1. In figure 3.8, \( \angle \) is an exterior angle of \( \triangle ABC \). \( \angle B = 40^\circ, \angle A = 70^\circ \). Find the measure of \( \angle ACD \).

**Solution:**

Given \( \angle B = 40^\circ \)
\( \angle A = 70^\circ \)

\[ \therefore \angle ACD = \angle A + \angle B \]  
[Exterior angle]

\[ \therefore \angle ACD = 70 + 40 = 110^\circ. \]

Hence measure of \( \angle ACD \) is 110\(^\circ\).

2. In \( \triangle PQR \), \( \angle P = 70^\circ, \angle Q = 65^\circ \) then find \( \angle R \).

**Solution:**

Given \( \angle P = 70^\circ \)
\( \angle Q = 65^\circ \)

\[ \angle P + \angle Q + \angle R = 180^\circ \]  
[Angle sum property of triangle]

70 + 65 + \( \angle R \) = 180

\[ \therefore \angle R = 180 - (70 + 65) \]

= 180 - 135 = 45\(^\circ\)

Hence measure of \( \angle R \) is 45\(^\circ\).

3. The measures of angles of a triangle are \( x^\circ, (x-20)^\circ, (x-40)^\circ \). Find the measure of each angle.

**Solution:**

Given measures of angles of a triangle are \( x^\circ, (x-20)^\circ, (x-40)^\circ \).

The sum of angles of a triangle is equal to 180\(^\circ\).

\[ x + (x-20) + (x-40) = 180 \]

\[ \therefore 3x = 180 \]

\[ 3x = 180 + 60 = 240 \]

\[ x = \frac{240}{3} = 80^\circ \]

\[ \therefore x-20 = 80-20 = 60^\circ \]

\[ \therefore x-40 = 80-40 = 40^\circ \]

Hence the measures of angles of a triangle are 80\(^\circ\), 60\(^\circ\) and 40\(^\circ\).

4. The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.

**Solution:**

Let the smallest angle be \( x \).
Then measure of second angle = 2x
Measure of third angle = 3x.
\[ x + 2x + 3x = 180^\circ \]
6x = 180°
x = \frac{180}{6} = 30°
\[ 2x = 2 \times 30 = 60° \]
\[ 3x = 3 \times 30 = 90° \]
Hence the measures of the angles are 30°, 60° and 90°.

5. In figure 3.9, measures of some angles are given. Using the measures find the values of x, y, z.

Solution:
Given \( \angle EMR = 140^\circ \)
\[ \therefore \angle z + \angle EMR = 180^\circ \]
[Linear pair]
\[ \therefore \angle z + 140^\circ = 180^\circ \]
\[ \therefore \angle z = 180^\circ - 140^\circ = 40^\circ \]
Given \( \angle TEN = 100^\circ \)
\[ \angle TEN + \angle y = 180^\circ \]
[Linear pair]
\[ 100^\circ + \angle y = 180^\circ \]
\[ \therefore \angle y = 180^\circ - 100^\circ = 80^\circ \]
\[ \therefore \angle x + \angle y + \angle z = 180^\circ \]
[Angle sum property of triangle]
\[ \therefore \angle x + 80^\circ + 40^\circ = 180^\circ \]
\[ \angle x + 120^\circ = 180^\circ \]
\[ \angle x = 180^\circ - 120^\circ = 60^\circ \]
Hence the measures of the angles x, y and z are 60°, 80° and 40° respectively.

6. In figure 3.10, line AB \parallel line DE. Find the measures of \( \angle DRE \) and \( \angle ARE \) using given measures of some angles.
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Chapter 3 Triangles

Solution:
Given \( AB \parallel DE \).
\( \therefore \) \( AD \) is the transversal.
Given \( \angle BAD = 70^\circ \)
\( \therefore \) \( \angle RDE = 70^\circ \) \hspace{1cm} [Alternate interior angles]
Given \( \angle DER = 40^\circ \)
In \( \triangle RDE \)
\( \angle RDE + \angle DRE + \angle DER = 180^\circ \) \hspace{1cm} [Angle Sum Property of triangle]
\( 70^\circ + \angle DRE + 40^\circ = 180^\circ \)
\( 110^\circ + \angle DRE = 180^\circ \)
\( \therefore \) \( \angle DRE = 180 - 110 = 70^\circ \)
\( \angle ARE + \angle DRE = 180^\circ \) \hspace{1cm} [Linear pair]
\( \angle ARE + 70^\circ = 180^\circ \)
\( \angle ARE = 180^\circ - 70^\circ = 110^\circ \)
Hence measures of \( \angle DRE \) and \( \angle ARE \) are 70° and 110° respectively.

7. In \( \triangle ABC \), bisectors of \( \angle A \) and \( \angle B \) intersect at point \( O \). If \( \angle C = 70^\circ \). Find measure of \( \angle AOB \).

Solution:
Given \( \angle C = 70^\circ \)
\( \angle OAB = \frac{1}{2} \angle CAB \) \hspace{1cm} \text{[AO is the bisector of } \angle CAB]\)
\( \angle OBA = \frac{1}{2} \angle CBA \) \hspace{1cm} \text{[BO is the bisector of } \angle CBA]\)

In \( \triangle ABC \)
\( \angle CAB + \angle CBA + \angle C = 180^\circ \) \hspace{1cm} \text{[Angle sum property of triangle]}
\( \angle CAB + \angle CBA + 70^\circ = 180^\circ \)
\( \angle CAB + \angle CBA = 110^\circ \)

Multiply both sides by \( \frac{1}{2} \)
\( \frac{1}{2} \angle CAB + \frac{1}{2} \angle CBA = 55^\circ \)
\( \therefore \angle OAB + \angle OBA = 55^\circ \) \hspace{1cm} \text{[From (i) and (ii)]}

In \( \triangle AOB \)
\( \angle AOB + \angle OAB + \angle OBA = 180^\circ \) \hspace{1cm} \text{[Angle sum property of triangle]}
\( \angle AOB + 55^\circ = 180^\circ \) \hspace{1cm} \text{[From (iii)]}
\( \therefore \angle AOB = 125^\circ \)

Hence measure of \( \angle AOB \) is 125\(^\circ\).

8. In Figure 3.11, line AB \parallel line CD and line PQ is the transversal. Ray PT and ray QT are bisectors of \( \angle BPQ \) and \( \angle PQD \) respectively. Prove that \( m\angle PTQ = 90^\circ \).

**Solution:**

Given: \( AB \parallel CD \). PQ is the transversal.

To prove: \( m\angle PTQ = 90^\circ \)

Proof:
\( \angle QPT = \frac{1}{2} \angle BPQ \) \hspace{1cm} \text{[PT is the bisector of } \angle BPQ]\)
\( \angle PQT = \frac{1}{2} \angle PQD \) \hspace{1cm} \text{[QT is the bisector of } \angle PQD]\)

Given \( AB \parallel CD \). PQ is the transversal.

\( \therefore \angle BPQ + \angle PQD = 180^\circ \) \hspace{1cm} \text{[Interior angles on same side of transversal are supplementary]}

Multiply both side by \( \frac{1}{2} \).
\( \frac{1}{2} \angle BPQ + \frac{1}{2} \angle PQD = 90^\circ \)
\( \therefore \angle QPT + \angle PQT = 90^\circ \) \hspace{1cm} \text{[From (i) and (ii)]}

In \( \triangle PTQ \)
\[ \angle QPT + \angle PQT + \angle PTQ = 180^\circ \]  
[Angle sum property of triangle]

\[ 90^\circ + \angle PTQ = 180^\circ \]  
[From (iii)]

\[ \angle PTQ = 180^\circ - 90^\circ \]

\[ \angle PTQ = 90^\circ \]

Hence proved.

9. Using the information in figure 3.12, find the measures of \( \angle a \), \( \angle b \) and \( \angle c \).

**Solution:**

\[ \angle b = 70^\circ \]  
[Vertically opposite angles]

\[ \angle c + 100^\circ = 180^\circ \]  
[Linear pair]

\[ \therefore \angle c = 180 - 100 = 80^\circ \]

\[ \angle a + \angle b + \angle c = 180^\circ \]  
[Angle sum property of triangle]

\[ \therefore \angle a + 70^\circ + 80^\circ = 180^\circ \]

\[ \therefore \angle a + 150^\circ = 180^\circ \]

\[ \therefore \angle a = 180 - 150 = 30^\circ \]

Hence \( \angle a = 30^\circ \), \( \angle b = 70^\circ \) and \( \angle c = 80^\circ \).

10. In figure 3.13, line DE \parallel line GF ray EG and ray FG are bisectors of \( \angle DEF \) and \( \angle DFM \) respectively. Prove that,

(i) \( \angle DEG = \frac{1}{2} \angle DEF \)

(ii) \( EF = FG \)

**Solution:**

(i) Given \( DE \parallel GF \)

\[ \angle DEG = \angle GEF = \frac{1}{2} \angle DEF \]  
[Ray EG bisects \( \angle DEF \)]

\[ \angle DFG = \angle GFM = \frac{1}{2} \angle DFM \]  
[Ray FG bisects \( \angle DFM \)]
ED \parallel FG
\therefore \angle DFG = \angle EDF \hspace{1cm} ...(iii) \hspace{1cm} \text{[Alternate interior angles]}

In \triangle DEF
\angle DFM = \angle DEF + \angle EDF
2\angle EDF = \angle DEF + \angle EDF
2\angle EDF - \angle EDF = \angle DEF
\angle EDF = \angle DEF
\therefore \angle EDF = 2\angle DEG
\therefore \angle DEG = \frac{1}{2} \angle EDF
Hence proved.

(ii) Given DE \parallel GF,
EG is the transversal.
\therefore \angle DEG = \angle EGF \hspace{1cm} ...(iv) \hspace{1cm} \text{[Alternate interior angles]}
\therefore \angle EGF = \angle GEF \hspace{1cm} \text{[From (i) and (iv)]}
\therefore EF = FG
[Sides opposite to equal angles of a triangle are equal]
Hence proved.
1. In each of the examples given below, a pair of triangles is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.

(i) From the figure, we know that the three sides of \( \triangle ABC \) are equal to the three sides of \( \triangle PQR \).
\[ \therefore \text{By SSS test, } \triangle ABC \cong \triangle PQR \]

(ii) From the figure, we know that the two sides and the included angle of \( \triangle XYZ \) are equal to the two sides and the included angle of \( \triangle LMN \).
\[ \therefore \text{By SAS test, } \triangle XYZ \cong \triangle LMN \]

(iii) From the figure, we know that the two angles and the included side of \( \triangle PRQ \) are equal to the two angles and the included side of \( \triangle STU \).
\[ \therefore \text{By ASA test, } \triangle PRQ \cong \triangle STU \]

(iv) From the figure, we know that the right angles, hypotenuse and a side of \( \triangle LMN \) are equal to the right angle, hypotenuse and a side of \( \triangle PTR \).
\[ \therefore \text{By \( \angle \angle \) test, } \triangle LMN \cong \triangle PTR \]

Solution:
(i) From the figure, we know that the three sides of \( \triangle ABC \) are equal to the three sides of \( \triangle PQR \).
\[ \therefore \text{By SSS test, } \triangle ABC \cong \triangle PQR \]

(ii) From the figure, we know that the two sides and the included angle of \( \triangle XYZ \) are equal to the two sides and the included angle of \( \triangle LMN \).
\[ \therefore \text{By SAS test, } \triangle XYZ \cong \triangle LMN \]

(iii) From the figure, we know that the two angles and the included side of \( \triangle PRQ \) are equal to the two angles and the included side of \( \triangle STU \).
\[ \therefore \text{By ASA test, } \triangle PRQ \cong \triangle STU \]

(iv) From the figure, we know that the right angles, hypotenuse and a side of \( \triangle LMN \) are equal to the right angle, hypotenuse and a side of \( \triangle PTR \).
\[ \therefore \text{By \( \angle \angle \) test, } \triangle LMN \cong \triangle PTR \]
2. Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.

(i)

From the information shown in the figure, in \( \triangle ABC \) and \( \triangle PQR \)

\[ \angle ABC \cong \angle PQR \]
\[ \text{seg } BC \cong \text{seg } QR \]
\[ \angle ACB \cong \angle PRQ \]
\[ \triangle ABC \cong \triangle PQR \quad \text{ASA test} \]

\[ \therefore \angle BAC \cong \angle QPR \]

\[ \text{seg } AB \cong \text{seg } PQ \]

and \[ \text{seg } AC \cong \text{seg } PR \quad \{ \text{corresponding sides of congruent triangles} \} \]

Solution:

From the information shown in the figure, in \( \triangle ABC \) and \( \triangle PQR \)

\[ \angle ABC \cong \angle PQR \]
\[ \text{seg } BC \cong \text{seg } QR \]
\[ \angle ACB \cong \angle PRQ \]
\[ \triangle ABC \cong \triangle PQR \quad \text{ASA test} \]

\[ \therefore \angle BAC \cong \angle QPR \quad \text{corresponding angles of congruent triangles.} \]

\[ \text{seg } AB \cong \text{seg } PQ \]

and \[ \text{seg } AC \cong \text{seg } PR \quad \{ \text{corresponding sides of congruent triangles} \} \]

(ii)

From the information shown in the figure,

In \( \triangle PTQ \) and \( \triangle STR \)

\[ \text{seg } PT \cong \text{seg } ST \]
\[ \angle PTQ \cong \angle STR \quad \{ \text{vertically opposite angles} \} \]

\[ \text{seg } TQ \cong \text{seg } TR \]
\( \triangle PTQ \cong \triangle STR \ldots \text{SAS test} \)
\( \angle TPQ \cong \angle TRS \)
and \( \angle TQP \cong \angle TRS \) \{corresponding angles of congruent triangles.\}
seg PQ \cong \text{corresponding sides of congruent triangles.}\)

**Solution:**
From the information shown in the figure,
In \( \triangle PTQ \) and \( \triangle STR \)
seg PT \( \cong \) seg ST
\( \angle PTQ \cong \angle STR \ldots \text{vertically opposite angles} \)
seg TQ \( \cong \) seg TR
\( \therefore \triangle PTQ \cong \triangle STR \ldots \text{SAS test} \)
\( \therefore \angle TPQ \cong \angle TSR \)
and \( \angle TQP \cong \angle TRS \) \{corresponding angles of congruent triangles.\}
seg PQ \( \cong \) seg SR \text{corresponding sides of congruent triangles.}\)

3. From the information shown in the figure, state the test assuring the congruence of \( \triangle ABC \) and \( \triangle PQR \). Write the remaining congruent parts of the triangles.

![Fig. 3.22](image)

**Solution:**
From given figure,
seg AB \( \cong \) seg QP
seg BC \( \cong \) seg PR
\( \angle A = \angle Q = 90^\circ \)
\( \therefore \triangle ABC \cong \triangle QPR \)
\( \therefore \) seg AC \( \cong \) seg QR \text{[c.s.c.t]}\)
\( \angle C = \angle R \) \text{[c.a.c.t]}\)
\( \angle B = \angle P \) \text{[c.a.c.t]}\)

4. As shown in the following figure, in \( \triangle LMN \) and \( \triangle PNM \), \( LM = PN \), \( LN = PM \). Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.

![Fig. 3.23](image)

**Solution:**
Given LM = PN
LN = PM  
MN ≅ NM  \[\text{Common side}\]

\[
\therefore \triangle LMN \cong \triangle PNM \quad \text{[SSS test]}
\]

\[
\angle LMN \cong \angle PNM \quad \text{[c.a.c.t]}
\]

\[
\angle MNL \cong \angle NMP \quad \text{[c.a.c.t]}
\]

\[
\angle NLM \cong \angle MPN \quad \text{[c.a.c.t]}
\]

5. In figure 3.24, seg AB \cong seg CB and seg AD \cong seg CD. Prove that \(\triangle ABD \cong \triangle CBD\)

\[\text{Fig. 3.24}\]

**Solution:**

**Proof:**
seg AB \cong seg CB \quad \text{[Given]}
seg AD \cong seg CD \quad \text{[Given]}
seg BD \cong seg BD \quad \text{[Common side]}

\[
\therefore \text{By SSS test, } \triangle ABD \cong \triangle CBD
\]
Hence proved.

6. In figure 3.25, \(\angle P \cong \angle R\), seg PQ \cong seg RQ. Prove that, \(\triangle PQT \cong \triangle RQS\)

\[\text{Fig. 3.25}\]

**Solution:**

\[
\angle P \cong \angle R \quad \text{[Given]}
\]
seg PQ \cong seg RQ \quad \text{[Given]}
\angle Q = \angle Q \quad \text{[common angle]}

\[
\therefore \text{By ASA test, } \triangle PQT \cong \triangle RQS
\]
Hence proved.
1. Find the values of \( x \) and \( y \) using the information shown in figure 3.37. Find the measure of \( \angle ABD \) and \( \angle ACD \).

\[ \text{Solution:} \]

In \( \triangle ABC \)
\[ \text{seg AB} \cong \text{seg AC} \quad \text{[Given]} \]
\[ \angle ACB = 50' \quad \text{[Given]} \]
\[ \therefore \angle x = \angle ACB = 50' \quad \text{[Isosceles angle theorem]} \]

In \( \triangle BCD \)
\[ \text{seg BD} \cong \text{seg CD} \quad \text{[Given]} \]
\[ \angle CBD = 60' \quad \text{[Given]} \]
\[ \therefore \angle y = \angle CBD = 60' \quad \text{[Isosceles angle theorem]} \]

Hence measure of \( x \) and \( y \) are 50' and 60' respectively.

\[ \angle ABD = \angle x + \angle CBD \quad \text{[Angle addition property]} \]
\[ \therefore \angle ABD = 50' + 60' = 110' \]

Hence measure of \( \angle ABD \) is 110'.

\[ \angle ACD = \angle ACB + \angle y \quad \text{[Angle addition property]} \]
\[ \therefore \angle ACD = 50' + 60' = 110' \]

Hence measure of \( \angle ACD \) is 110'.

2. The length of hypotenuse of a right angled triangle is 15. Find the length of median of its hypotenuse.

\[ \text{Solution:} \]

In a right angled triangle, the length of the median on its hypotenuse is half the length of the hypotenuse.

Given length of hypotenuse = 15

\[ \therefore \text{Length of median on its hypotenuse} = \frac{15}{2} = 7.5 \]

Hence the length of median on its hypotenuse is 7.5 units.

3. In \( \triangle PQR \), \( \angle Q = 90° \), \( PQ = 12 \), \( QR = 5 \) and \( QS \) is a median. Find \( l(QS) \).

\[ \text{Solution:} \]
In $\triangle PQR$
Given $PQ = 12$
$QR = 5$
$\angle Q = 90^\circ$

$\therefore PR^2 = PQ^2 + QR^2$ [Pythagoras theorem]

$\therefore PR^2 = 12^2 + 5^2$

$\therefore PR^2 = 144 + 25 = 169$
Taking square root
$PR = \sqrt{169} = 13$

In a right angled triangle, the length of the median on its hypotenuse is half the length of the hypotenuse.
$\therefore QS = \frac{1}{2} PR$
$= \frac{1}{2} \times 13 = 6.5$
$\therefore l(QS)$ is 6.5 units.

4. In figure 3.38, point $G$ is the point of concurrence of the medians of $\triangle PQR$. If $GT = 2.5$, find the lengths of $PG$ and $PT$.

Solution:
Given $G$ is the point of concurrence of the medians of $\triangle PQR$.
The point of concurrence of medians of a triangle divides each median in the ratio $2 : 1$. 

https://byjus.com
\[ \text{PG:GT} = 2:1 \]
\[ \frac{\text{PG}}{\text{GT}} = \frac{2}{1} \]
\[ \frac{\text{PG}}{2.5} = \frac{2}{1} \quad [\text{Given GT} = 2.5] \]
\[ \Rightarrow \text{PG} = 2.5 \times 2 = 5 \]
\[ \text{PT} = \text{PG} + \text{GT} \quad [\text{P-G-T}] \]
\[ \therefore \text{PT} = 5 + 2.5 = 7.5 \]
Hence length of PG and PT are 5 and 7.5 units respectively.
1. In figure 3.48, point A is on the bisector of \( \angle XYZ \). If AX = 2 cm then find AZ.

![Fig. 3.48](image-url)

**Solution:**

Given AX = 2 cm
Point A is on the bisector of \( \angle XYZ \).
Every point on the bisector of an angle is equidistant from the sides of the angle.
\[ \therefore AZ = AX = 2 \text{ cm}. \]
Hence length of AZ is 2cm.

2. In figure 3.49, \( \angle RST = 56^\circ \), seg PT \( \perp \) ray ST, seg PR \( \perp \) ray SR and seg PR \( \cong \) seg PT Find the measure of \( \angle RSP \). State the reason for your answer.

![Fig. 3.49](image-url)

**Solution:**

Given seg PT \( \perp \) ray ST
seg PR \( \perp \) ray SR
seg PR \( \cong \) seg PT
Any point equidistant from sides of an angle is on the bisector of the angle.
\[ \therefore P \text{ is a point on the bisector of } \angle TSR. \]
\[ \angle RST = 56^\circ \quad [\text{Given}] \]
\[ \therefore \angle RSP = \frac{1}{2} \angle RST \quad [\text{Bisector of an angle divides it into two equal angles}] \]
\[ \therefore \angle RSP = \frac{1}{2} \times 56^\circ = 28^\circ \]
Hence measure of \( \angle RSP \) is 28°.

3. In \( \triangle PQR \), PQ = 10 cm, QR = 12 cm, PR = 8 cm. Find out the greatest and the smallest angle of the triangle.

**Solution:**

[Further solution steps here]
Given $PQ = 10 \text{ cm}$, $QR = 12 \text{ cm}$, $PR = 8 \text{ cm}$. Here $QR$ is the longest side and $PR$ is the shortest side.

The angle opposite to the longest side is the largest angle and the angle opposite to the smallest side is the smallest angle.

$\therefore \angle RPQ$ is the largest angle and $\angle PQR$ is the smallest angle.

4. In $\triangle FAN$, $\angle F = 80^\circ$, $\angle A = 40^\circ$. Find out the greatest and the smallest side of the triangle. State the reason.

Solution:
Given \( \angle F = 80^\circ \)
\( \angle A = 40^\circ \)
\( \angle F + \angle A + \angle N = 180^\circ \) \[\text{[Angle sum property of triangle]}\]
80° + 40° + \( \angle N \) = 180°
120° + \( \angle N \) = 180°
\[\therefore \angle N = 180^\circ - 120^\circ = 60^\circ\]
The side opposite to the largest angle is the largest side and the side opposite to the smallest angle is the smallest side.
Since 80° > 60° > 40°
\( \angle F > \angle N > \angle A \)
\[\therefore \text{AN is the largest side and FN is the smallest side.}\]

5. Prove that an equilateral triangle is equiangular.

Solution:

Given: \( \triangle PQR \) is an equilateral triangle.
To prove: \( \triangle PQR \) is equiangular.
Proof:
\[\text{seg PQ = seg PR} \quad \text{...(i)} \quad \text{[Sides of equilateral triangle]}\]
\[\therefore \angle R = \angle Q \quad \text{...(i)} \quad \text{[Isosceles triangle theorem]}\]
\[\text{seg PQ = seg QR} \quad \text{[Sides of equilateral triangle]}\]
\[\therefore \angle R = \angle P \quad \text{...(ii)} \quad \text{[Isosceles triangle theorem]}\]
\[\therefore \angle R = \angle Q = \angle P \quad \text{[From (i) and (ii)]}\]
Hence \( \triangle PQR \) is equiangular.

6. Prove that, if the bisector of \( \angle BAC \) of \( \triangle ABC \) is perpendicular to side \( BC \), then \( \triangle ABC \) is an isosceles triangle.
Solution:
Given AD bisects \( \angle BAC \).

\( AD \perp BC \)

In \( \triangle ADB \) and \( \triangle ADC \),

\( \angle BAD \cong \angle CAD \) \[\text{[AD bisects } \angle BAC\] 

\( AD \cong AD \) \[\text{[Common side]}\]

\( \angle ADB \cong \angle ADC \) \[\because AD \perp BC\]

\( \therefore \) By ASA test, \( \triangle ADB \cong \triangle ADC \).

\( \therefore \) AB \( \cong \) AC \[\text{[c.s.c.t]}\]

\( \therefore \) \( \triangle ABC \) is an isosceles triangle.

Hence proved.

7. In figure 3.50, if \( \text{seg PR} \cong \text{seg PQ} \), show that \( \text{seg PS} > \text{seg PQ} \).

Solution:

Given: \( \text{seg PR} \cong \text{seg PQ} \)

To prove: \( \text{seg PS} > \text{seg PQ} \)

Proof:

\( \text{seg PQ} \cong \text{seg PR} \) \[\text{[Given]}\]

\( \angle PQR \cong \angle PRQ \ldots(i) \) \[\text{[Isosceles triangle theorem]}\]
\( \angle PRQ \) is the exterior angle of \( \trianglePRS \).

\[
\therefore \angle PRQ > \angle PSR \ldots \text{(ii)} \quad \text{[Property of Exterior angle of a triangle]}
\]

\[
\therefore \angle PQR > \angle PSR \quad \text{[From (i) and (ii)]}
\]

\[
\therefore \angle Q > \angle S \quad \text{...(iii)}
\]

In \( \triangle PQS \),

\[
\angle Q > \angle S \quad \text{[The side opposite to the largest angle is the largest side]}
\]

Hence proved.

8. In figure 3.51, in \( \triangle ABC \), seg AD and seg BE are altitudes and AE = BD. Prove that seg AD \( \cong \) seg BE.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.51.png}
\caption{Fig. 3.51}
\end{figure}

\textbf{Solution:}

\textbf{Proof:}

In \( \triangle BAE \) and \( \triangle ABD \),

\[
\text{seg } AE \cong \text{seg BD} \quad \text{[Given]}
\]

\[
\angle BEA \cong \angle ADB \cong 90^\circ \quad \text{[Given]}
\]

\[
\text{seg BA} \cong \text{seg AB} \quad \text{[Common side]}
\]

\[
\therefore \triangle BAE \cong \triangle ABD \quad \text{[By Hypotenuse side test]}
\]

\[
\therefore \text{seg AD} \cong \text{seg BE} \quad \text{[c.s.c.t]}
\]

Hence proved.
1. If \( \triangle XYZ \sim \triangle LMN \), write the corresponding angles of the two triangles and also write the ratios of corresponding sides.

Solution:
Given \( \triangle XYZ \sim \triangle LMN \)

\( \angle X \cong \angle L \)

\( \angle Y \cong \angle M \)

\( \angle Z \cong \angle N \)

Ratio of corresponding sides = \( \frac{XY}{LM} = \frac{YZ}{MN} = \frac{XZ}{LN} \)

2. In \( \triangle XYZ \), \( XY = 4 \text{ cm}, YZ = 6 \text{ cm}, XZ = 5 \text{ cm} \), If \( \triangle XYZ \sim \triangle PQR \) and \( PQ = 8 \text{ cm} \) then find the lengths of remaining sides of \( \triangle PQR \).

Solution:
Given \( \triangle XYZ \sim \triangle PQR \)

If two triangles are similar then their corresponding sides are in proportion and corresponding angles are congruent.

\[ \frac{XY}{PQ} = \frac{YZ}{QR} = \frac{XZ}{PR} \]  
[Corresponding sides of similar triangles]

\[ \frac{4}{8} = \frac{6}{QR} = \frac{5}{PR} \]  
..(i)

\[ \frac{4}{8} = \frac{6}{QR} \]

\[ 4 \times QR = 8 \times 6 \]

\[ QR = \frac{8 \times 6}{4} = 12 \text{ cm} \]

Also, \( \frac{4}{8} = \frac{5}{PR} \)  
[From (i)]

\[ 4 \times PR = 8 \times 5 \]

\[ PR = \frac{8 \times 5}{4} = 10 \text{ cm} \]

Hence measure of QR and PR are 12 cm and 10 cm respectively.

3. Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.

Solution:
Two similar triangles are shown below.

\[ \triangle ABC \sim \triangle DEF \]
1. Choose the correct alternative answer for the following questions.

(i) If two sides of a triangle are 5 cm and 1.5 cm, the length of its third side cannot be . . . . . . .
(A) 3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

Solution:
The sum of any two sides of a triangle is greater than the third side.
Here 1.5 + 3.4 = 4.9 < 5.
∴ 3.4 cm cannot be length of third side.
Hence Option D is the answer.

(ii) In \(\triangle PQR\), if \(\angle R > \angle Q\) then . . . . . . .
(A) QR > PR (B) PQ > PR (C) PQ < PR (D) QR < PR

Solution:
The side opposite to the largest angle is the largest side.
∴ If \(\angle R > \angle Q\), then PQ > PR.
Hence Option B is the answer.

(iii) In \(\triangle TPQ\), \(\angle T = 65^\circ\), \(\angle P = 95^\circ\) which of the following is a true statement?
(A) PQ < TP (B) PQ < TQ (C) TQ < TP < PQ (D) PQ < TP < TQ

Solution:
The angle opposite to the largest side is the largest angle.
∴ PQ < TQ
Hence Option B is the answer.

2. \(\triangle ABC\) is isosceles in which \(AB = AC\). Seg BD and seg CE are medians. Show that BD = CE.

Solution:

[Diagram of \(\triangle ABC\) with medians BD and CE marked]
Given: \( \triangle ABC \) is an isosceles triangle.
AB = AC
BD and CE are the medians.

To Prove: BD = CE

Proof:
AD = \( \frac{1}{2} \) AC .....(i)  
[\( D \) is the midpoint of AC]
AE = \( \frac{1}{2} \) AB .....(ii)  
[\( E \) is the midpoint of AB]

Given \( AB = AC \)....(iii)

\[ \therefore \ AE = AD ... (iv) \]
[From (i), (ii) and (iii)]

In \( \triangle ABD \) and \( \triangle ACE \)
seg AB = seg AC [Given]
\( \angle BAD = \angle CAE \) [Common angle]
seg AE = seg AD [From (iv)]

\[ \therefore \text{By SAS test } \triangle ABD \cong \triangle ACE. \]

\[ \therefore \text{BD = CE } \text{ [c.s.c.t]} \]

Hence proved.

3. In \( \triangle PQR \), If \( PQ > PR \) and bisectors of \( \angle Q \) and \( \angle R \) intersect at \( S \). Show that \( SQ > SR \).

Solution:
Given: In \( \triangle PQR \), \( PQ > PR \) and bisectors of \( \angle Q \) and \( \angle R \) intersect at \( S \).
To prove: \( SQ > SR \)

Proof:
\( \angle SQR = \frac{1}{2} \angle PQR \) ....(i)  
[\( QS \) is the bisector of \( \angle Q \)]
\( \angle SRQ = \frac{1}{2} \angle PRQ \) ....(ii)  
[\( RS \) is the bisector of \( \angle R \)]

In \( \triangle PQR \),
PQ > PR [Given]

\[ \therefore \angle R > \angle Q \]
[Angle opposite to larger side is larger.]

Multiply both sides by \( \frac{1}{2} \)

\[ \therefore \frac{1}{2} (\angle R) > \frac{1}{2} (\angle Q) \]

\[ \therefore \angle SRQ > \angle SQR \) ....(iii)  
[From (i) and (ii)]

In \( \triangle SQR \), from (iii)

\[ \therefore \text{SQ > SR } \text{[Side opposite to larger angle is larger]} \]

Hence proved.

4. In figure 3.59, point D and E are on side BC of \( \triangle ABC \), such that \( BD = CE \) and \( AD = AE \). Show that \( \triangle ABD \cong \triangle ACE \).
Solution:
Given: Point D and E are on side BC of \( \triangle ABC \)
BD = CE
AD = AE

To Prove: \( \triangle ABD \cong \triangle ACE \)

Proof:
In \( \triangle ADE \)

\[ \text{Given seg } AD = \text{seg } AE \]

\[ \therefore \angle ADE = \angle AED \quad \text{(i)} \]  
[Isosceles triangle theorem]

\[ \angle ADB + \angle ADE = 180^\circ \quad \text{(ii)} \]  
[Linear Pair]

\[ \angle AED + \angle AEC = 180^\circ \quad \text{(iii)} \]  
[Linear Pair]

Equate (ii) and (iii)

\[ \angle ADB + \angle ADE = \angle AED + \angle AEC \quad \text{(iv)} \]

Substitute (i) in (iv)

\[ \angle ADB + \angle AED = \angle AED + \angle AEC \quad \text{(v)} \]

In \( \triangle ABD \) and \( \triangle ACE \),

\[ \text{seg } BD \cong \text{seg } CE \quad \text{(Given)} \]

\[ \angle ADB = \angle AEC \quad \text{(from(v))} \]

\[ \text{seg } AD \cong \text{seg } AE \quad \text{(Given)} \]

\[ \therefore \text{By SAS test, } \triangle ABD \cong \triangle ACE. \]

Hence proved.

5. In figure 3.60, point S is any point on side QR of \( \triangle PQR \) Prove that: \( PQ + QR + RP > 2PS \).

Solution:
In $\triangle PSR$, \\
RP+SR > PS \ldots (i) \quad [\text{Sum of any two sides of a triangle is greater than the third side}]

In $\triangle PQS$, \\
PQ+QS > PS \ldots (ii) \quad [\text{Sum of any two sides of a triangle is greater than the third side}]

Adding (i) and (ii) \\
\therefore RP+SR+PQ+QS > PS+PS \\
\therefore RP+PQ+QR > 2PS \quad [QS+SR = QR \quad \text{Q-S-R}]

Re-arranging the terms \\
PQ+QR+RP > 2PS \\
Hence proved.

6. In figure 3.61, bisector of $\angle BAC$ intersects side BC at point D. Prove that $AB > BD$

**Solution:**

Given: Bisector of $\angle BAC$ intersects side BC at point D.

To prove: $AB > BD$

Proof:

$\angle DAB = \angle CAD$ \ldots (i) \quad [\text{AD bisects } \angle BAC] \\
$\angle ADB$ is the exterior angle of $\triangle ADC$. \\

$\therefore \angle ADB > \angle CAD$ \ldots (ii) \quad [\text{Property of exterior angle of triangle}] \\
Also $\angle ADB > \angle DAB$ \ldots (iii) \quad [\text{From (i) and (ii)}] \\
In $\triangle ABD$, \\
$\angle ADB > \angle DAB$ \quad [\text{From (iii)}] \\
\therefore AB > BD \quad [\text{Side opposite to larger angle is larger}] \\
Hence proved.

7. In figure 3.62, seg PT is the bisector of $\angle QPR$. A line through R intersects ray QP at point S. Prove that $PS = PR$
Solution:
Given: seg PT is the bisector of \( \angle QPR \)
To Prove: PS = PR

Proof:

1. PT \parallel SR, PR is the transversal. [From figure]
2. \( \angle RPT = \angle PRS \) ……(i) [Alternate interior angles]
3. PT \parallel SR, PS is the transversal. [From figure]
4. \( \angle TPQ = \angle PSR \) …..(ii) [Corresponding angles]

Given seg PT is the bisector of \( \angle QPR \).
5. \( \angle TPQ = \angle RPT \) …..(iii)

From (i) and (ii)
6. \( \angle PSR = \angle PRS \)
7. \( \angle PR = PS \)
8. PS = PR

Hence proved.

8. In figure 3.63, seg AD \perp seg BC, seg AE is the bisector of \( \angle CAB \) and C - E - D. Prove that \( \angle DAE = \frac{1}{2} (\angle C - \angle B) \).
Solution:
\[ \angle CAE = \frac{1}{2} \angle A \]  
……(i)

In \( \triangle DAE \),
\[ \angle DAE + \angle AED + 90^\circ = 180^\circ \]
\[ \angle DAE = 180^\circ - 90^\circ - \angle AED \]  
= 90^\circ - \angle AED  
……(ii)

In \( \triangle ACE \),
\[ \angle ACE + \angle CEA + \angle CAE = 180^\circ \]
\[ \angle C + \frac{1}{2} \angle A + \angle AED = 180^\circ \]  
[AED = AEC  Since C-D-E]
\[ \angle AED = 180^\circ - \angle C - \frac{1}{2} \angle A \]  
…..(iii)

Substitute \( \angle AED \) in (ii)
\[ \angle DAE = 90^\circ - (180^\circ - \angle C - \frac{1}{2} \angle A) \]
= -90^\circ + \angle C + \frac{1}{2} \angle A  
…..(iv)

In \( \triangle ABC \),
\[ \angle A + \angle B + \angle C = 180^\circ \]
Divide both sides by 2.
\[ \frac{1}{2} (\angle A + \angle B + \angle C) = 90^\circ \]
\[ \frac{1}{2} \angle A = 90^\circ - \frac{1}{2} \angle B - \frac{1}{2} \angle C \]  
……(v)

Substitute (v) in (iv)
\[ \angle DAE = -90^\circ + \angle C + 90^\circ - \frac{1}{2} \angle B - \frac{1}{2} \angle C \]
= \frac{1}{2} \angle C - \frac{1}{2} \angle B
= \frac{1}{2} (\angle C - \angle B)
Hence proved.