
Consider line XY parallel ray AB.
BC is the transversal.

$\angle BCX + \angle ABC = 180^\circ$ [Interior angles are supplementary]
$\therefore \angle ABC = 180 - \angle BCX$
$= 180 - 50 = 130^\circ$
Hence $\angle ABC$ is 130°.

5. In figure 2.22, ray AE || ray BD, ray AF is the bisector of ∠EAB and ray BC is the bisector of ∠ABD. Prove that line AF || line BC.

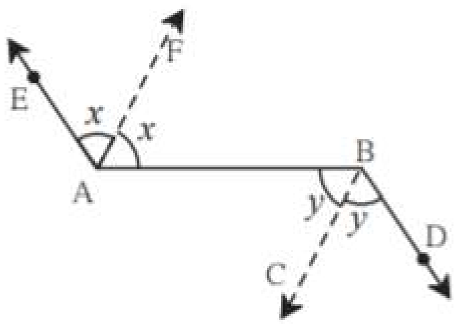


Fig. 2.22

Solution:
Given: Ray AE ∥ ray BD.
Ray AF is the bisectors of ∠EAB
Ray BC is the bisector of ∠ABD
To prove: line AF ∥ line BC
Proof:
Ray AE ∥ ray BD
seg AB is their transversal.
$\therefore \angle DBA = \angle EAB$…..(i) [Alternate interior angles]
$\angle FAB = ½ \angle EAB$ [AF is the bisector of ∠EAB]
$\therefore \angle EAB = 2\angle FAB$…..(ii)
$\angle ABC = ½ \angle DBA$ [BC is the bisector of ∠ABD]
$\therefore \angle DBA = 2\angle ABC$ ……(iii)
$\therefore 2\angle FAB = 2\angle ABC$ [From (i), (ii) and (iii)]
$\therefore \angle FAB = \angle ABC$ [Divide both sides of equation by 2]
∠FAB and ∠ABC are alternate interior angles on lines AF and BC when AB is the transversal.
∴line AF ∥ line BC [Alternate angles test]
Hence proved.

6. A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of ∠BPQ and ∠PQC respectively. Prove that line AB || line CD.

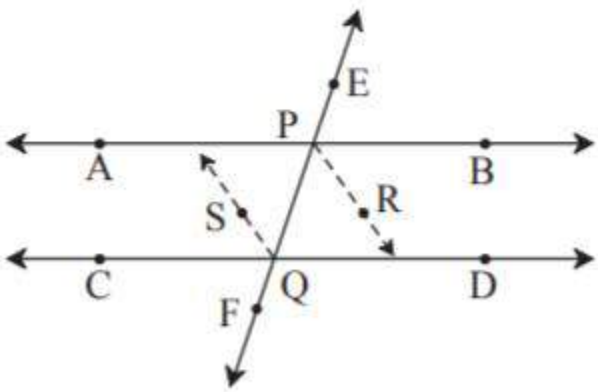


Fig. 2.23

Solution:

Given : Ray PR ∥ Ray QS
Ray PR is the bisector of ∠BPQ.
Ray QS is the bisector of ∠PQC.
To Prove : Line AB ∥ line CD
Proof:
Ray PR ∥ Ray QS
PQ is the transversal.
∴∠QPR = ∠PQS…..(i) [Alternate interior angles]
∠QPR = ½ ∠BPQ…..(ii) [Ray PR is the bisector of ∠BPQ]
∠PQS = ½ ∠PQC……(iii) [Ray QS is the bisector of ∠PQC]
Substitute (ii) and (iii) in (i)
½∠ BPQ = ½ ∠PQC
∴∠ BPQ = ∠PQC
∠ BPQ and ∠PQC are alternate interior angles on AB and CD when line EF is the transversal.
∴line AB ∥ line CD [Alternate angles test]
Hence proved.

Problem Set 2 Page 22

1. Select the correct alternative and fill in the blanks in the following statements.
(i) If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is ....
(A) 0° (B) 90° (C) 180° (D) 360°

Solution:
The sum of interior angles on the same side of the transversal is 180˚.
Hence Option C is the answer.

(ii) The number of angles formed by a transversal of two lines is ....
(A) 2 (B) 4 (C) 8 (D) 16

Solution:
The number of angles formed by a transversal of two lines is 8.
Hence Option C is the answer.

(iii) A transversal intersects two parallel lines. If the measure of one of the angles is 40°then the measure of its corresponding angle is ....
(A) 40° (B) 140° (C) 50° (D) 180°

Solution:
Corresponding angles formed by transversal of two parallel lines are equal in measure.
$\therefore$Measure of corresponding angle is 40˚.
Hence Option A is the answer.

(iv) In $\triangle ABC$, $\angle A = 76°$, $\angle B = 48°$, $\therefore$ $\angle C$ = ...
(A) 66° (B) 56° (C) 124° (D) 28°

Solution:
Sum of angles of a triangle is equal to 180˚.
Given $\angle A = 76^\circ$ and $\angle B = 48^\circ$
$\therefore \angle A+\angle B+\angle C = 180^\circ$
$76+48+\angle C = 180$
$\angle C = 180-(76+48) = 180-124 = 56^\circ$
Hence Option B is the answer.

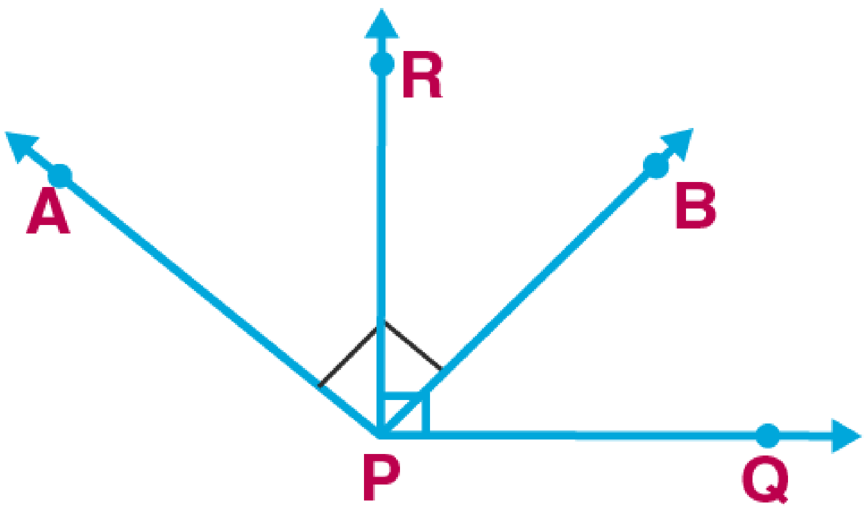
(v) Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is ....
(A) 105° (B) 15° (C) 75° (D) 45°

Solution:
The alternate interior angles formed by a transversal of two parallel lines are of equal measure.
So measure of other angle is 75˚.
Hence Option C is the answer.

2* . Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of $\angle QPR$ respectively. Ray PB and ray PA are perpendicular to each other. Draw a figure showing all these

rays and write -
(i) A pair of complementary angles
(ii) A pair of supplementary angles.
(iii) A pair of congruent angles.

Solution:



(i) Two angles are complementary if their sum is equal to 90°.
Ray PR $\perp$ Ray PQ
$\therefore \angle RPQ = 90^\circ$
$\angle RPB + \angle BPQ = 90^\circ$ [Angle addition property]
$\therefore \angle RPB$ and $\angle BPQ$ are a pair of complementary angles.

(ii) Two angles are supplementary if their sum is equal to 180°.
Ray PA $\perp$ Ray PB [Given]
$\therefore \angle APB = 90^\circ$
$\angle APB + \angle RPQ = 90+90 = 180^\circ$
$\therefore \angle APB$ and $\angle RPQ$ are a pair of supplementary angles.

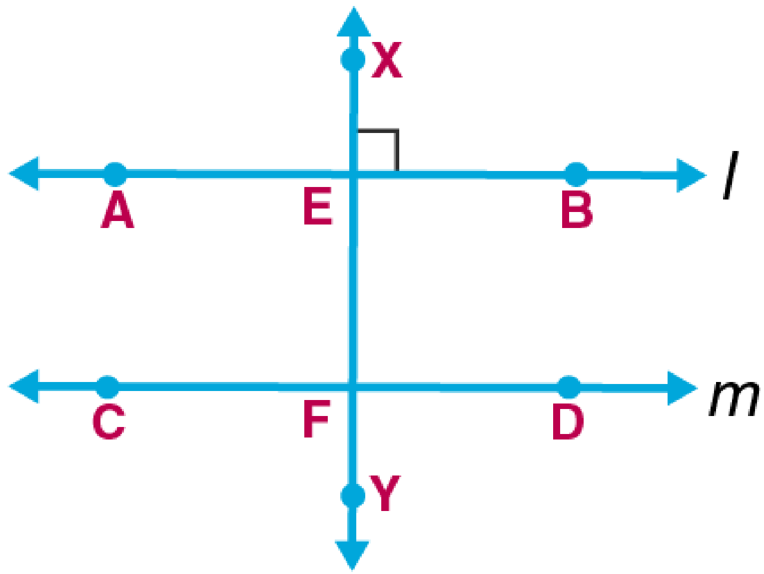
(iii)Two angles are congruent if they have same measures.
$\angle RPQ = \angle APB = 90^\circ$
$\therefore \angle RPQ \cong \angle APB$
$\therefore \angle RPQ$ and $\angle APB$ are a pair of congruent angles.

3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

Solution:

Construction:

Draw two parallel lines AB and CD. Draw line XY intersects them at E and F.



Given : Line AB || line CD.
Line XY intersects them at E and F respectively.
Line XY $\perp$ line AB
To prove: Line XY $\perp$ line CD
Proof:
Given line XY $\perp$ line AB
$\therefore \angle XEB = 90^\circ$….(i)
Line AB || line CD and XY is the transversal.
$\therefore \angle XEB = \angle EFD$ ….(ii) [corresponding angles]
$\therefore \angle EFD = 90^\circ$ [From (i) and (ii)]
$\therefore$ line XY $\perp$ line CD
Hence proved.

4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of $\angle x$ and $\angle y$ and hence show that line l || line m

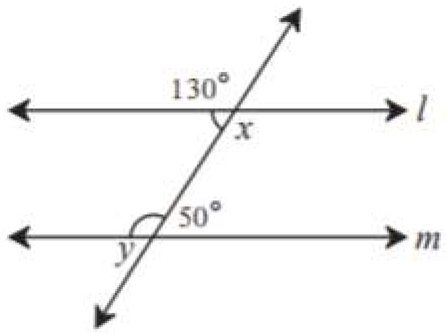


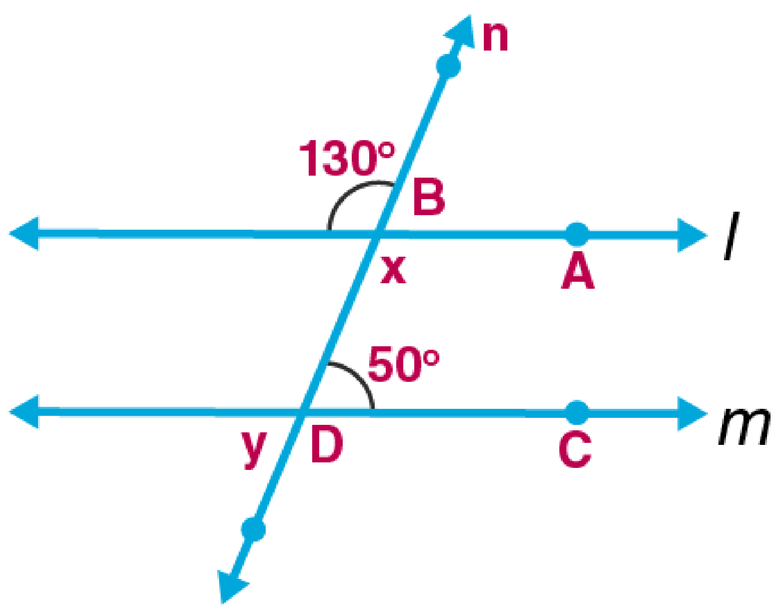
Fig. 2.24

Solution:

Proof:

∠x = 130˚ [∠x and 130 ˚vertically opposite angles]

∠y = 50˚ [∠y and 50 ˚vertically opposite angles]



Here ∠ABD +∠CDB = 130+50 = 180

But, ∠ ABD and ∠CDB are a pair of interior angles on lines l and m when line n is the transversal.

∴line *l* || line m [Interior angles test]

Hence proved.

5. Line AB || line CD || line EF and line QP is their transversal. If y : z = 3 : 7 then find the measure of ∠x. (See figure 2.25.)

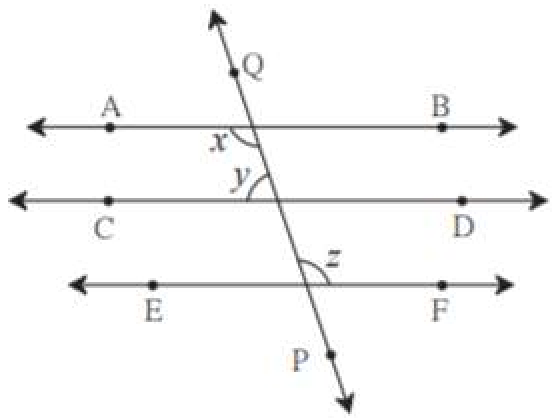


Fig. 2.25

Solution:
Given y:z = 3:7
Let k be a common multiple.
∴ y = 3k
z = 7k……(i)
Given line AB ∥ line EF.
PQ is the transversal.
∴ ∠x = ∠z [Alternate interior angles]
∴ x = 7k…..(ii)
Given line AB ∥ line CD.
PQ is the transversal.
∴ x+y = 180° [Interior angles on same side of transversal are supplementary]
7k+3k = 180
10k = 180
⇒k = 180/10 = 18
∴ ∠x = 7k [From (ii)]
= 7×18 = 126°
Hence measure of ∠x is 126°.

6. In figure 2.26, if line q || line r, line p is their transversal and if a = 80° find the values of f and g.

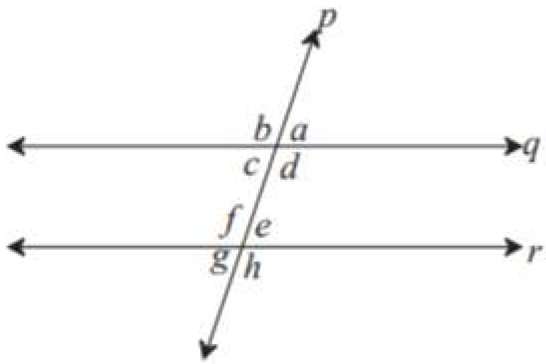


Fig. 2.26

Solution:
Given line q ∥ line r.
Line p is the transversal.
$\angle a = 80^\circ$
$\therefore \angle c = \angle a = 80^\circ$ [a and c are vertically opposite angles]
$\angle f + \angle c = 180$ [f and c are interior angles on same side of transversal]
$\therefore \angle f = 180-80 = 100^\circ$
$\angle g + \angle f = 180$ [Linear pair]
$\therefore \angle g = 180-100 = 80^\circ$
Hence measure of f and g are 100° and 80° respectively.

7. In figure 2.27, if line AB || line CF and line BC || line ED then prove that ∠ABC = ∠FDE

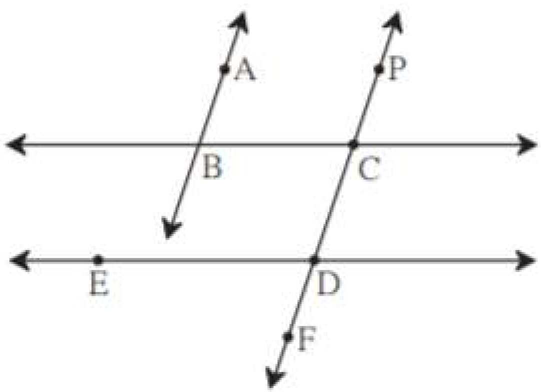


Fig. 2.27

Solution:
Given: Line AB ∥ line CF and BC is their transversal.
To prove : ∠ABC = ∠FDE
Proof:
Given line AB ∥ line CF and BC is their transversal.

$\therefore \angle ABC = \angle DCB$ ….(i) [Alternate interior angles]
Given line BC $\parallel$ line ED and CD is their transversal.
$\therefore \angle DCB = \angle FDE$….(ii) [Corresponding angles]
$\therefore \angle ABC = \angle FDE$ [from (i) and (ii)]
Hence proved.

8. In figure 2.28, line PS is a transversal of parallel line AB and line CD. If Ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that □ QXRY is a rectangle.

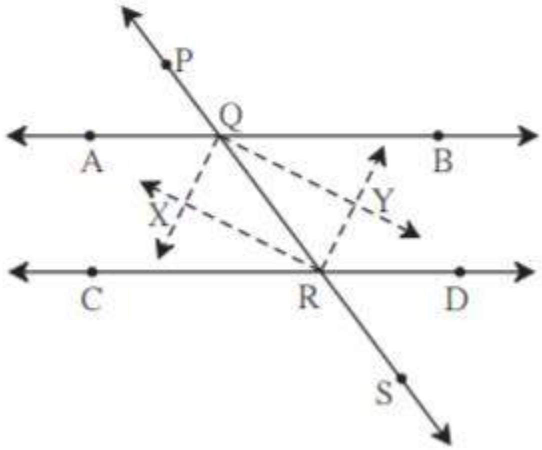
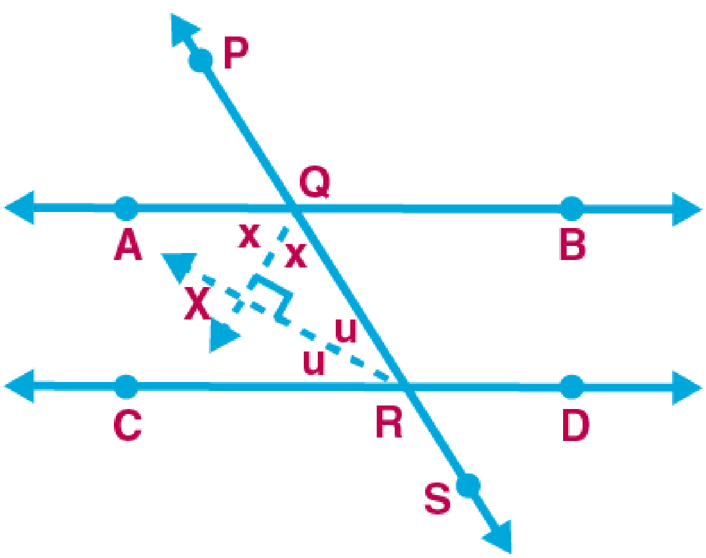
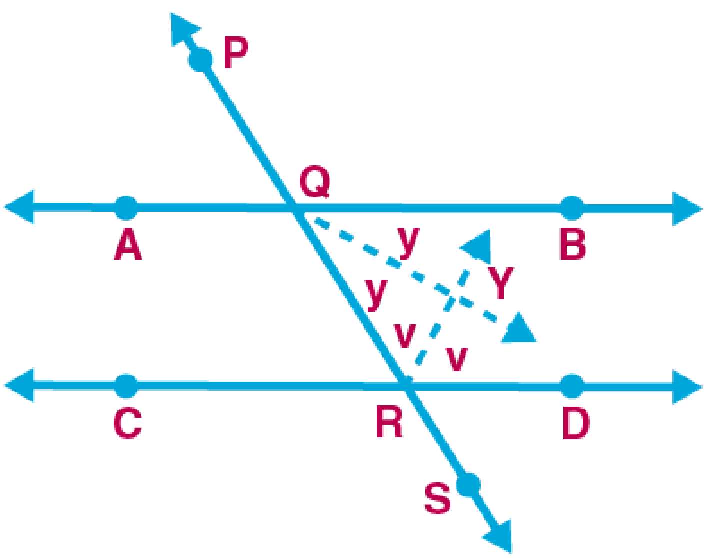


Fig. 2.28

Solution:
Given PS is a transversal of parallel line AB and CD.
Ray QX, ray QY, ray RX, ray RY are angle bisectors.
To Prove: QXRY is a rectangle.
Proof:



$\angle XQA = \angle XQR = x°$…(i) [Ray QX bisects $\angle AQR$]
$\angle YQR = \angle YQB = y°$….(ii) [Ray QY bisects $\angle BQR$]
$\angle XRQ = \angle XRC = u°$ ….. (iii) [Ray RX bisects $\angle CRQ$]
$\angle YRQ = \angle YRD = v°$ …(iv) [Ray RY bisects $\angle DRQ$]
line AB $\parallel$ line CD .
Line PS is their transversal.
$\angle AQR + \angle CRQ = 180°$ [Interior angles on same side of transversal]
$\angle AQR = \angle XQA + \angle XQR$ [Angle addition property]
$\angle CRQ = \angle XRC + \angle XRQ$ [Angle addition property]
$\therefore (\angle XQA + \angle XQR) + (\angle XRC + \angle XRQ) = 180°$
$\therefore (x+x)+(u+u) = 180°$ [From (i) and (ii)]
$\therefore 2x+2u = 180°$
$\therefore 2(x+u) = 180°$
$\Rightarrow x+u = 90°$ ……..(v)
In $\triangle XQR$
$\angle XQR + \angle XRQ + \angle QXR = 180°$ [Angle sum property of triangle]
$\therefore x+u + \angle QXR = 180°$ [From (i) and (iii)]
$\therefore 90 + \angle QXR = 180°$ [From (v)]
$\therefore \angle QXR = 180° - 90°$
$\therefore \angle QXR = 90°$ …..(vi)
Similarily we can prove that,
$\therefore y+v = 90°$



Hence $\angle QYR = 90°$ ……(vii)
Now, $\angle AQR + \angle BQR = 180°$ [Linear pair]
$\angle AQR = \angle XQA + \angle XQR$ [Angle addition property]
$\angle BQR = \angle YQR + \angle YQB$ [Angle addition property]
$\therefore (\angle XQA + \angle XQR) + (\angle YQR + \angle YQB) = 180°$

$\therefore$ (x+x)+(y+y) = 180° [From (i) and (ii)]
$\therefore$ 2x+2y = 180°
$\therefore$ 2(x+y) = 180°
$\Rightarrow$ x+y = 90°
i.e. ∠XQR +∠YQR = 90° [From (i) and (ii)]
$\therefore$ ∠XQY = 90° ….(viii) [Angle addition property]
Similarly we can prove that,
∠XRY = 90° …(ix)
In □QXRY
∠QXR = ∠QYR = ∠XQY = ∠XRY = 90° [From (vi), (vii), (viii) and (ix)]
$\therefore$ □QXRY is a parallelogram with all angles are right angles.
Rectangle is a parallelogram in which all angles are right angles.
□QXRY is a rectangle.
Hence proved.